

A top-down view of a wooden desk with a light brown grain. On the left side, a black laptop is partially visible, showing its keyboard and trackpad. Below the laptop, a black smartphone lies horizontally. To the right of the phone, a pair of blue-rimmed glasses is placed. Further up and to the right, a black smartwatch with a black strap is visible. The right half of the image is mostly empty, showing the texture of the wooden desk.

Lógica Computacional

by Professor Pantoja

A top-down view of a wooden desk. On the right side, there is a black laptop with its keyboard visible. Below the laptop, a pair of blue-rimmed glasses lies on the desk. To the left of the glasses, a black pen is positioned diagonally. Further down and to the right, a black smartphone is lying flat. The background is the natural wood grain of the desk.

Lógica Computacional
by Professor Pantoja

Introdução a Lógica

Aula 1

ARGUMENTO

ARGUMENTO

- conjunto de fatos (*premissas*)
- uma conclusão (*V ou F*)

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- conjunto de fatos (*premissas*)
- uma conclusão (*V ou F*)



INDUTIVO

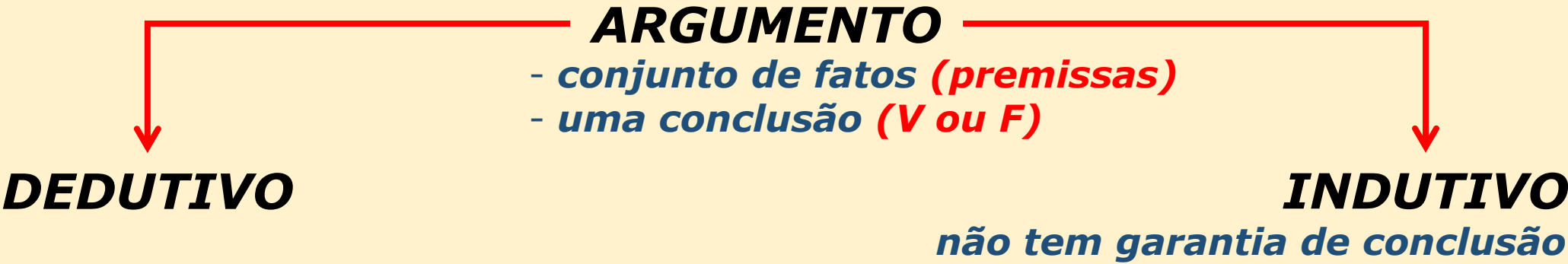
ARGUMENTO

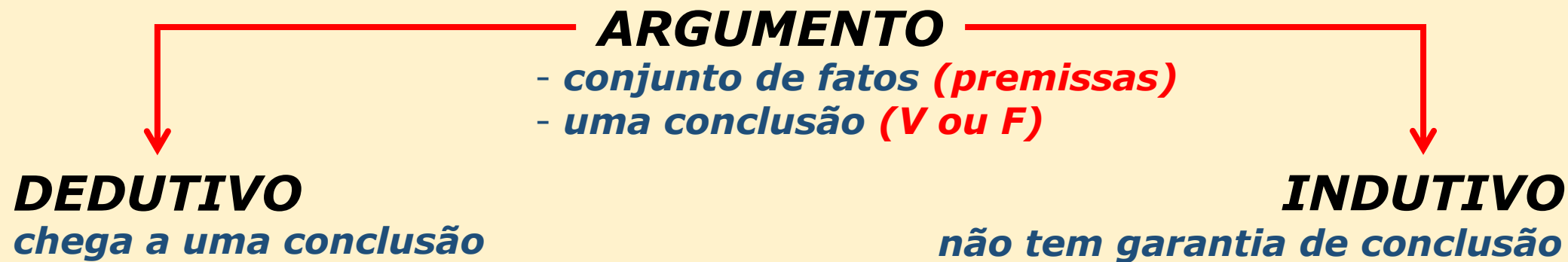
- conjunto de fatos (*premissas*)
- uma conclusão (*V ou F*)

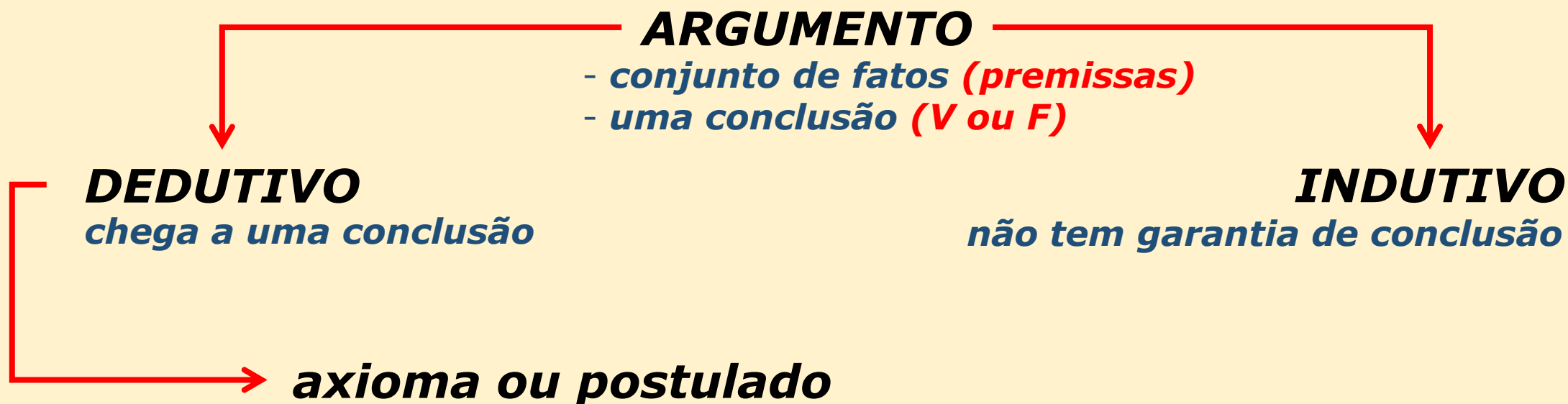


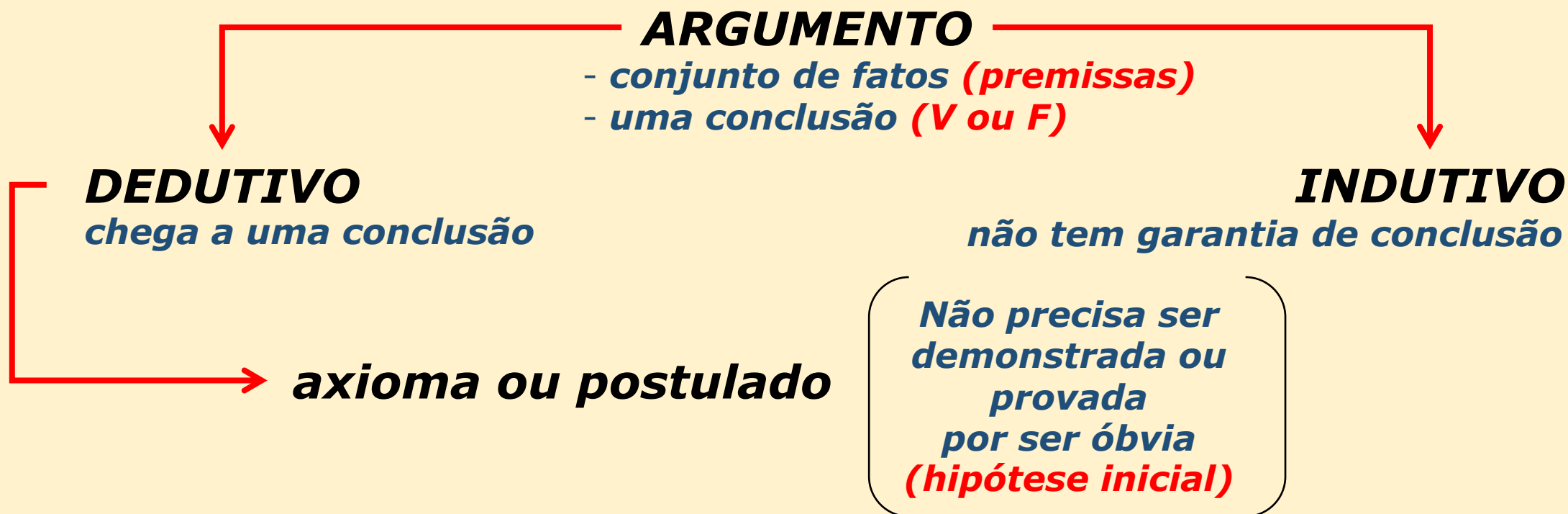
INDUTIVO

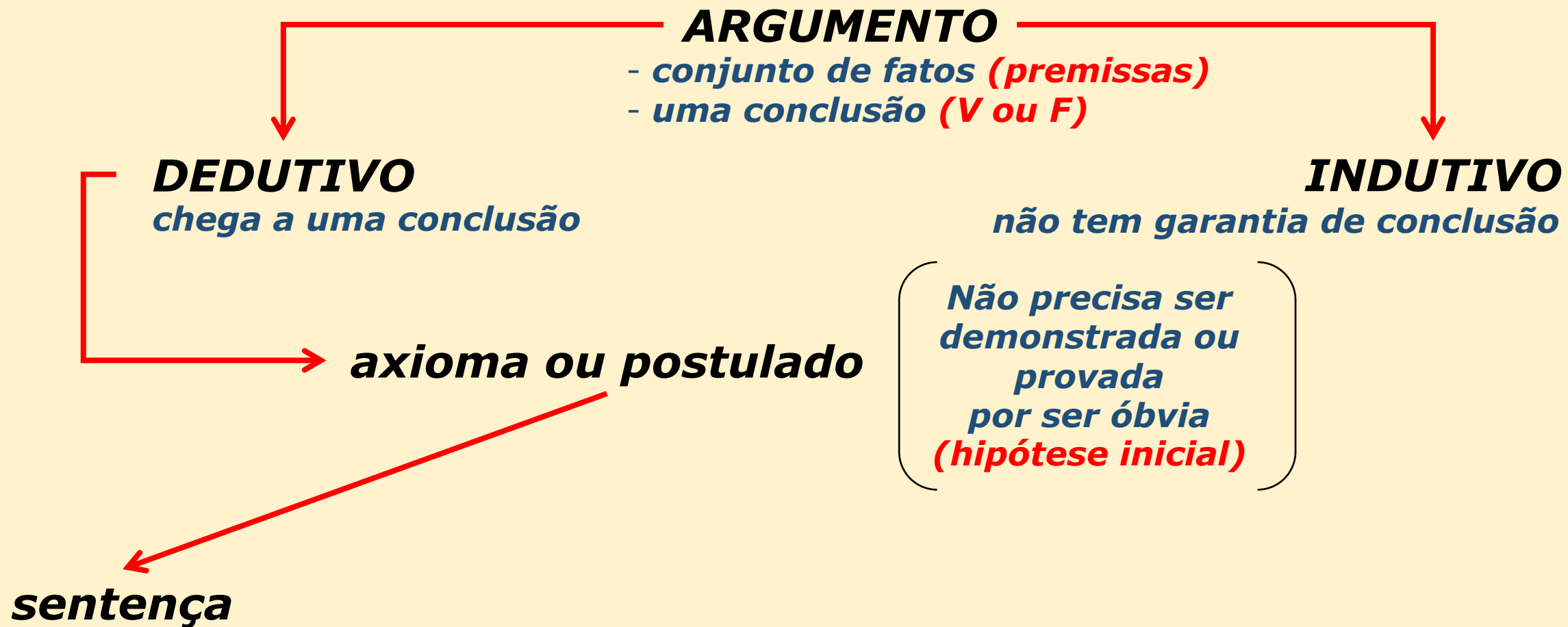
não tem garantia de conclusão

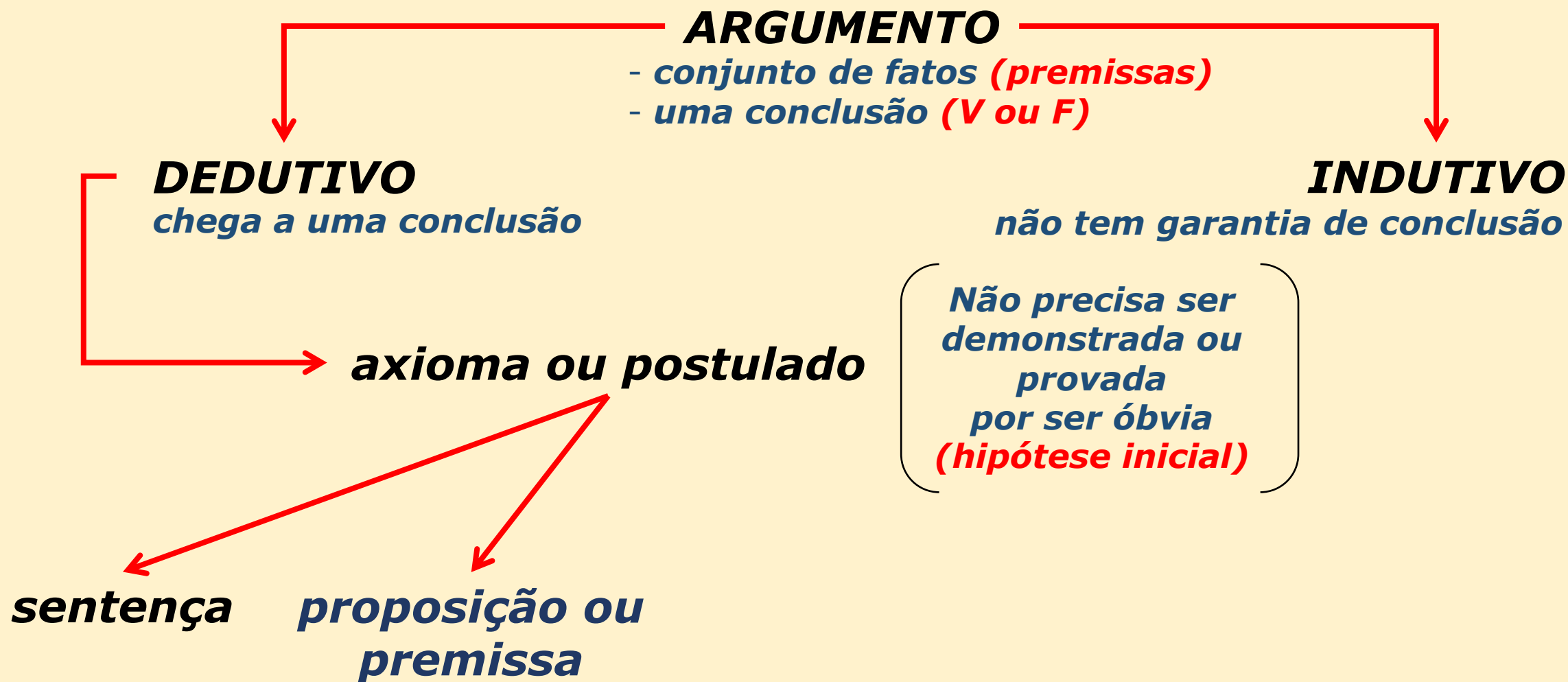


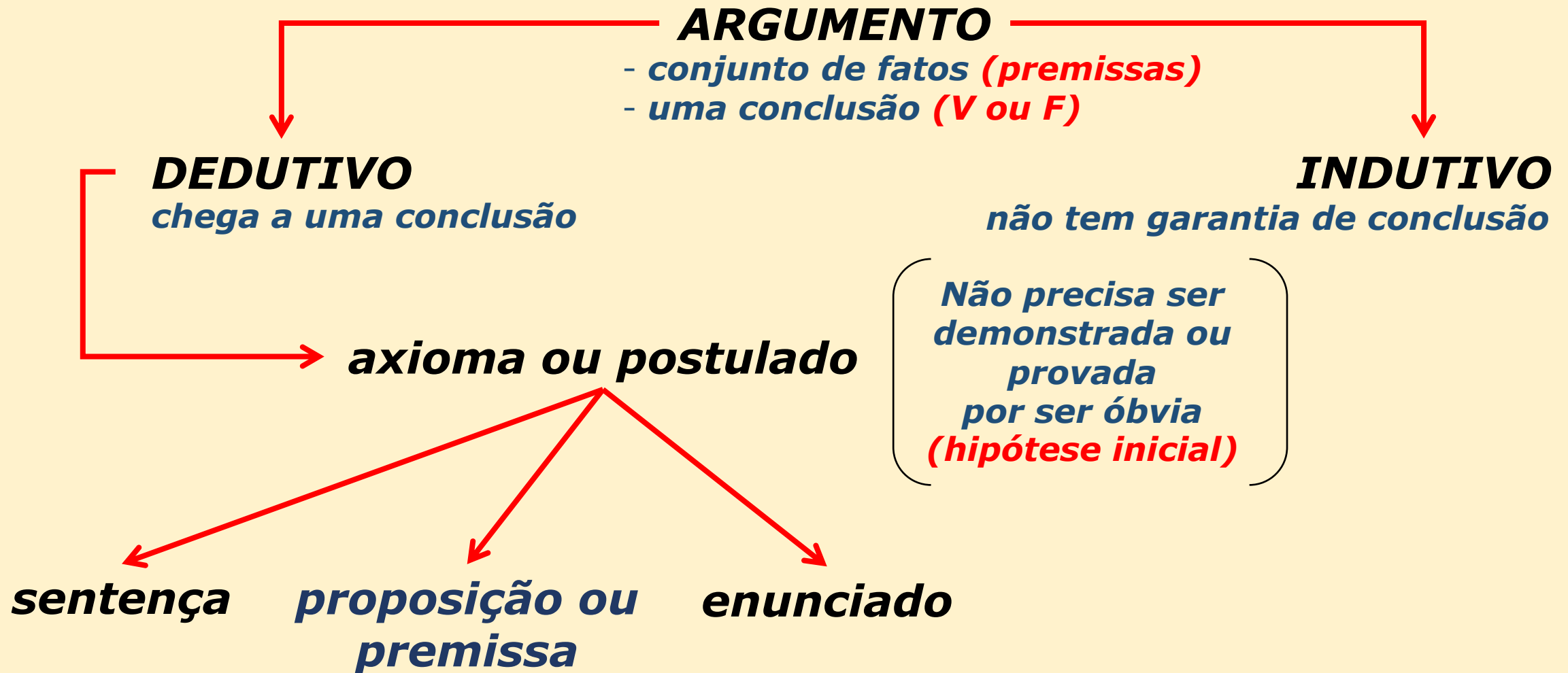


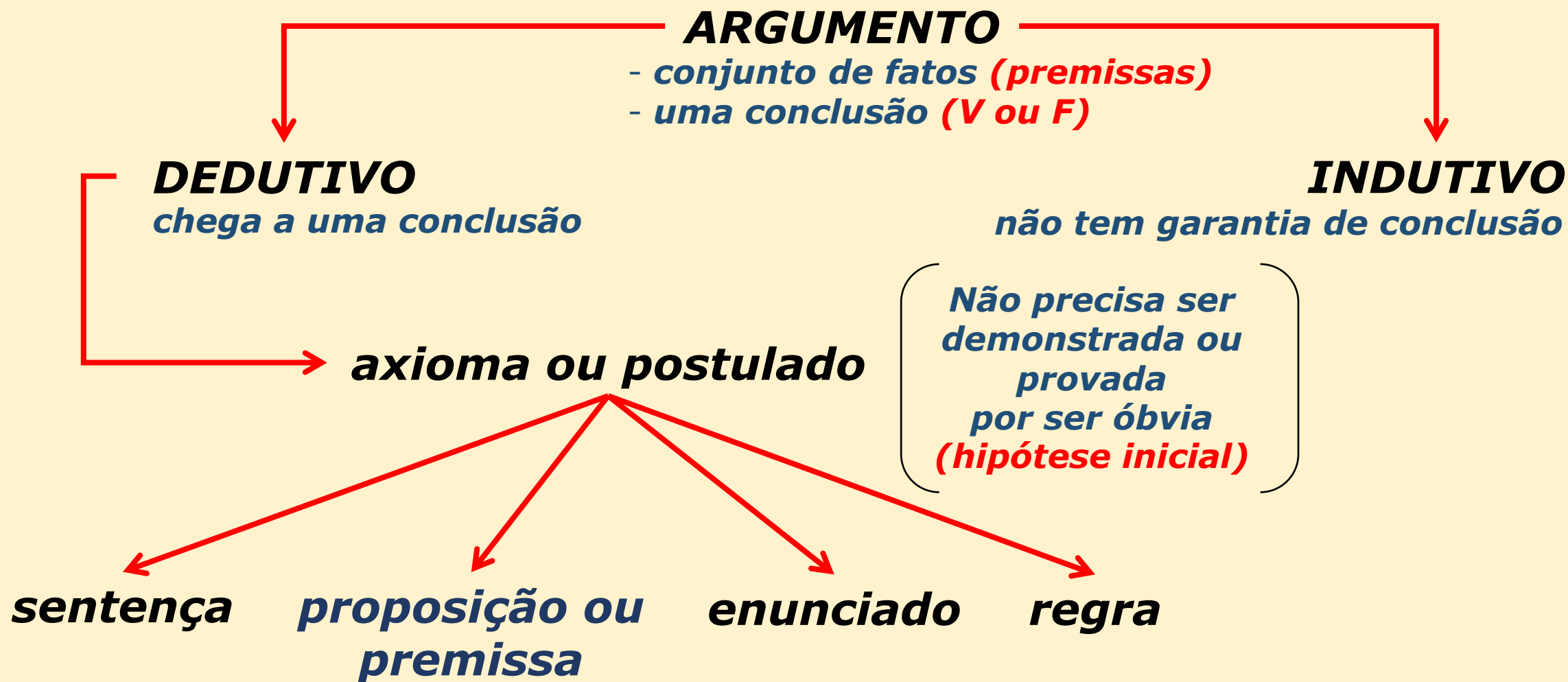












Os postulados clássicos são:

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- ***Princípio da Identidade***

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$$A = A$$

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- ***Princípio da Identidade***
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$$A = A$$

$$p = p$$

Os postulados clássicos são:

- ***Princípio da Identidade***
 - ***Tudo é igual a si mesmo.***

$$A = A$$

$$p = p$$

$$p = q$$

Os postulados clássicos são:

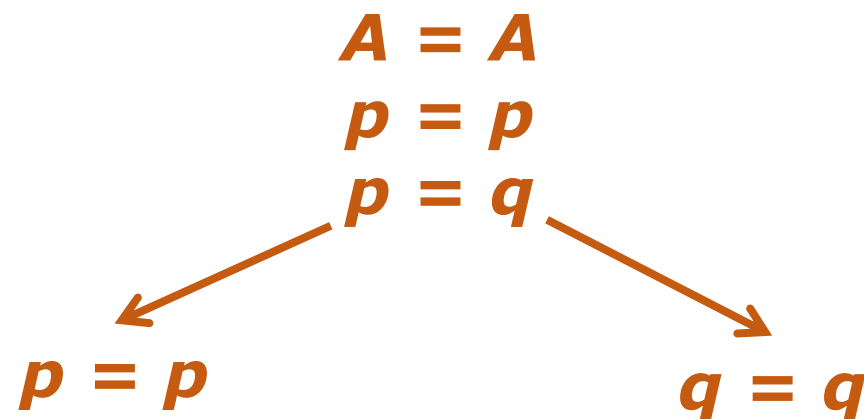
- ***Princípio da Identidade***
 - ***Tudo é igual a si mesmo.***

$$\begin{array}{l} A = A \\ p = p \\ p = q \end{array}$$

$p = p$

Os postulados clássicos são:

- ***Princípio da Identidade***
 - *Tudo é igual a si mesmo.*



Os postulados clássicos são:

- ***Princípio da Identidade***
- ***Princípio da Não-Contradição***

Os postulados clássicos são:

- ***Princípio da Identidade***
- ***Princípio da Não-Contradição***
 - ***duas afirmações contraditórias não podem ser verdadeiras ao mesmo tempo.***

Os postulados clássicos são:

- ***Princípio da Identidade***
- ***Princípio da Não-Contradição***
 - ***duas afirmações contraditórias não podem ser verdadeiras ao mesmo tempo.***

$$A \neq \neg A$$

Os postulados clássicos são:

- ***Princípio da Identidade***
- ***Princípio da Não-Contradição***
 - ***duas afirmações contraditórias não podem ser verdadeiras ao mesmo tempo.***

$$\begin{aligned} A &\neq \neg A \\ \neg B &\neq B \end{aligned}$$

Os postulados clássicos são:

- ***Princípio da Identidade***
- ***Princípio da Não-Contradição***
 - ***duas afirmações contraditórias não podem ser verdadeiras ao mesmo tempo.***

$$\begin{aligned} A &\neq \neg A \\ \neg B &\neq B \end{aligned}$$

$$\neg(p \wedge \neg p)$$

Os postulados clássicos são:

- ***Princípio da Identidade***
- ***Princípio da Não-Contradição***
- ***Princípio do Terceiro Excluído***

Os postulados clássicos são:

- ***Princípio da Identidade***
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 - ***Ou a afirmação é verdadeira ou sua negação é verdadeira.***

Os postulados clássicos são:

- ***Princípio da Identidade***
- ***Princípio da Não-Contradição***
- ***Princípio do Terceiro Excluído***
 - ***Ou a afirmação é verdadeira ou sua negação é verdadeira.***

$$A = V \text{ ou } \neg A = V$$

A top-down view of a wooden desk. On the right side, there is a black laptop with its keyboard visible. Below the laptop, a pair of blue-rimmed glasses lies on the desk. To the left of the glasses, a black pen is positioned diagonally. Further down and to the right, a black smartphone is lying flat. The text is overlaid on the left side of the image.

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Conectores Lógicos

Aula 2

Negação

Negação

Conjunção

Negação

Conjunção

Disjunção

Negação

Conjunção

Disjunção

Ou Exclusivo

Negação

Conjunção

Disjunção

Ou Exclusivo

***Condicional ou
Implicação***

Negação

Conjunção  **Não E**

Disjunção

Ou Exclusivo

**Condicional ou
Implicação**

Negação

Conjunção $\longrightarrow$ **Não E**

Disjunção $\longrightarrow$ **Não Ou**

Ou Exclusivo

**Condicional ou
Implicação**

Negação

Conjunção $\longrightarrow$ **Não E**

Disjunção $\longrightarrow$ **Não Ou**

Ou Exclusivo $\longrightarrow$ **Bi-implicação**

**Condicional ou
Implicação**

$\neg$ ou **$\sim$**

$\neg$ ou **$\sim$**

p	$\neg p$
F	
V	

$\neg$ ou **$\sim$**

p	$\neg p$
F	V
V	

$\neg$ ou **$\sim$**

p	$\neg p$
F	V
V	F

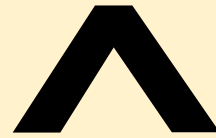


$\wedge$

p	q	$p \wedge q$
F	F	
F	V	
V	F	
V	V	

$\wedge$

p	q	$p \wedge q$
F	F	F
F	V	
V	F	
V	V	



p	q	$p \wedge q$
F	F	F
F	V	F
V	F	
V	V	

$\wedge$

p	q	$p \wedge q$
F	F	F
F	V	F
V	F	F
V	V	V

$\wedge$

p	q	$p \wedge q$
F	F	F
F	V	F
V	F	F
V	V	V



$\vee$

p	q	$p \vee q$
F	F	
F	V	
V	F	
V	V	

V

p	q	$p \vee q$
F	F	F
F	V	
V	F	
V	V	

V

p	q	$p \vee q$
F	F	F
F	V	V
V	F	
V	V	

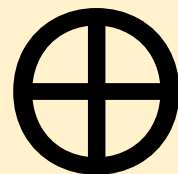
V

p	q	$p \vee q$
F	F	F
F	V	V
V	F	V
V	V	

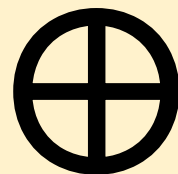
$\vee$

p	q	$p \vee q$
F	F	F
F	V	V
V	F	V
V	V	V

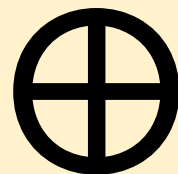




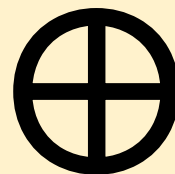
p	q	$p \oplus q$
F	F	
F	V	
V	F	
V	V	



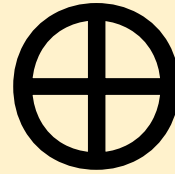
p	q	$p \oplus q$
F	F	F
F	V	
V	F	
V	V	



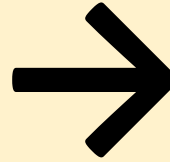
p	q	$p \oplus q$
F	F	F
F	V	V
V	F	
V	V	

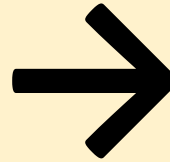


p	q	$p \oplus q$
F	F	F
F	V	V
V	F	V
V	V	

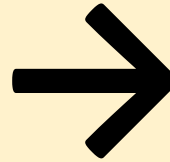


p	q	$p \oplus q$
F	F	F
F	V	V
V	F	V
V	V	F

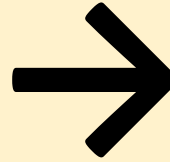




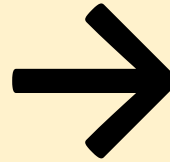
p	q	$p \rightarrow q$
F	F	
F	V	
V	F	
V	V	



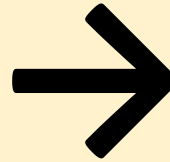
p	q	$p \rightarrow q$
F	F	V
F	V	
V	F	
V	V	



p	q	$p \rightarrow q$
F	F	V
F	V	V
V	F	
V	V	



p	q	$p \rightarrow q$
F	F	V
F	V	V
V	F	F
V	V	



p	q	$p \rightarrow q$
F	F	V
F	V	V
V	F	F
V	V	V

$$\neg (p \wedge q)$$

$$\neg (p \wedge q)$$

p	q	$p \wedge q$
F	F	
F	V	
V	F	
V	V	

$$\neg (p \wedge q)$$

p	q	$p \wedge q$
F	F	F
F	V	F
V	F	F
V	V	V

$$\neg(p \wedge q)$$

p	q	$p \wedge q$	$\neg(p \wedge q)$
F	F	F	
F	V	F	
V	F	F	
V	V	V	

$$\neg(p \wedge q)$$

p	q	$p \wedge q$	$\neg(p \wedge q)$
F	F	F	V
F	V	F	
V	F	F	
V	V	V	

$$\neg(p \wedge q)$$

p	q	$p \wedge q$	$\neg(p \wedge q)$
F	F	F	V
F	V	F	V
V	F	F	
V	V	V	

$$\neg(p \wedge q)$$

p	q	$p \wedge q$	$\neg(p \wedge q)$
F	F	F	V
F	V	F	V
V	F	F	V
V	V	V	

$$\neg(p \wedge q)$$

p	q	$p \wedge q$	$\neg(p \wedge q)$
F	F	F	V
F	V	F	V
V	F	F	V
V	V	V	F

$$\neg (p \vee q)$$

$$\neg (p \vee q)$$

p	q	$p \vee q$
F	F	
F	V	
V	F	
V	V	

$$\neg (p \vee q)$$

p	q	$p \vee q$
F	F	F
F	V	V
V	F	V
V	V	V

$$\neg (p \vee q)$$

p	q	$p \vee q$
F	F	F
F	V	V
V	F	V
V	V	V

$$\neg (p \vee q)$$

p	q	$p \vee q$	$\neg (p \vee q)$
F	F	F	
F	V	V	
V	F	V	
V	V	V	

$$\neg (p \vee q)$$

p	q	$p \vee q$	$\neg (p \vee q)$
F	F	F	V
F	V	V	
V	F	V	
V	V	V	

$$\neg (p \vee q)$$

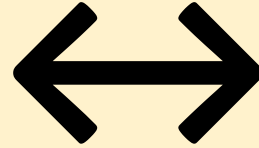
p	q	$p \vee q$	$\neg (p \vee q)$
F	F	F	V
F	V	V	F
V	F	V	
V	V	V	

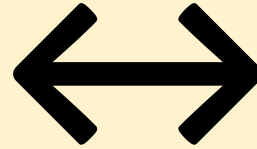
$$\neg (p \vee q)$$

p	q	$p \vee q$	$\neg (p \vee q)$
F	F	F	V
F	V	V	F
V	F	V	F
V	V	V	

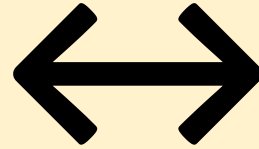
$$\neg (p \vee q)$$

p	q	$p \vee q$	$\neg (p \vee q)$
F	F	F	V
F	V	V	F
V	F	V	F
V	V	V	F

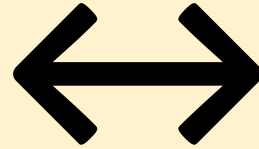




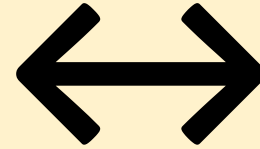
p	q	$p \leftrightarrow q$
F	F	
F	V	
V	F	
V	V	



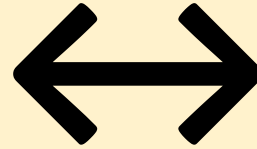
p	q	$p \leftrightarrow q$
F	F	V
F	V	
V	F	
V	V	



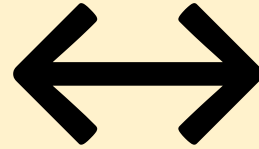
p	q	$p \leftrightarrow q$
F	F	V
F	V	F
V	F	
V	V	



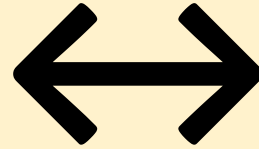
p	q	$p \leftrightarrow q$
F	F	V
F	V	F
V	F	F
V	V	



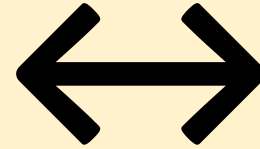
p	q	$p \leftrightarrow q$
F	F	V
F	V	F
V	F	F
V	V	V



p	q	$p \oplus q$	$\neg(p \oplus q)$
F	F	F	
F	V	V	
V	F	V	
V	V	F	

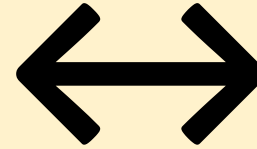


p	q	$p \oplus q$	$\neg(p \oplus q)$
F	F	F	V
F	V	V	F
V	F	V	F
V	V	F	V



p	q	$p \leftrightarrow q$
F	F	V
F	V	F
V	F	F
V	V	V

p	q	$p \oplus q$	$\neg(p \oplus q)$
F	F	F	V
F	V	V	F
V	F	V	F
V	V	F	V



p	q	$p \leftrightarrow q$
F	F	V
F	V	F
V	F	F
V	V	V

p	q	$p \oplus q$	$\neg(p \oplus q)$
F	F	F	V
F	V	V	F
V	F	V	F
V	V	F	V

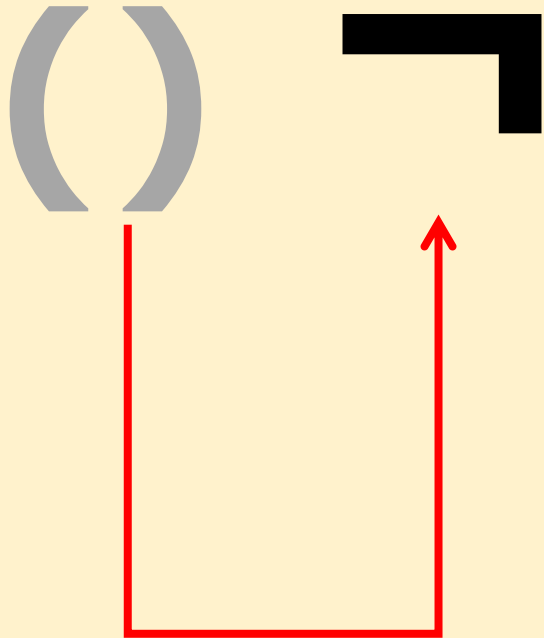
A top-down view of a wooden desk. On the right side, there is a black laptop with its keyboard visible. Below the laptop, a pair of blue-rimmed glasses lies on the desk. To the left of the glasses, a black pen is positioned diagonally. Further down and to the right, a black smartphone is lying flat. The text is overlaid on the left side of the image.

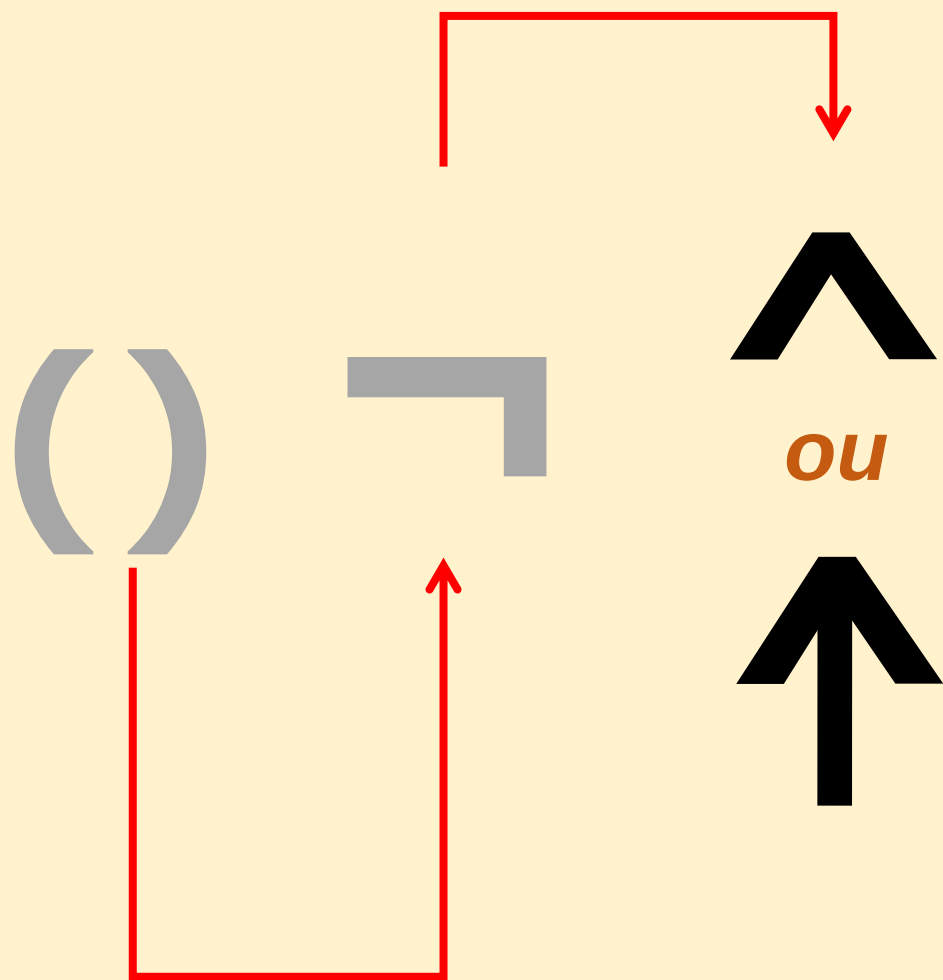
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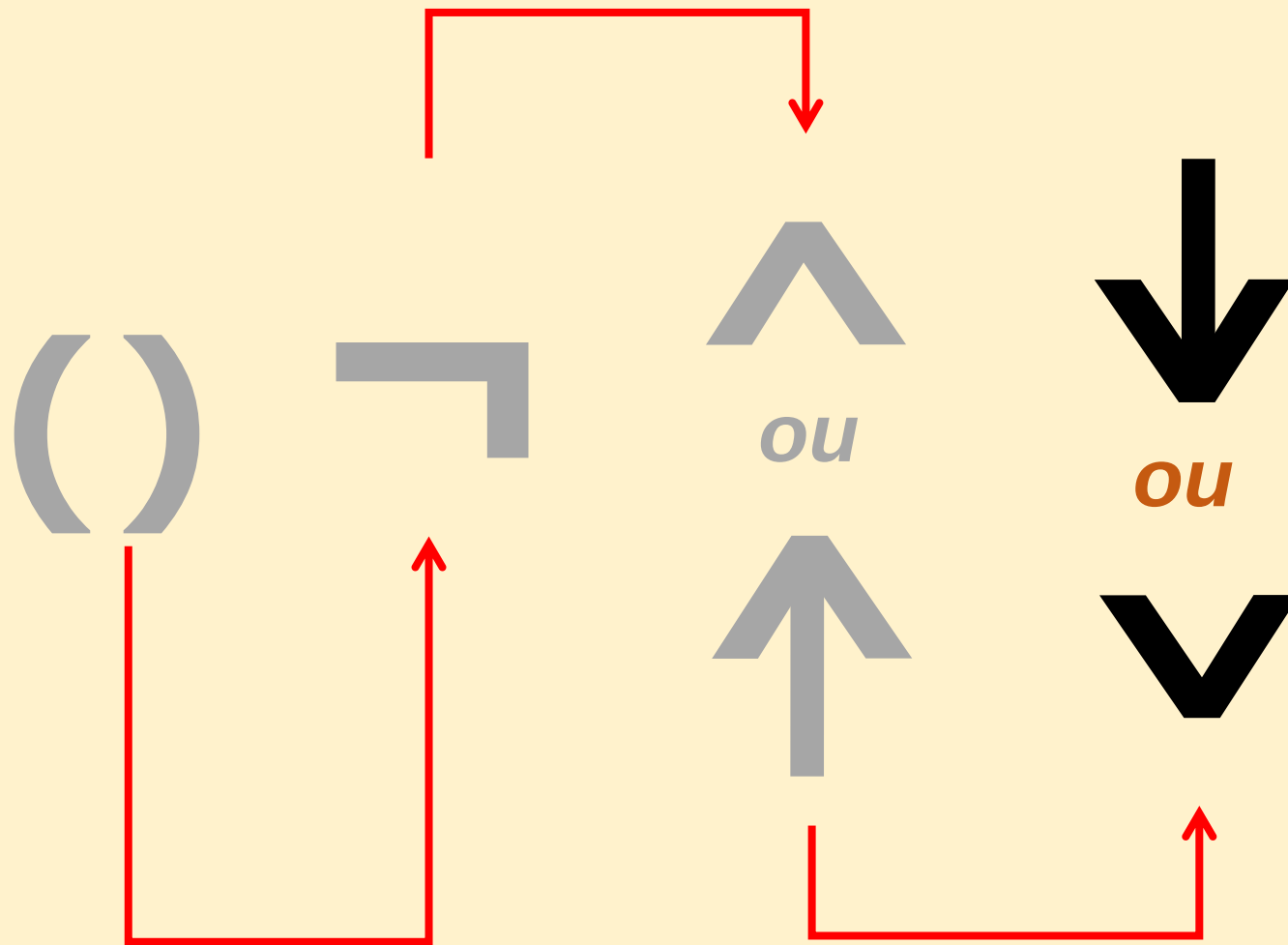
Fórmulas Proposicionais

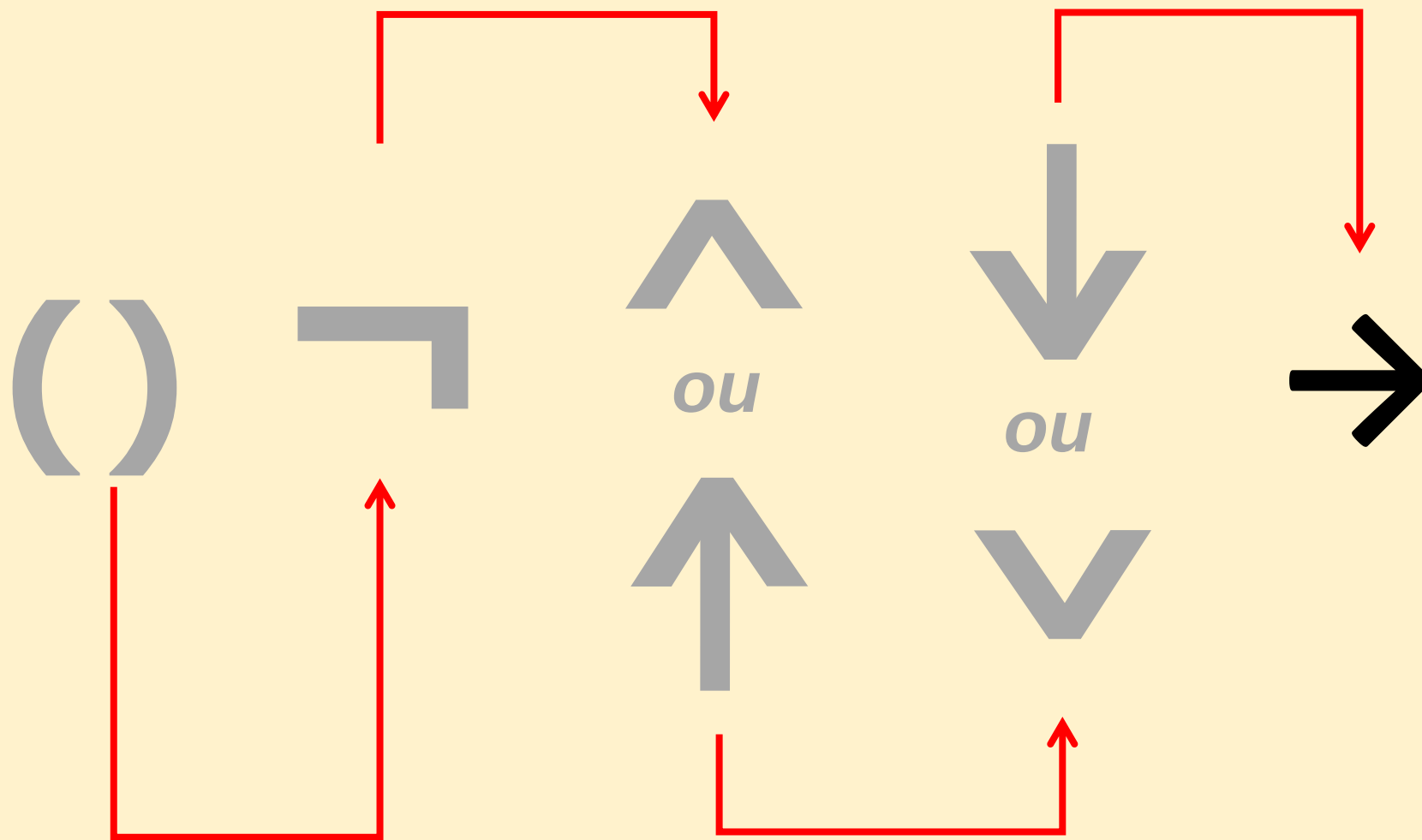
Aula 3

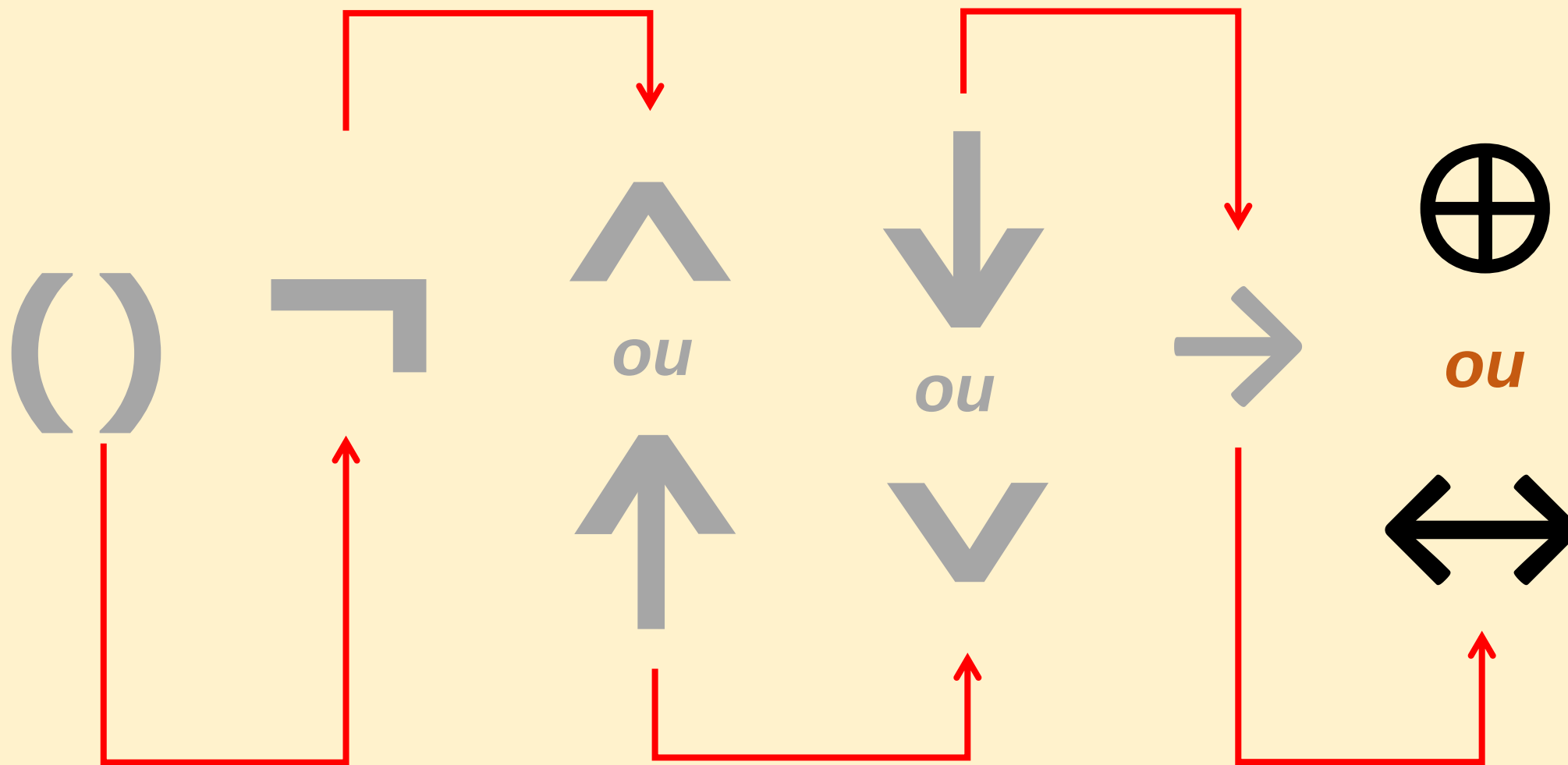
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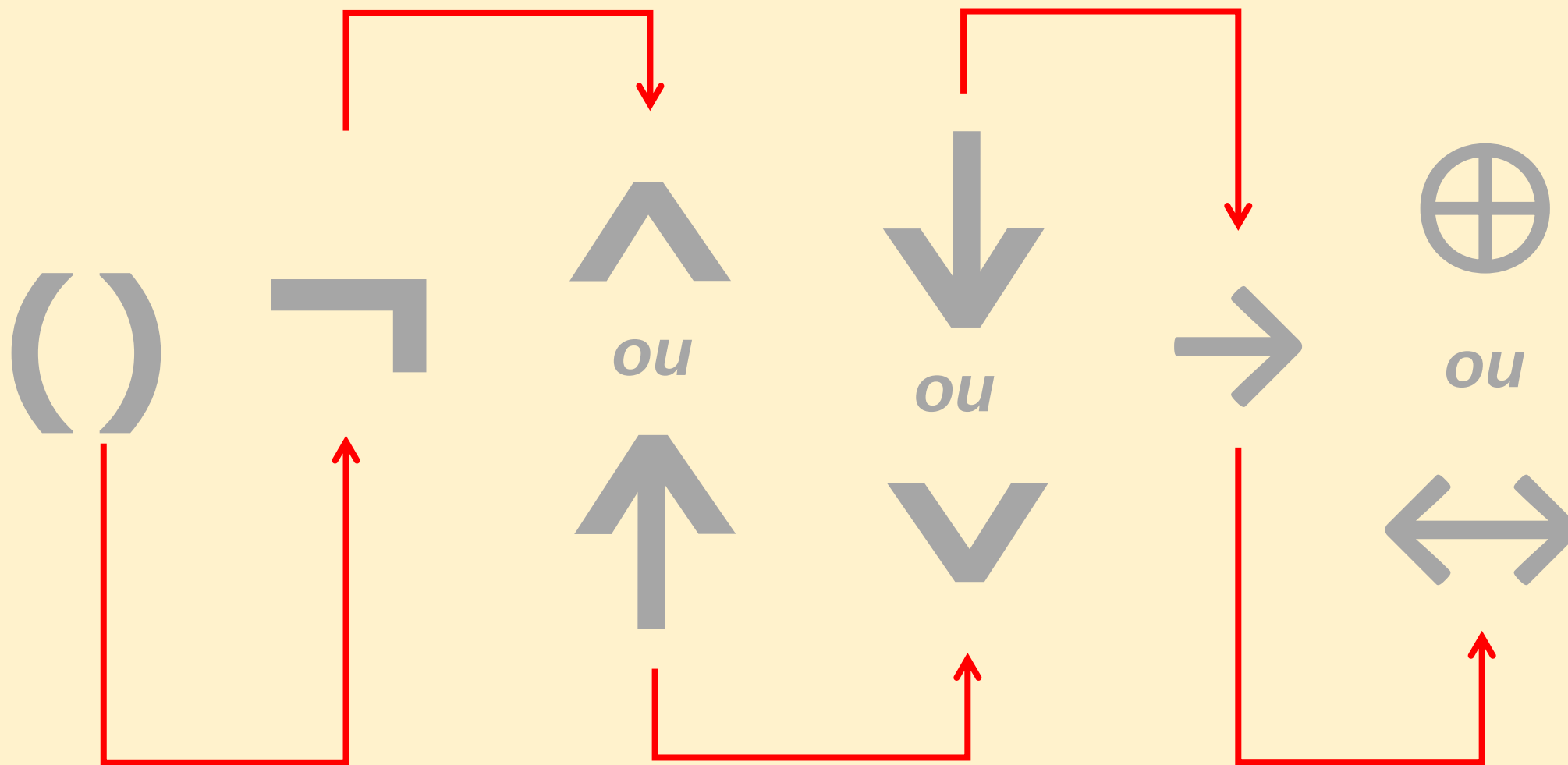












fml ::= p

p

q

$fml ::= p$

$fml ::= \neg fml$

$\neg p$

$\neg q$

$fml ::= p$

$fml ::= \neg fml$

$fml ::= fml \text{ op } fml$

$op ::= \{\text{conectores}\}$

$p \wedge q$

$p \wedge \neg q$

$p \oplus q$

$\neg p \oplus q$

$\neg(p \oplus q)$

$\neg(\neg p \oplus q)$

$fml ::= p$

$fml ::= \neg fml$

$fml ::= fml \text{ op } fml$

$op ::= \{\text{conectores}\}$

$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$

$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

$$\begin{array}{l} p = F \\ q = V \end{array}$$

$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

$$\begin{array}{l} p = F \\ q = V \end{array}$$

$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

$$\neg(F \oplus V) \wedge \neg(\neg F \oplus V)$$

$$\begin{array}{l} p = F \\ q = V \end{array}$$

$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

$$\neg(F \oplus V) \wedge \neg(\neg F \oplus V)$$

$$\neg(V) \wedge \neg(V \oplus V)$$

$$\begin{array}{l} p = F \\ q = V \end{array}$$

$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

$$\neg(F \oplus V) \wedge \neg(\neg F \oplus V)$$

$$\neg(V) \wedge \neg(V \oplus V)$$

$$F \wedge \neg(F)$$

$$\begin{array}{l} p = F \\ q = V \end{array}$$

$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

$$\neg(F \oplus V) \wedge \neg(\neg F \oplus V)$$

$$\neg(V) \wedge \neg(V \oplus V)$$

$$F \wedge \neg(F)$$

$$F \wedge V$$

$$\begin{array}{l} p = F \\ q = V \end{array}$$

$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

$$\neg(F \oplus V) \wedge \neg(\neg F \oplus V)$$

$$\neg(V) \wedge \neg(V \oplus V)$$

$$F \wedge \neg(F)$$

$$F \wedge V$$

$$F$$

$$\begin{array}{l} p = F \\ q = V \end{array}$$

$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

$$\neg(F \oplus V) \wedge \neg(\neg F \oplus V)$$

$$\neg(V) \wedge \neg(V \oplus V)$$

$$F \wedge \neg(F)$$

$$F \wedge V$$

$$F$$

$$\begin{array}{l} p = F \\ q = V \end{array}$$

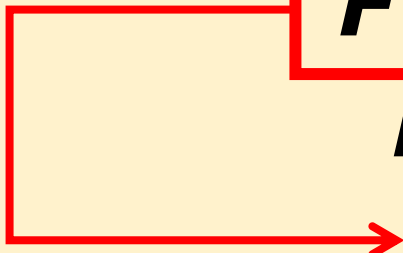
$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

$$\neg(F \oplus V) \wedge \neg(\neg F \oplus V)$$

$$\neg(V) \wedge \neg(V \oplus V)$$

$$F \wedge \neg(F)$$

$$F \wedge V$$



F

$$\begin{array}{l} p = F \\ q = V \\ r = V \end{array}$$

$$(p \rightarrow q) \leftrightarrow ((p \vee r) \rightarrow q)$$

$$\begin{array}{l} p = F \\ q = V \\ r = V \end{array}$$

$$\begin{array}{l} (p \rightarrow q) \leftrightarrow ((p \vee r) \rightarrow q) \\ (F \rightarrow V) \leftrightarrow ((F \vee V) \rightarrow V) \end{array}$$

$$p = F$$

$$q = V$$

$$r = V$$

$$(p \rightarrow q) \leftrightarrow ((p \vee r) \rightarrow q)$$

$$(F \rightarrow V) \leftrightarrow ((F \vee V) \rightarrow V)$$

$$(V) \leftrightarrow ((V) \rightarrow V)$$

$$p = F$$

$$q = V$$

$$r = V$$

$$(p \rightarrow q) \leftrightarrow ((p \vee r) \rightarrow q)$$

$$(F \rightarrow V) \leftrightarrow ((F \vee V) \rightarrow V)$$

$$(V) \leftrightarrow ((V) \rightarrow V)$$

$$(V) \leftrightarrow (V)$$

$$p = F$$

$$q = V$$

$$r = V$$

$$(p \rightarrow q) \leftrightarrow ((p \vee r) \rightarrow q)$$

$$(F \rightarrow V) \leftrightarrow ((F \vee V) \rightarrow V)$$

$$(V) \leftrightarrow ((V) \rightarrow V)$$

$$(V) \leftrightarrow (V)$$

V

A top-down view of a wooden desk. On the right side, there is a black laptop with its keyboard visible. Below the laptop, a pair of blue-rimmed glasses lies on the desk. To the left of the glasses is a black pen. Further down and to the right is a black smartphone. The background is a light-colored wooden surface with a visible grain.

Lógica Computacional
by Professor Pantoja

Tabela Verdade

Aula 4

$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

p	q

$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

p	q
F	F
F	V
V	F
V	V

$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

p	q	$p \oplus q$
F	F	
F	V	
V	F	
V	V	

$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

p	q	$p \oplus q$
F	F	F
F	V	
V	F	
V	V	

$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

p	q	$p \oplus q$
F	F	F
F	V	V
V	F	
V	V	

$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

p	q	$p \oplus q$
F	F	F
F	V	V
V	F	V
V	V	

$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

p	q	$p \oplus q$
F	F	F
F	V	V
V	F	V
V	V	F

$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

p	q	$p \oplus q$	$\neg(p \oplus q)$
F	F	F	
F	V	V	
V	F	V	
V	V	F	

$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

p	q	$p \oplus q$	$\neg(p \oplus q)$
F	F	F	V
F	V	V	
V	F	V	
V	V	F	

$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

p	q	$p \oplus q$	$\neg(p \oplus q)$
F	F	F	V
F	V	V	F
V	F	V	
V	V	F	

$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

p	q	$p \oplus q$	$\neg(p \oplus q)$
F	F	F	V
F	V	V	F
V	F	V	F
V	V	F	

$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

p	q	$p \oplus q$	$\neg(p \oplus q)$
F	F	F	V
F	V	V	F
V	F	V	F
V	V	F	V

$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

p	q	$p \oplus q$	$\neg(p \oplus q)$	$\neg p$
F	F	F	V	
F	V	V	F	
V	F	V	F	
V	V	F	V	

$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

p	q	$p \oplus q$	$\neg(p \oplus q)$	$\neg p$
F	F	F	V	V
F	V	V	F	
V	F	V	F	
V	V	F	V	

$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

p	q	$p \oplus q$	$\neg(p \oplus q)$	$\neg p$
F	F	F	V	V
F	V	V	F	V
V	F	V	F	
V	V	F	V	

$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

p	q	$p \oplus q$	$\neg(p \oplus q)$	$\neg p$
F	F	F	V	V
F	V	V	F	V
V	F	V	F	F
V	V	F	V	

$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

p	q	$p \oplus q$	$\neg(p \oplus q)$	$\neg p$
F	F	F	V	V
F	V	V	F	V
V	F	V	F	F
V	V	F	V	F

$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

p	q	$p \oplus q$	$\neg(p \oplus q)$	$\neg p$	$\neg p \oplus q$
F	F	F	V	V	
F	V	V	F	V	
V	F	V	F	F	
V	V	F	V	F	

$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

p	q	$p \oplus q$	$\neg(p \oplus q)$	$\neg p$	$\neg p \oplus q$
F	F	F	V	V	V
F	V	V	F	V	
V	F	V	F	F	
V	V	F	V	F	

$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

p	q	$p \oplus q$	$\neg(p \oplus q)$	$\neg p$	$\neg p \oplus q$
F	F	F	V	V	V
F	V	V	F	V	F
V	F	V	F	F	
V	V	F	V	F	

$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

p	q	$p \oplus q$	$\neg(p \oplus q)$	$\neg p$	$\neg p \oplus q$
F	F	F	V	V	V
F	V	V	F	V	F
V	F	V	F	F	F
V	V	F	V	F	

$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

p	q	$p \oplus q$	$\neg(p \oplus q)$	$\neg p$	$\neg p \oplus q$
F	F	F	V	V	V
F	V	V	F	V	F
V	F	V	F	F	F
V	V	F	V	F	V

$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

p	q	$p \oplus q$	$\neg(p \oplus q)$	$\neg p$	$\neg p \oplus q$	$\neg(\neg p \oplus q)$
F	F	F	V	V	V	
F	V	V	F	V	F	
V	F	V	F	F	F	
V	V	F	V	F	V	

$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

p	q	$p \oplus q$	$\neg(p \oplus q)$	$\neg p$	$\neg p \oplus q$	$\neg(\neg p \oplus q)$
F	F	F	V	V	V	F
F	V	V	F	V	F	
V	F	V	F	F	F	
V	V	F	V	F	V	

$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

p	q	$p \oplus q$	$\neg(p \oplus q)$	$\neg p$	$\neg p \oplus q$	$\neg(\neg p \oplus q)$
F	F	F	V	V	V	F
F	V	V	F	V	F	V
V	F	V	F	F	F	
V	V	F	V	F	V	

$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

p	q	$p \oplus q$	$\neg(p \oplus q)$	$\neg p$	$\neg p \oplus q$	$\neg(\neg p \oplus q)$
F	F	F	V	V	V	F
F	V	V	F	V	F	V
V	F	V	F	F	F	V
V	V	F	V	F	V	

$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

p	q	$p \oplus q$	$\neg(p \oplus q)$	$\neg p$	$\neg p \oplus q$	$\neg(\neg p \oplus q)$
F	F	F	V	V	V	F
F	V	V	F	V	F	V
V	F	V	F	F	F	V
V	V	F	V	F	V	F

$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

p	q	$p \oplus q$	$\neg(p \oplus q)$	$\neg p$	$\neg p \oplus q$	$\neg(\neg p \oplus q)$	$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$
F	F	F	V	V	V	F	
F	V	V	F	V	F	V	
V	F	V	F	F	F	V	
V	V	F	V	F	V	F	

$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

p	q	$p \oplus q$	$\neg(p \oplus q)$	$\neg p$	$\neg p \oplus q$	$\neg(\neg p \oplus q)$	$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$
F	F	F	V	V	V	F	F
F	V	V	F	V	F	V	
V	F	V	F	F	F	V	
V	V	F	V	F	V	F	

$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

p	q	$p \oplus q$	$\neg(p \oplus q)$	$\neg p$	$\neg p \oplus q$	$\neg(\neg p \oplus q)$	$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$
F	F	F	V	V	V	F	F
F	V	V	F	V	F	V	F
V	F	V	F	F	F	V	
V	V	F	V	F	V	F	

$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

p	q	$p \oplus q$	$\neg(p \oplus q)$	$\neg p$	$\neg p \oplus q$	$\neg(\neg p \oplus q)$	$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$
F	F	F	V	V	V	F	F
F	V	V	F	V	F	V	F
V	F	V	F	F	F	V	F
V	V	F	V	F	V	F	

$$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$$

p	q	$p \oplus q$	$\neg(p \oplus q)$	$\neg p$	$\neg p \oplus q$	$\neg(\neg p \oplus q)$	$\neg(p \oplus q) \wedge \neg(\neg p \oplus q)$
F	F	F	V	V	V	F	F
F	V	V	F	V	F	V	F
V	F	V	F	F	F	V	F
V	V	F	V	F	V	F	F

$$p \wedge (q \vee r)$$

p	q	r

$$p \wedge (q \vee r)$$

p	q	r
F	F	F
F	F	V
F	V	F
F	V	V
V	F	F
V	F	V
V	V	F
V	V	V

$$p \wedge (q \vee r)$$

p	q	r	$q \vee r$
F	F	F	
F	F	V	
F	V	F	
F	V	V	
V	F	F	
V	F	V	
V	V	F	
V	V	V	

$$p \wedge (q \vee r)$$

p	q	r	$q \vee r$
F	F	F	F
F	F	V	
F	V	F	
F	V	V	
V	F	F	
V	F	V	
V	V	F	
V	V	V	

$$p \wedge (q \vee r)$$

p	q	r	$q \vee r$
F	F	F	F
F	F	V	V
F	V	F	
F	V	V	
V	F	F	
V	F	V	
V	V	F	
V	V	V	

$$p \wedge (q \vee r)$$

p	q	r	$q \vee r$
F	F	F	F
F	F	V	V
F	V	F	V
F	V	V	
V	F	F	
V	F	V	
V	V	F	
V	V	V	

$$p \wedge (q \vee r)$$

p	q	r	$q \vee r$
F	F	F	F
F	F	V	V
F	V	F	V
F	V	V	V
V	F	F	
V	F	V	
V	V	F	
V	V	V	

$$p \wedge (q \vee r)$$

p	q	r	$q \vee r$
F	F	F	F
F	F	V	V
F	V	F	V
F	V	V	V
V	F	F	F
V	F	V	
V	V	F	
V	V	V	

$$p \wedge (q \vee r)$$

p	q	r	$q \vee r$
F	F	F	F
F	F	V	V
F	V	F	V
F	V	V	V
V	F	F	F
V	F	V	V
V	V	F	
V	V	V	

$$p \wedge (q \vee r)$$

p	q	r	$q \vee r$
F	F	F	F
F	F	V	V
F	V	F	V
F	V	V	V
V	F	F	F
V	F	V	V
V	V	F	V
V	V	V	

$$p \wedge (q \vee r)$$

p	q	r	$q \vee r$
F	F	F	F
F	F	V	V
F	V	F	V
F	V	V	V
V	F	F	F
V	F	V	V
V	V	F	V
V	V	V	V

$$p \wedge (q \vee r)$$

p	q	r	$q \vee r$	$p \wedge (q \vee r)$
F	F	F	F	
F	F	V	V	
F	V	F	V	
F	V	V	V	
V	F	F	F	
V	F	V	V	
V	V	F	V	
V	V	V	V	

$$p \wedge (q \vee r)$$

p	q	r	$q \vee r$	$p \wedge (q \vee r)$
F	F	F	F	F
F	F	V	V	
F	V	F	V	
F	V	V	V	
V	F	F	F	
V	F	V	V	
V	V	F	V	
V	V	V	V	

$$p \wedge (q \vee r)$$

p	q	r	$q \vee r$	$p \wedge (q \vee r)$
F	F	F	F	F
F	F	V	V	F
F	V	F	V	
F	V	V	V	
V	F	F	F	
V	F	V	V	
V	V	F	V	
V	V	V	V	

$$p \wedge (q \vee r)$$

p	q	r	$q \vee r$	$p \wedge (q \vee r)$
F	F	F	F	F
F	F	V	V	F
F	V	F	V	F
F	V	V	V	
V	F	F	F	
V	F	V	V	
V	V	F	V	
V	V	V	V	

$$p \wedge (q \vee r)$$

p	q	r	$q \vee r$	$p \wedge (q \vee r)$
F	F	F	F	F
F	F	V	V	F
F	V	F	V	F
F	V	V	V	F
V	F	F	F	
V	F	V	V	
V	V	F	V	
V	V	V	V	

$$p \wedge (q \vee r)$$

p	q	r	$q \vee r$	$p \wedge (q \vee r)$
F	F	F	F	F
F	F	V	V	F
F	V	F	V	F
F	V	V	V	F
V	F	F	F	F
V	F	V	V	
V	V	F	V	
V	V	V	V	

$$p \wedge (q \vee r)$$

p	q	r	$q \vee r$	$p \wedge (q \vee r)$
F	F	F	F	F
F	F	V	V	F
F	V	F	V	F
F	V	V	V	F
V	F	F	F	F
V	F	V	V	V
V	V	F	V	
V	V	V	V	

$$p \wedge (q \vee r)$$

p	q	r	$q \vee r$	$p \wedge (q \vee r)$
F	F	F	F	F
F	F	V	V	F
F	V	F	V	F
F	V	V	V	F
V	F	F	F	F
V	F	V	V	V
V	V	F	V	V
V	V	V	V	V

$$p \wedge (q \vee r)$$

p	q	r	$q \vee r$	$p \wedge (q \vee r)$
F	F	F	F	F
F	F	V	V	F
F	V	F	V	F
F	V	V	V	F
V	F	F	F	F
V	F	V	V	V
V	V	F	V	V
V	V	V	V	V

$$(p \vee q) \rightarrow p$$

p	q
F	F
F	V
V	F
V	V

$$(p \vee q) \rightarrow p$$

p	q	p ∨ q
F	F	
F	V	
V	F	
V	V	

$$(p \vee q) \rightarrow p$$

p	q	p $\vee$ q
F	F	F
F	V	
V	F	
V	V	

$$(p \vee q) \rightarrow p$$

p	q	p ∨ q
F	F	F
F	V	V
V	F	
V	V	

$$(p \vee q) \rightarrow p$$

p	q	p $\vee$ q
F	F	F
F	V	V
V	F	V
V	V	

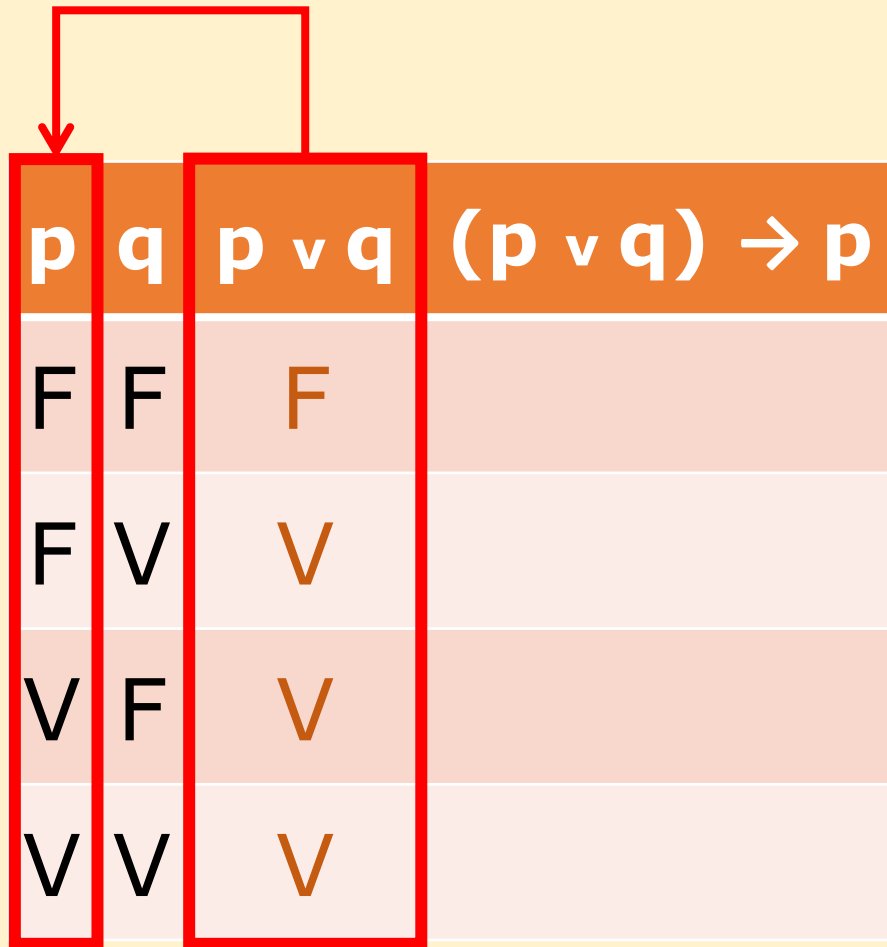
$$(p \vee q) \rightarrow p$$

p	q	$p \vee q$
F	F	F
F	V	V
V	F	V
V	V	V

$$(p \vee q) \rightarrow p$$

p	q	$p \vee q$	$(p \vee q) \rightarrow p$
F	F	F	
F	V	V	
V	F	V	
V	V	V	

$$(p \vee q) \rightarrow p$$



p	q	p ∨ q	(p ∨ q) → p
F	F	F	
F	V	V	
V	F	V	
V	V	V	

$$(p \vee q) \rightarrow p$$

p	q	$p \vee q$	$(p \vee q) \rightarrow p$
F	F	F	V
F	V	V	
V	F	V	
V	V	V	

$$(p \vee q) \rightarrow p$$

p	q	$p \vee q$	$(p \vee q) \rightarrow p$
F	F	F	V
F	V	V	F
V	F	V	
V	V	V	

$$(p \vee q) \rightarrow p$$

p	q	$p \vee q$	$(p \vee q) \rightarrow p$
F	F	F	V
F	V	V	F
V	F	V	V
V	V	V	V

$$(p \vee q) \rightarrow p$$

p	q	$p \vee q$	$(p \vee q) \rightarrow p$
F	F	F	V
F	V	V	F
V	F	V	V
V	V	V	V

A top-down view of a wooden desk. On the right side, there is a black laptop with its keyboard visible. Below the laptop, a pair of blue-rimmed glasses lies on the desk. To the left of the glasses, a black pen is positioned diagonally. Further down and to the right, a black smartphone is lying flat. The background is the natural wood grain of the desk.

Lógica Computacional
by Professor Pantoja

Tautologia, Contradição e Contingência

Aula 5

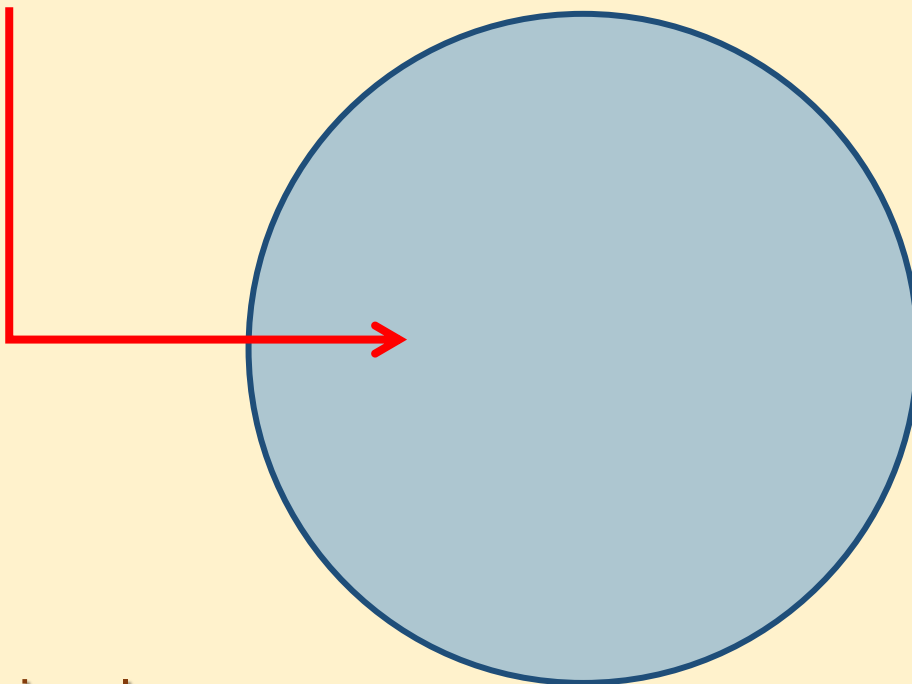
Tautologia

Tautologia

***Fórmula proposicional
que independente dos
valores de entrada de
suas proposições, o
resultado é sempre
Verdadeiro.***

Tautologia

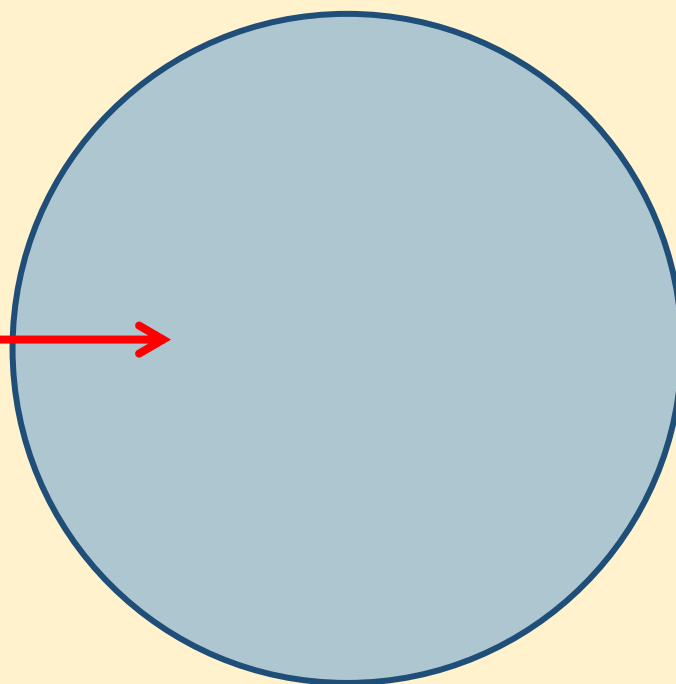
*Fórmula proposicional
que independente dos
valores de entrada de
suas proposições, o
resultado é sempre
Verdadeiro.*



Tautologia

*Fórmula proposicional
que independente dos
valores de entrada de
suas proposições, o
resultado é sempre
Verdadeiro.*

Contradição ou Falácia

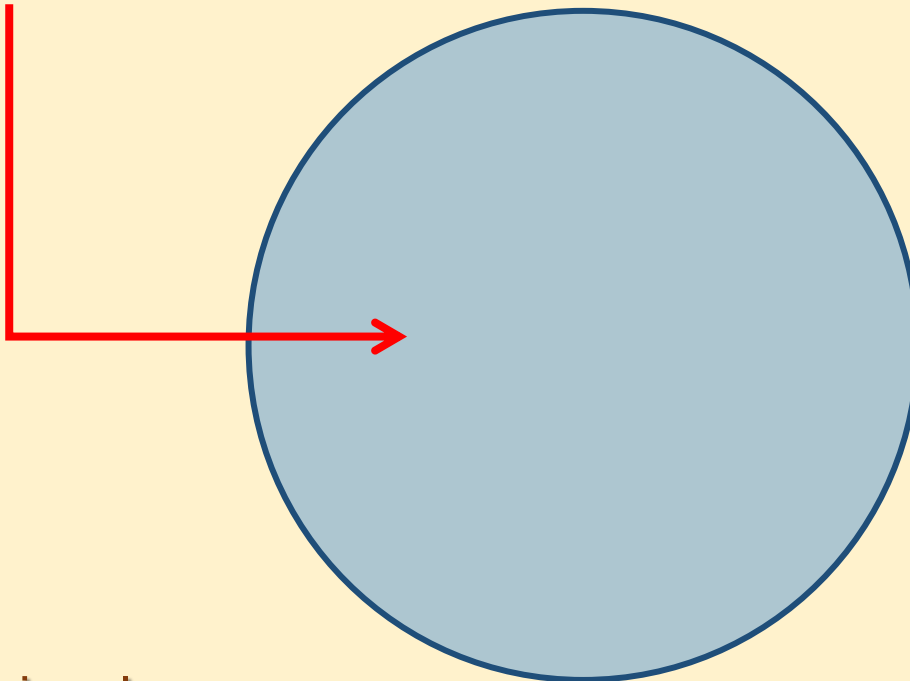


Tautologia

Fórmula proposicional que independente dos valores de entrada de suas proposições, o resultado é sempre Verdadeiro.

Contradição ou Falácia

Fórmula proposicional que independente dos valores de entrada de suas proposições, o resultado é sempre Falso.

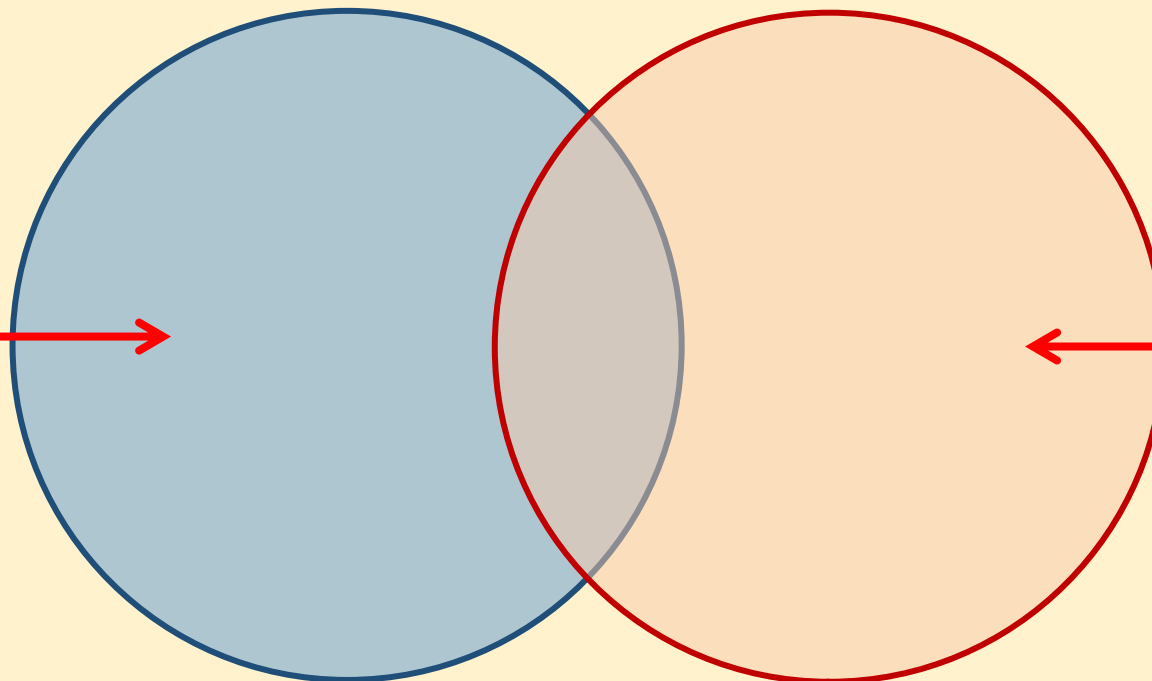


Tautologia

Fórmula proposicional que independente dos valores de entrada de suas proposições, o resultado é sempre Verdadeiro.

Contradição ou Falácia

Fórmula proposicional que independente dos valores de entrada de suas proposições, o resultado é sempre Falso.



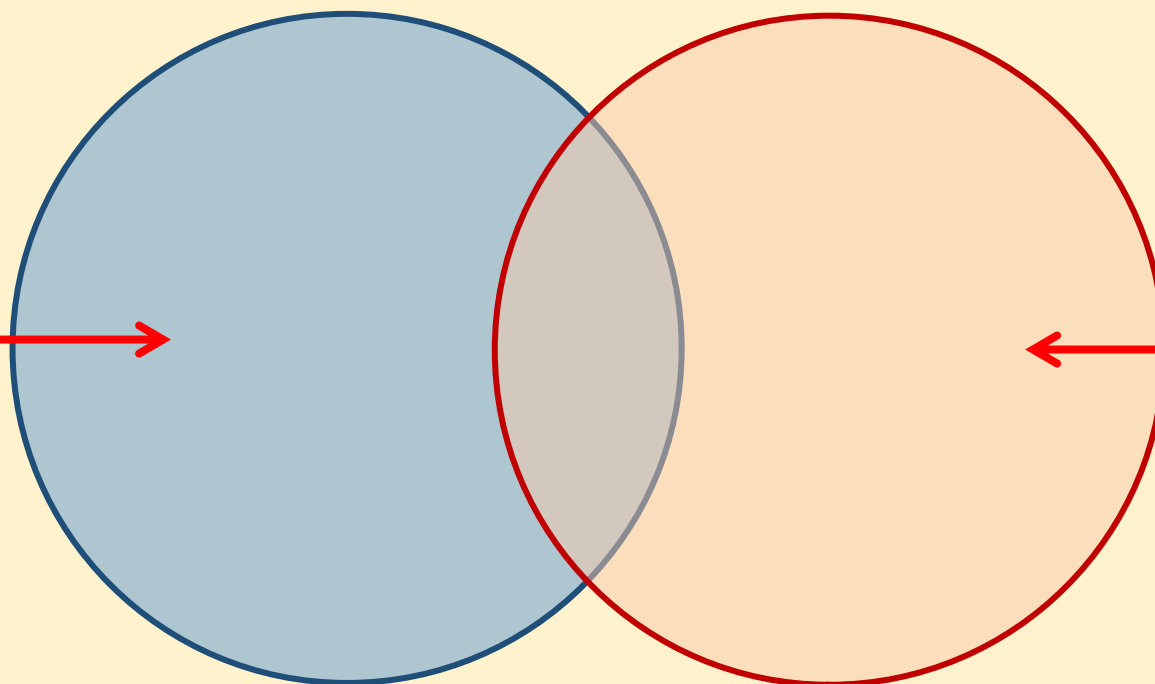
Tautologia

Fórmula proposicional que independente dos valores de entrada de suas proposições, o resultado é sempre Verdadeiro.

Contingência

Contradição ou Falácia

Fórmula proposicional que independente dos valores de entrada de suas proposições, o resultado é sempre Falso.



Tautologia

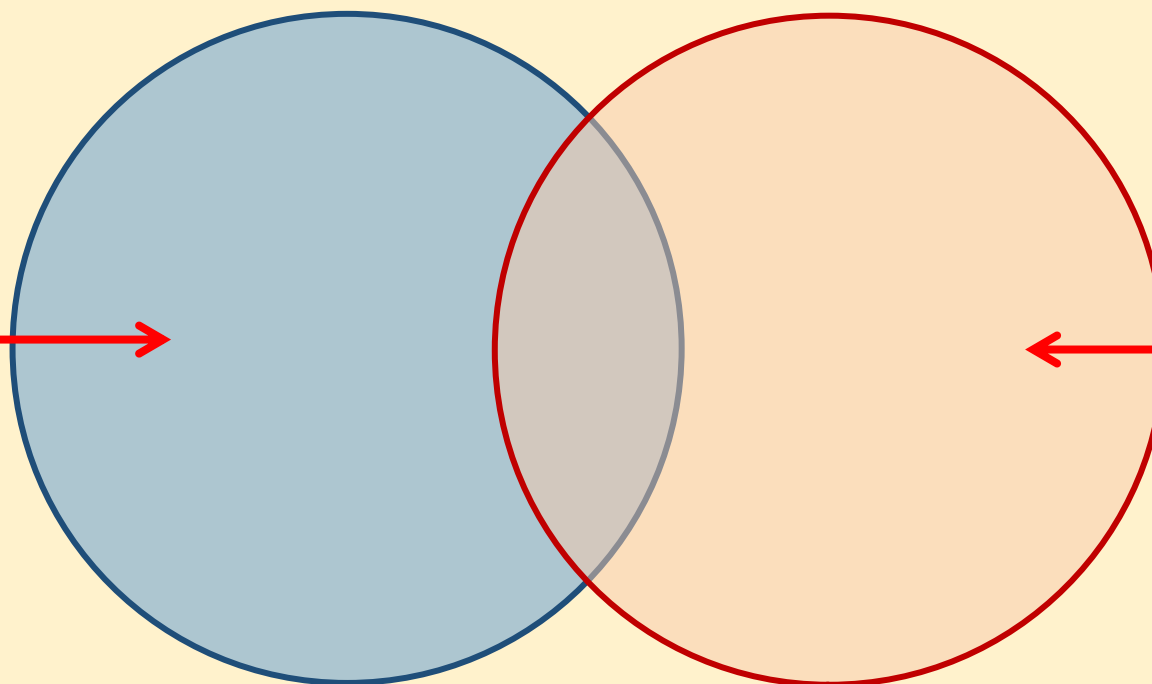
Fórmula proposicional que independente dos valores de entrada de suas proposições, o resultado é sempre Verdadeiro.

Contingência

chega a uma conclusão que pode ser tanto Verdadeiro quanto Falso.

Contradição ou Falácia

Fórmula proposicional que independente dos valores de entrada de suas proposições, o resultado é sempre Falso.



Tautologia

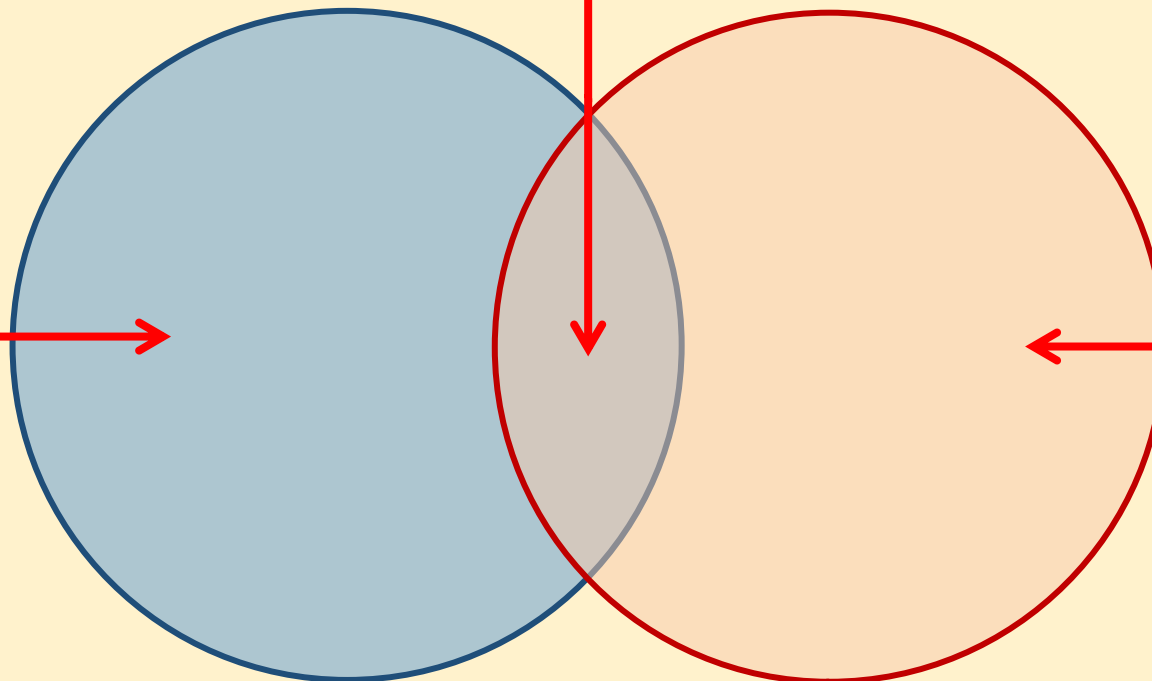
Fórmula proposicional que independente dos valores de entrada de suas proposições, o resultado é sempre Verdadeiro.

Contingência

chega a uma conclusão que pode ser tanto Verdadeiro quanto Falso.

Contradição ou Falácia

Fórmula proposicional que independente dos valores de entrada de suas proposições, o resultado é sempre Falso.



$$p \rightarrow (p \vee q)$$

p	q
F	F
F	V
V	F
V	V

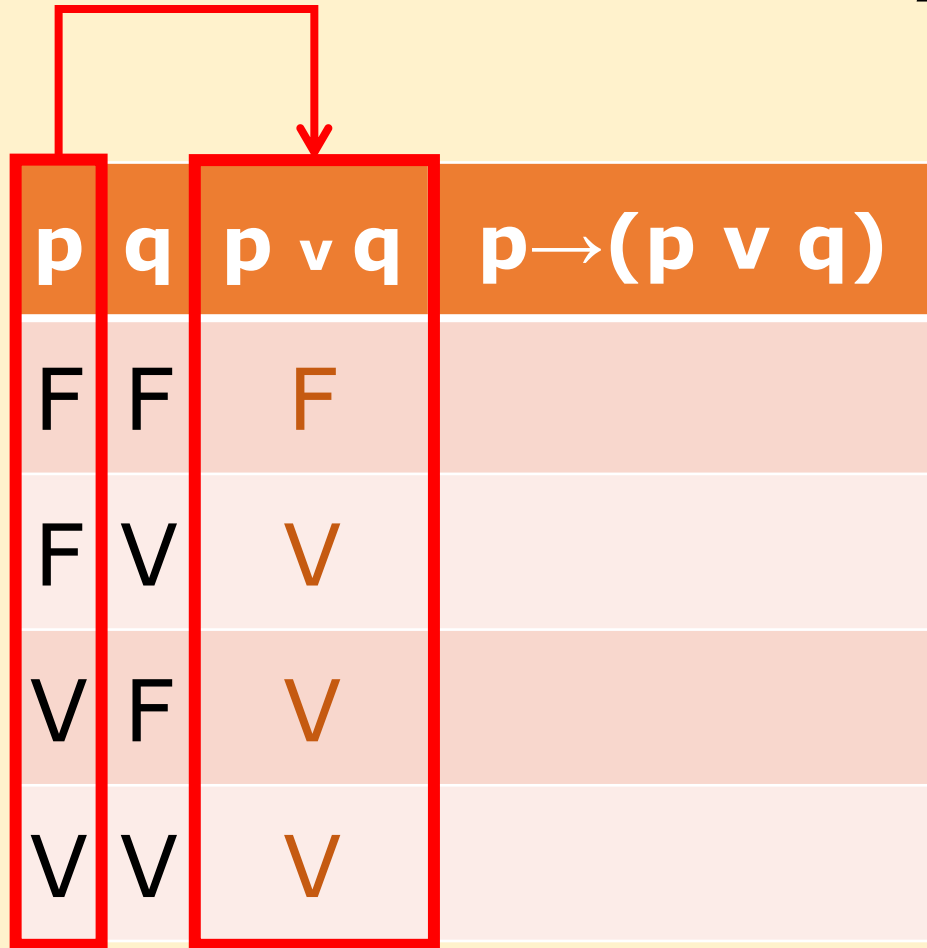
$$p \rightarrow (p \vee q)$$

p	q	p ∨ q
F	F	
F	V	
V	F	
V	V	

$$p \rightarrow (p \vee q)$$

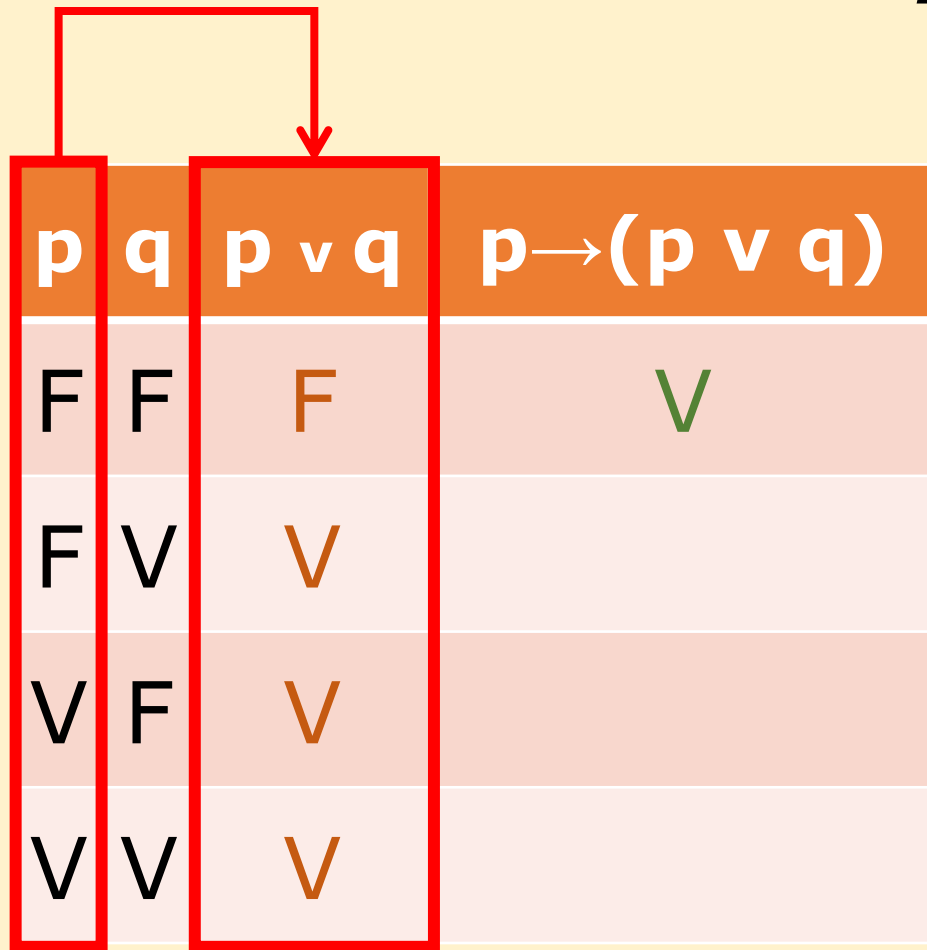
p	q	$p \vee q$
F	F	F
F	V	V
V	F	V
V	V	V

$$p \rightarrow (p \vee q)$$



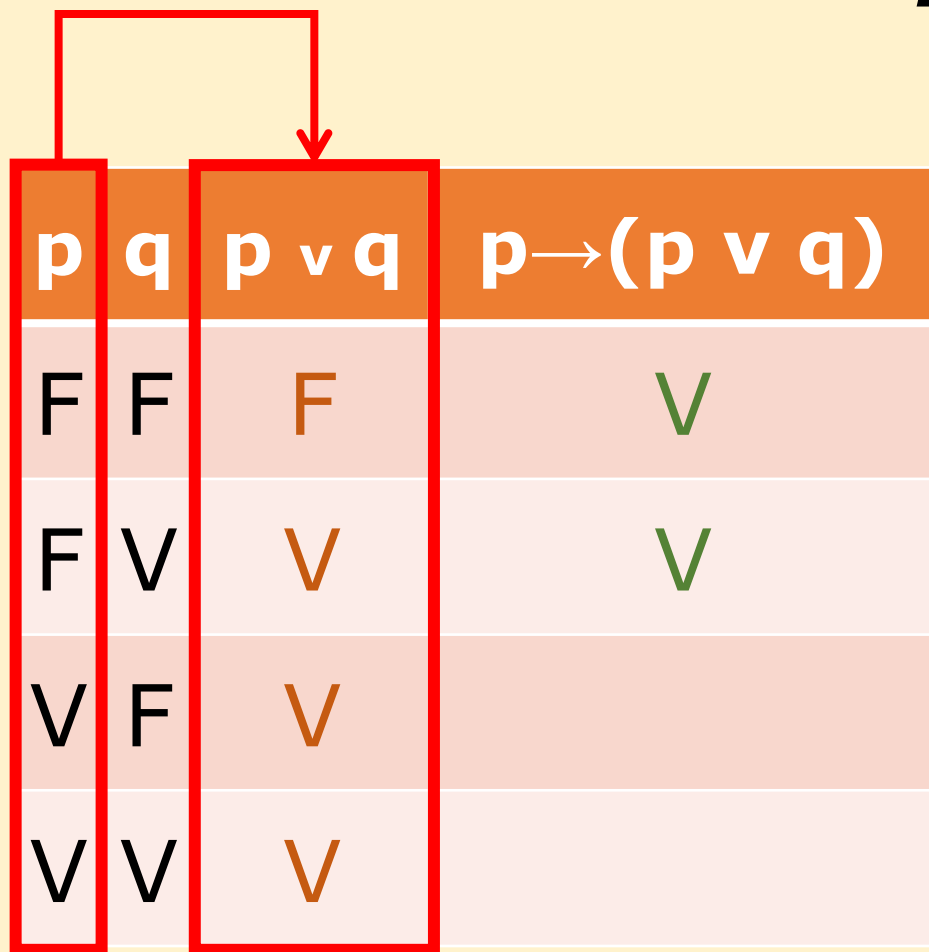
p	q	p v q	p → (p v q)
F	F	F	
F	V	V	
V	F	V	
V	V	V	

$$p \rightarrow (p \vee q)$$



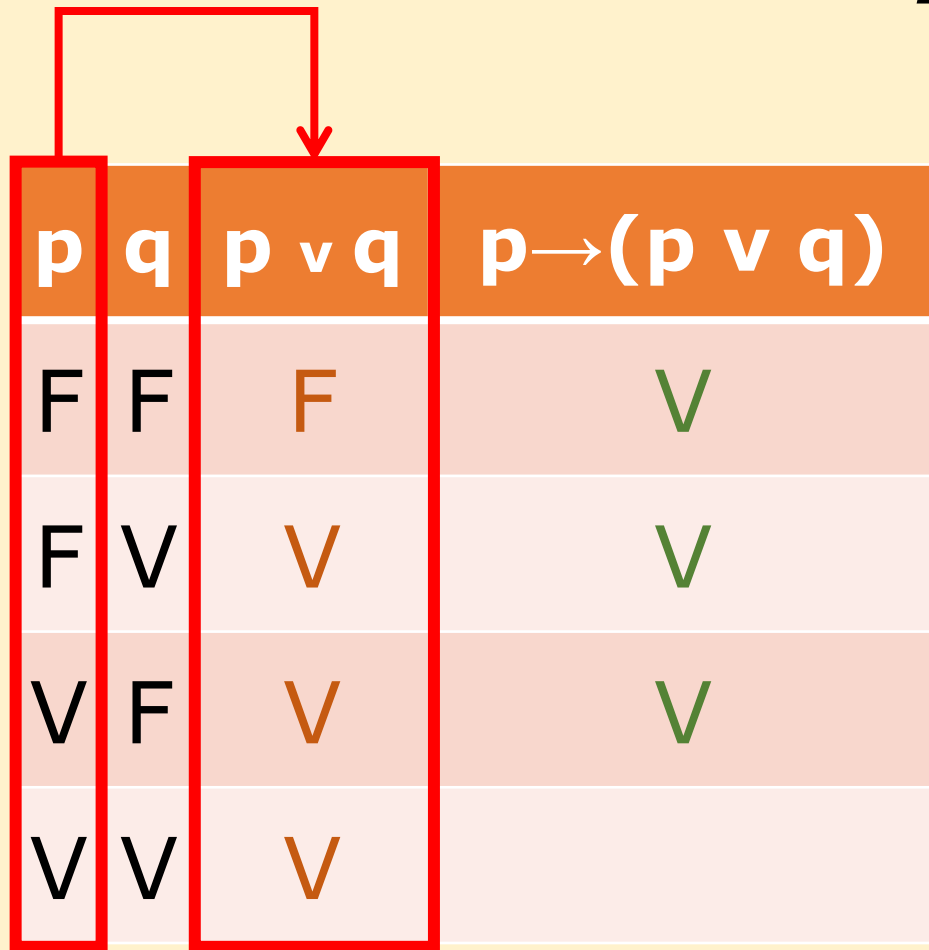
p	q	p v q	p → (p v q)
F	F	F	V
F	V	V	
V	F	V	
V	V	V	

$$p \rightarrow (p \vee q)$$



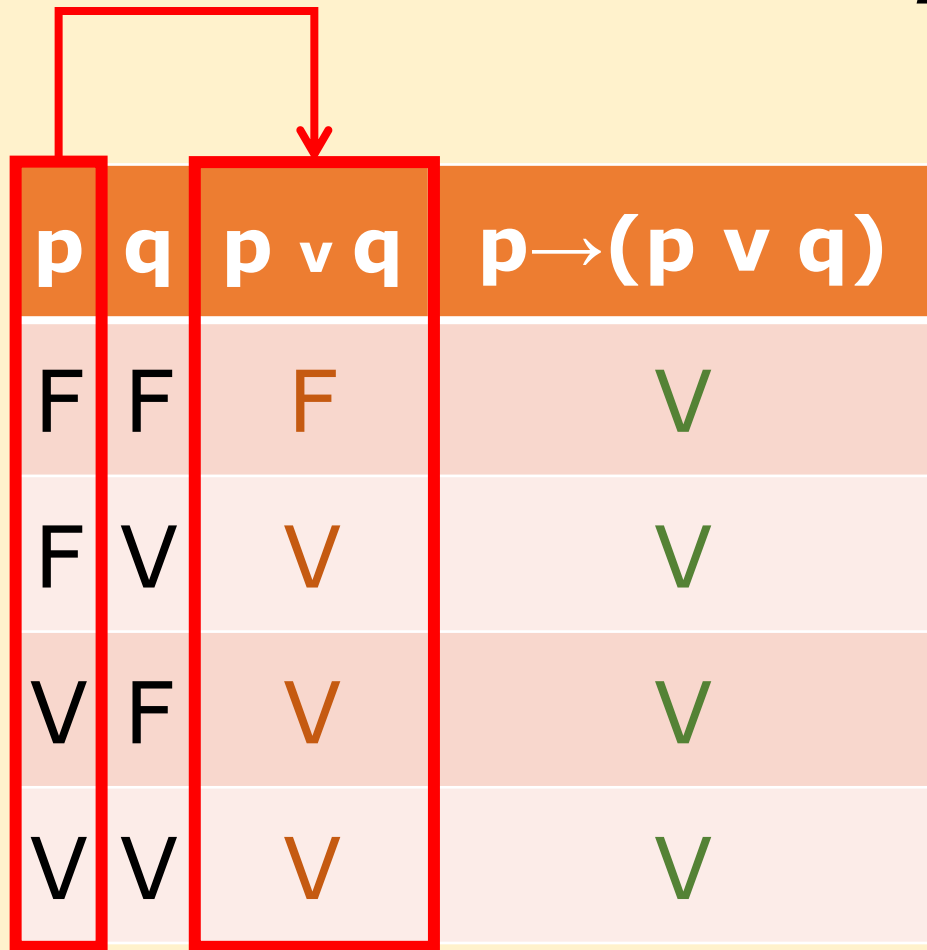
p	q	p v q	p → (p v q)
F	F	F	V
F	V	V	V
V	F	V	V
V	V	V	V

$$p \rightarrow (p \vee q)$$



p	q	p v q	p → (p v q)
F	F	F	V
F	V	V	V
V	F	V	V
V	V	V	V

$$p \rightarrow (p \vee q)$$



p	q	p ∨ q	p → (p ∨ q)
F	F	F	V
F	V	V	V
V	F	V	V
V	V	V	V

$$\neg(p \vee q) \wedge p$$

p	q
F	F
F	V
V	F
V	V

$$\neg(p \vee q) \wedge p$$

p	q	p v q
F	F	
F	V	
V	F	
V	V	

$$\neg(p \vee q) \wedge p$$

p	q	p ∨ q
F	F	F
F	V	V
V	F	V
V	V	V

$$\neg(p \vee q) \wedge p$$

p	q	p ∨ q	¬(p ∨ q)
F	F	F	
F	V	V	
V	F	V	
V	V	V	

$$\neg(p \vee q) \wedge p$$

p	q	$p \vee q$	$\neg(p \vee q)$
F	F	F	V
F	V	V	F
V	F	V	F
V	V	V	F

$$\neg(p \vee q) \wedge p$$

p	q	p ∨ q	¬(p ∨ q)	¬(p ∨ q) ∧ p
F	F	F	V	
F	V	V	F	
V	F	V	F	
V	V	V	F	

$$\neg(p \vee q) \wedge p$$

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg(p \vee q) \wedge p$
F	F	F	V	F
F	V	V	F	
V	F	V	F	
V	V	V	F	

$$\neg(p \vee q) \wedge p$$

p	q	p ∨ q	¬(p ∨ q)	¬(p ∨ q) ∧ p
F	F	F	V	F
F	V	V	F	F
V	F	V	F	
V	V	V	F	

$$\neg(p \vee q) \wedge p$$

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg(p \vee q) \wedge p$
F	F	F	V	F
F	V	V	F	F
V	F	V	F	F
V	V	V	F	F

$$\neg(p \vee q) \wedge p$$

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg(p \vee q) \wedge p$
F	F	F	V	F
F	V	V	F	F
V	F	V	F	F
V	V	V	F	F

$$(p \vee q) \rightarrow p$$

p	q
F	F
F	V
V	F
V	V

$$(p \vee q) \rightarrow p$$

p	q	p $\vee$ q
F	F	
F	V	
V	F	
V	V	

$$(p \vee q) \rightarrow p$$

p	q	$p \vee q$
F	F	F
F	V	V
V	F	V
V	V	V

$$(p \vee q) \rightarrow p$$

p	q	$p \vee q$	$(p \vee q) \rightarrow p$
F	F	F	
F	V	V	
V	F	V	
V	V	V	

$$(p \vee q) \rightarrow p$$



p	q	$p \vee q$	$(p \vee q) \rightarrow p$
F	F	F	
F	V	V	
V	F	V	
V	V	V	

$$(p \vee q) \rightarrow p$$



p	q	$p \vee q$	$(p \vee q) \rightarrow p$
F	F	F	V
F	V	V	
V	F	V	
V	V	V	

$$(p \vee q) \rightarrow p$$



p	q	$p \vee q$	$(p \vee q) \rightarrow p$
F	F	F	V
F	V	V	F
V	F	V	
V	V	V	

$$(p \vee q) \rightarrow p$$

p	q	$p \vee q$	$(p \vee q) \rightarrow p$
F	F	F	V
F	V	V	F
V	F	V	V
V	V	V	

$$(p \vee q) \rightarrow p$$



p	q	$p \vee q$	$(p \vee q) \rightarrow p$
F	F	F	V
F	V	V	F
V	F	V	V
V	V	V	V

A top-down view of a wooden desk. On the right side, there is a black laptop with its keyboard visible. Below the laptop, a pair of blue-rimmed glasses lies on the desk. To the left of the glasses, a black pen is positioned diagonally. Further down and to the right, a black smartphone is lying flat. The background is the natural wood grain of the desk.

Lógica Computacional
by Professor Pantoja

Satisfatibilidade e Validade

Aula 6

A é Satisfatível sse:

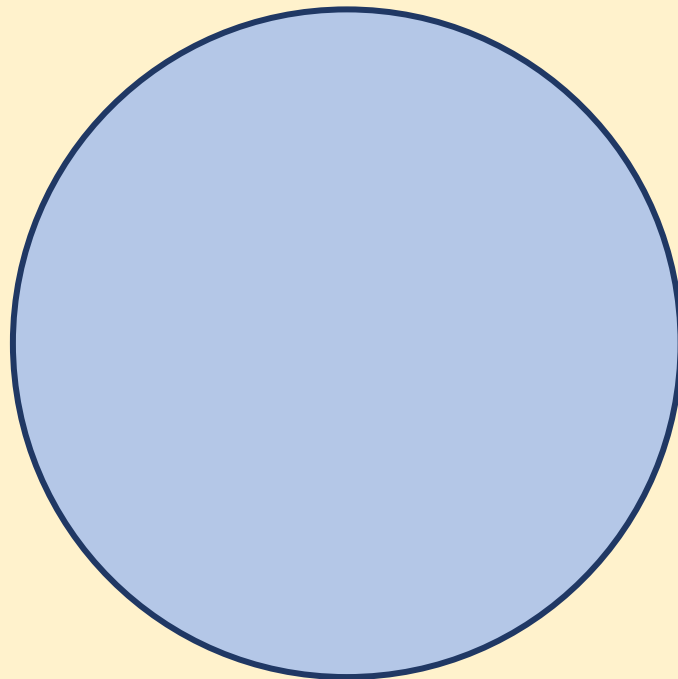
A é Satisfatível sse:

- ***$(A) = V$, para alguma interpretação;***

A é Satisfatível sse:

- ***$(A) = V$, para alguma interpretação;***
- ***é um modelo de/para A.***

Satisfável



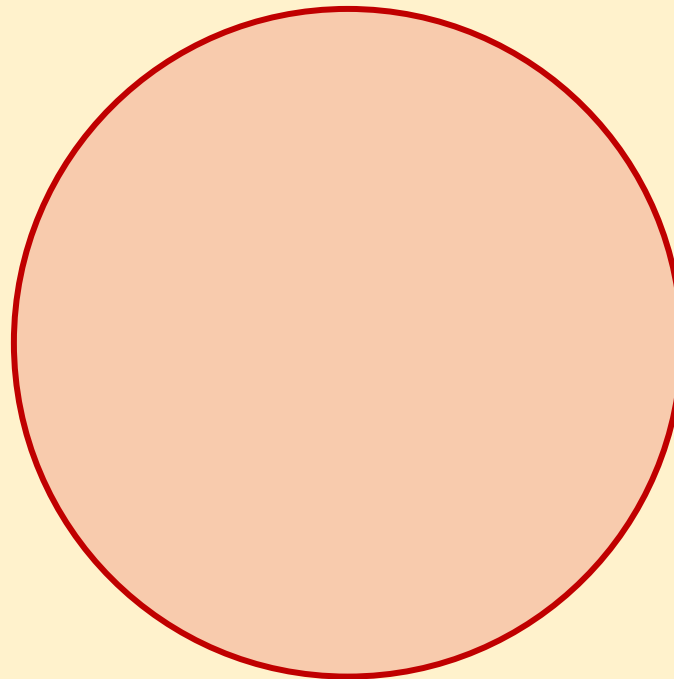
A é *Falsificável*, $\models$, sse:

A é Falsificável, $\models$, sse:

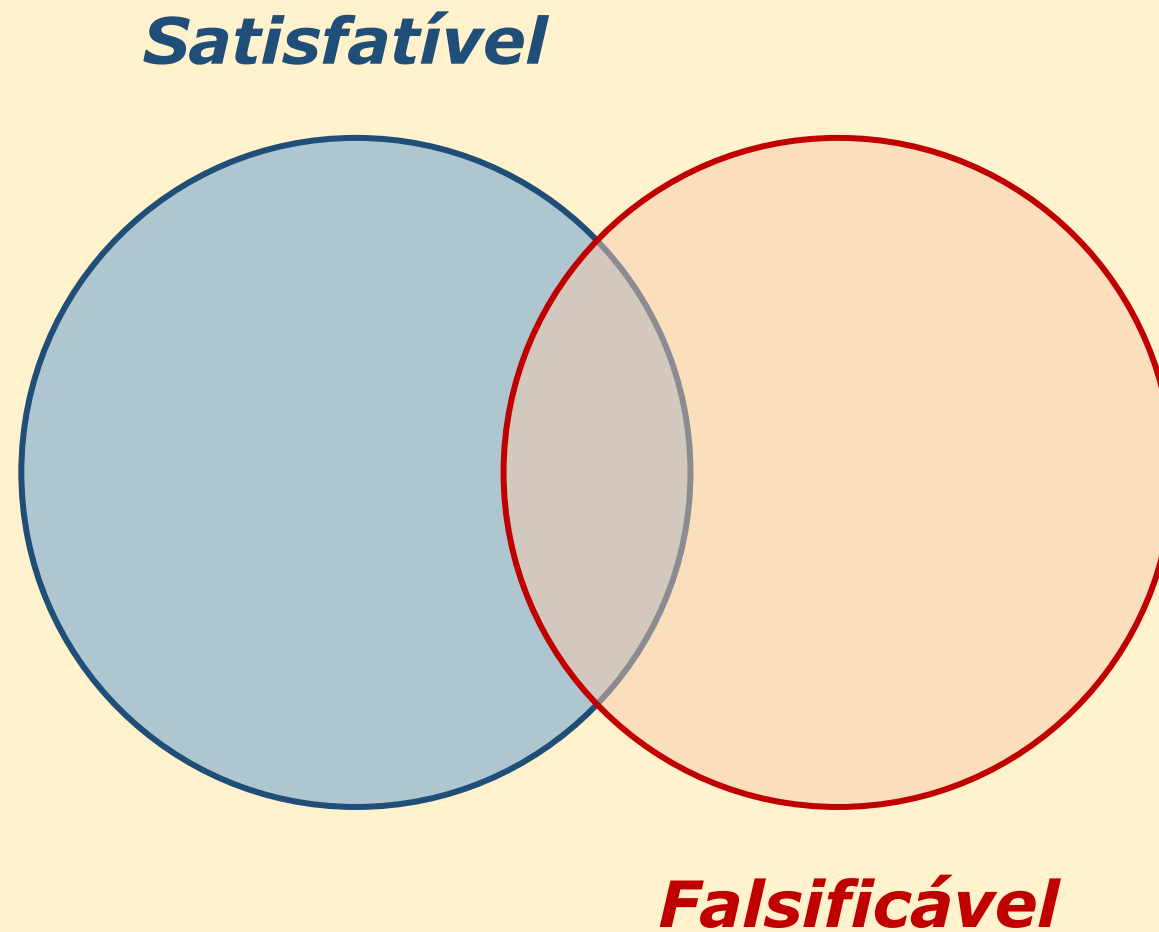
- ***A não é Válida;***

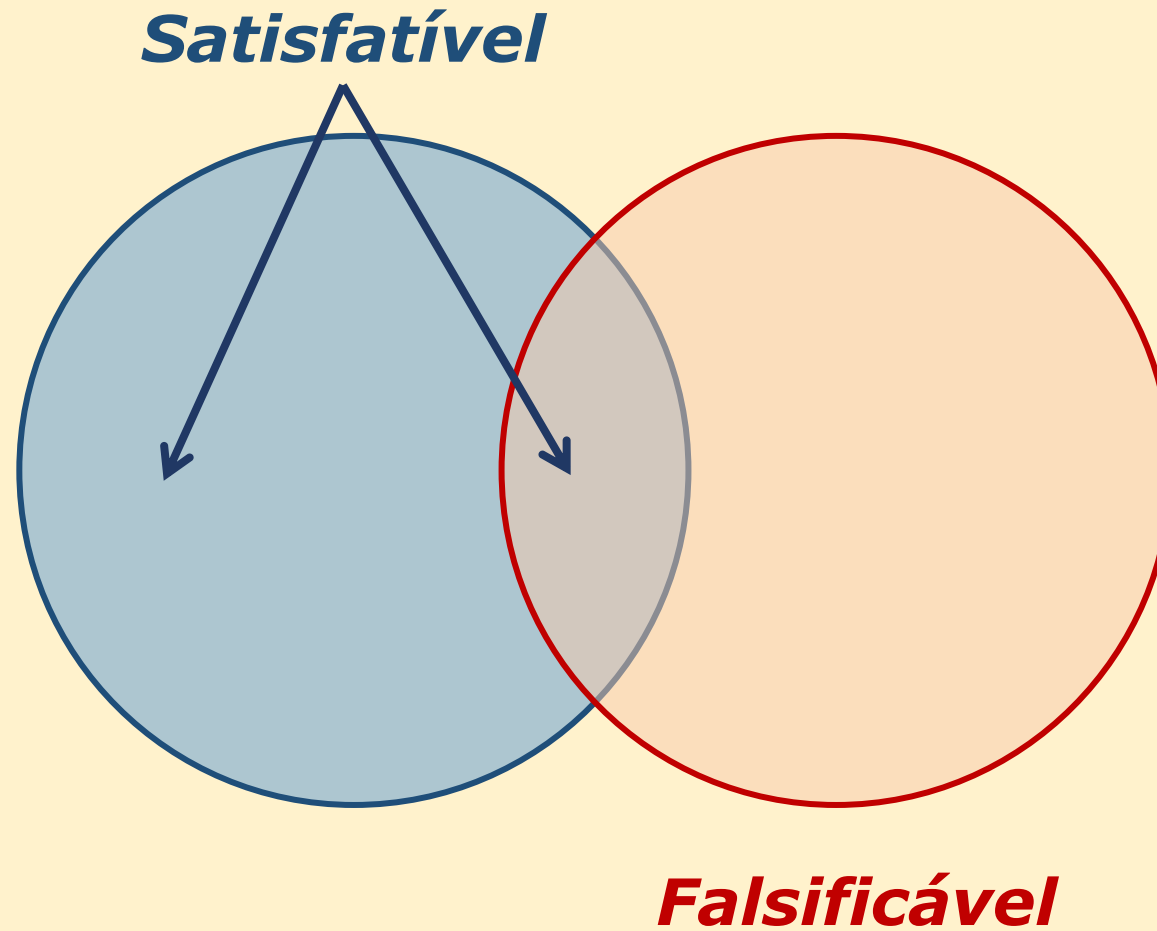
A é *Falsificável*, $\models$, sse:

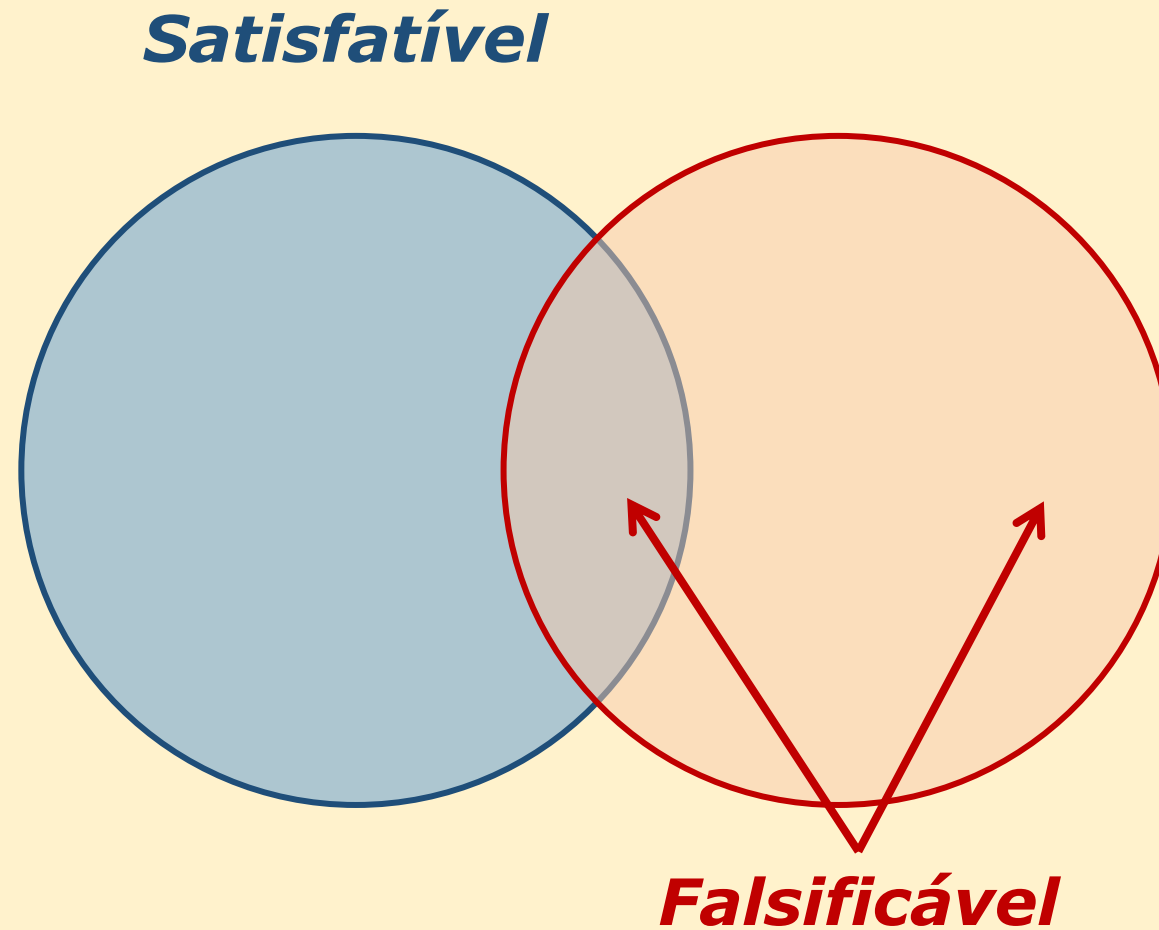
- **A não é *Válida*;**
- **$(A) = F$, para alguma interpretação.**

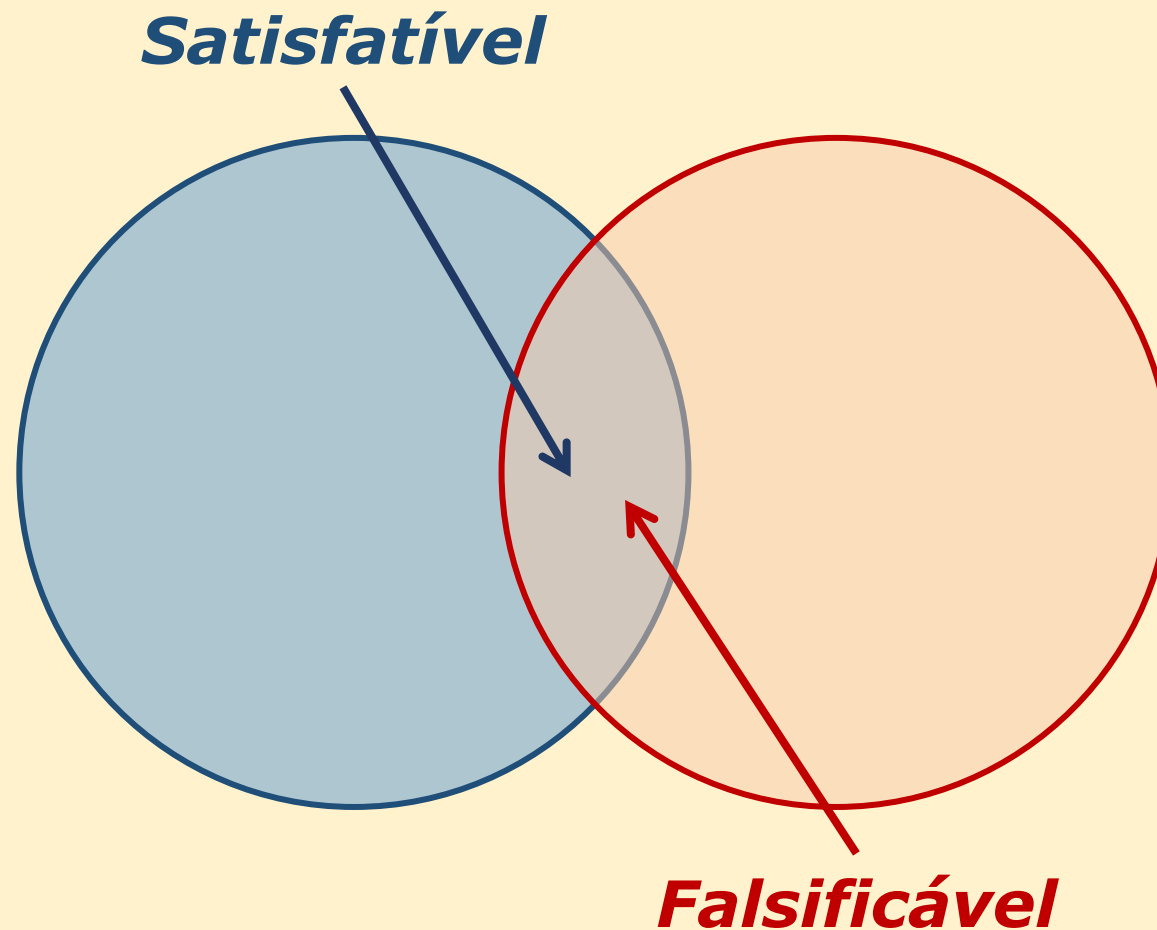


Falsificável









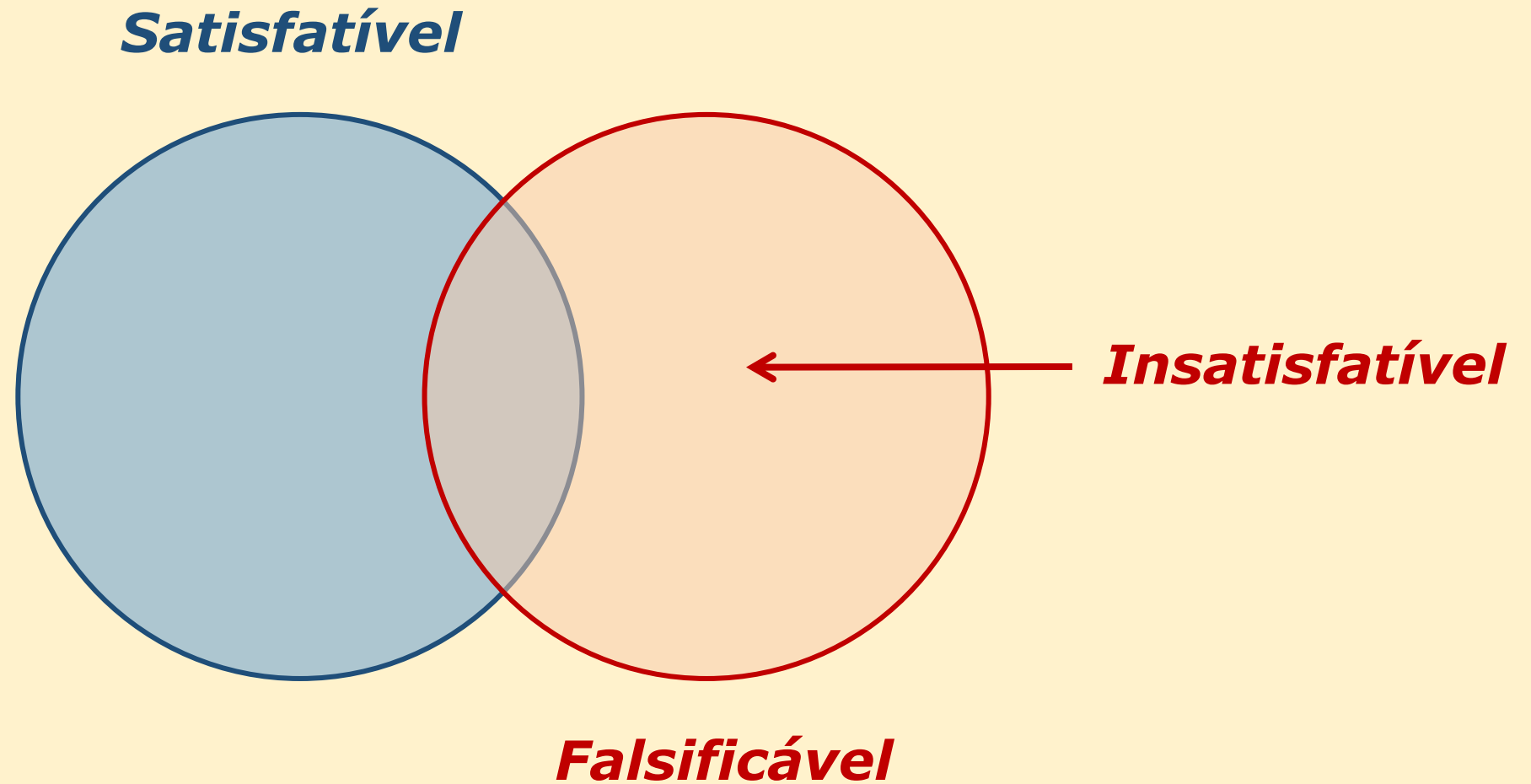
A é *Insatisfatível* sse:

A é *Insatisfatível* sse:

- **A não for *Satisfatível*;**

A é **Insatisfável** sse:

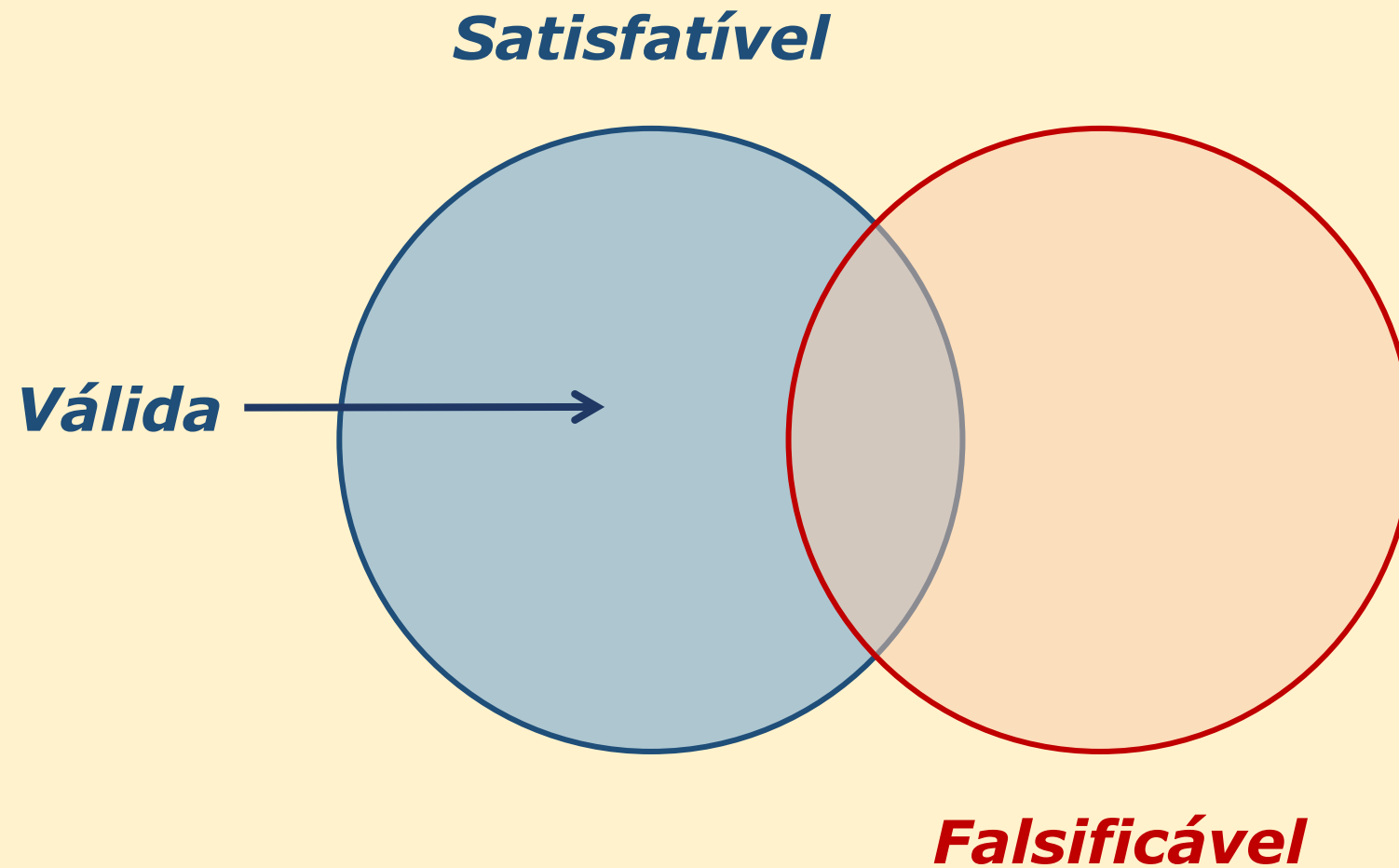
- ***A não for Satisfável;***
- ***(A) = F, para todas as interpretações.***

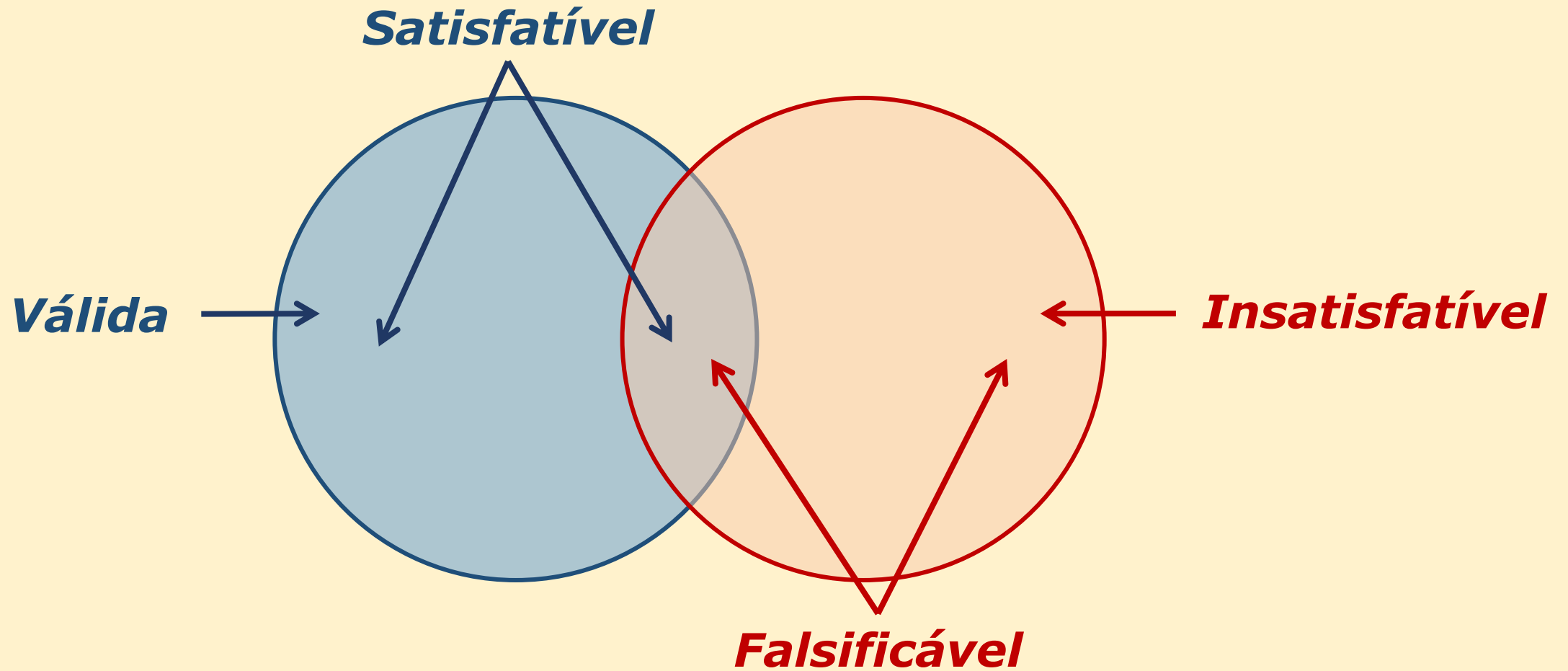


A é Válida, $\models$, sse:

A é Válida, $\models$, sse:

- ***$(A) = V$, para todas as interpretações.***





Pra decidir se A é **Válida**:

Pra decidir se A é Válida:

- ***Aplicar $\neg A$***

Pra decidir se A é Válida:

- ***Aplicar $\neg A$***
- ***Se $\neg A$ é Satisfatível, A não é Válida.***

Pra decidir se A é **Válida**:

- ***Aplicar $\neg A$***
 - ***Se $\neg A$ é **Satisfatível**, A não é **Válida**.***
 - ***Se $\neg A$ é **Insatisfatível**, A é **Válida**.***

Pra decidir se A é **Válida**, onde $A = p \rightarrow (p \vee q)$

p	q	$p \vee q$	$p \rightarrow (p \vee q)$
F	F	F	V
F	V	V	V
V	F	V	V
V	V	V	V

Pra decidir se A é **Válida**, onde $A = p \rightarrow (p \vee q)$

p	q	$p \vee q$	$p \rightarrow (p \vee q)$	$\neg(p \rightarrow (p \vee q))$
F	F	F	V	
F	V	V	V	
V	F	V	V	
V	V	V	V	

Pra decidir se A é **Válida**, onde $A = p \rightarrow (p \vee q)$

p	q	$p \vee q$	$p \rightarrow (p \vee q)$	$\neg(p \rightarrow (p \vee q))$
F	F	F	V	F
F	V	V	V	F
V	F	V	V	F
V	V	V	V	F

Pra decidir se A é **Válida**, onde $A = (p \vee q) \rightarrow p$

p	q	$p \vee q$	$(p \vee q) \rightarrow p$
F	F	F	V
F	V	V	F
V	F	V	V
V	V	V	V

Pra decidir se A é **Válida**, onde $A = (p \vee q) \rightarrow p$

p	q	$p \vee q$	$(p \vee q) \rightarrow p$	$\neg((p \vee q) \rightarrow p)$
F	F	F	V	
F	V	V	F	
V	F	V	V	
V	V	V	V	

Pra decidir se A é **Válida**, onde $A = (p \vee q) \rightarrow p$

p	q	$p \vee q$	$(p \vee q) \rightarrow p$	$\neg((p \vee q) \rightarrow p)$
F	F	F	V	F
F	V	V	F	V
V	F	V	V	F
V	V	V	V	F

A top-down view of a wooden desk. On the right side, there is a black laptop with its keyboard visible. Below the laptop, a pair of blue-rimmed glasses lies on the desk. To the left of the glasses is a black pen. Further down and to the right is a black smartphone. The text is overlaid on the left side of the image.

Lógica Computacional
by Professor Pantoja

Satisfatibilidade de Um Conjunto

Aula 7

$$U = \{A_1, A_2, A_3, \dots, A_n\}$$

$$U = \{A_1, A_2, A_3, \dots, A_n\}$$

- **É simultaneamente Satisfatível sse**
 $A_i = V$ para todo i

$$U = \{A_1, A_2, A_3, \dots, A_n\}$$

- É simultaneamente **Satisfatível** sse
 $A_i = V$ para todo i
- É **Insatisfatível** sse para cada interpretação existe um i
 $A_i = F$

$$U_1 = \{p, \neg p \vee q, q \wedge r\}$$

$$U_2 = \{p, \neg p \vee q, \neg p\}$$

$$p = V$$

$$q = V$$

$$r = V$$

$$U_1 = \{p, \neg p \vee q, q \wedge r\}$$

$$U_2 = \{p, \neg p \vee q, \neg p\}$$

$$U_1 = \{V, \neg V \vee V, V \wedge V\}$$

$$p = V$$

$$q = V$$

$$r = V$$

$$U_1 = \{p, \neg p \vee q, q \wedge r\}$$

$$U_1 = \{V, \neg V \vee V, V \wedge V\}$$

$$U_1 = \{V, F \vee V, V\}$$

$$U_2 = \{p, \neg p \vee q, \neg p\}$$

$$p = V$$

$$q = V$$

$$r = V$$

$$U_1 = \{p, \neg p \vee q, q \wedge r\}$$

$$U_1 = \{V, \neg V \vee V, V \wedge V\}$$

$$U_1 = \{V, F \vee V, V\}$$

$$U_1 = \{V, V, V\}$$

$$U_2 = \{p, \neg p \vee q, \neg p\}$$

$$p = V$$

$$q = V$$

$$r = V$$

$$U_1 = \{p, \neg p \vee q, q \wedge r\}$$

$$U_1 = \{V, \neg V \vee V, V \wedge V\}$$

$$U_1 = \{V, F \vee V, V\}$$

$$U_1 = \{V, V, V\}$$

$$U_2 = \{p, \neg p \vee q, \neg p\}$$

$$U_2 = \{V, \neg V \vee V, \neg V\}$$

$$p = V$$

$$q = V$$

$$r = V$$

$$U_1 = \{p, \neg p \vee q, q \wedge r\}$$

$$U_1 = \{V, \neg V \vee V, V \wedge V\}$$

$$U_1 = \{V, F \vee V, V\}$$

$$U_1 = \{V, V, V\}$$

$$U_2 = \{p, \neg p \vee q, \neg p\}$$

$$U_2 = \{V, \neg V \vee V, \neg V\}$$

$$U_2 = \{V, F \vee V, F\}$$

$$p = V$$

$$q = V$$

$$r = V$$

$$U_1 = \{p, \neg p \vee q, q \wedge r\}$$

$$U_1 = \{V, \neg V \vee V, V \wedge V\}$$

$$U_1 = \{V, F \vee V, V\}$$

$$U_1 = \{V, V, V\}$$

$$U_2 = \{p, \neg p \vee q, \neg p\}$$

$$U_2 = \{V, \neg V \vee V, \neg V\}$$

$$U_2 = \{V, F \vee V, F\}$$

$$U_2 = \{V, V, F\}$$

$$p = V$$

$$q = V$$

$$r = V$$

$$U_1 = \{p, \neg p \vee q, q \wedge r\}$$

$$U_1 = \{V, \neg V \vee V, V \wedge V\}$$

$$U_1 = \{V, F \vee V, V\}$$

$$U_1 = \{V, V, V\}$$

$$U_2 = \{p, \neg p \vee q, \neg p\}$$

$$U_2 = \{V, \neg V \vee V, \neg V\}$$

$$U_2 = \{V, F \vee V, F\}$$

$$U_2 = \{V, V, F\}$$

$$\begin{aligned} p &= V \\ q &= V \\ r &= V \end{aligned}$$

→ **Satisfatível**

$$U_1 = \{p, \neg p \vee q, q \wedge r\}$$

$$U_1 = \{V, \neg V \vee V, V \wedge V\}$$

$$U_1 = \{V, F \vee V, V\}$$

$$U_1 = \{V, V, V\}$$

$$U_2 = \{p, \neg p \vee q, \neg p\}$$

$$U_2 = \{V, \neg V \vee V, \neg V\}$$

$$U_2 = \{V, F \vee V, F\}$$

$$U_2 = \{V, V, F\}$$

$$\begin{aligned} p &= V \\ q &= V \\ r &= V \end{aligned}$$

Insatisfável

Satisfável

1. Se U é Satisfatível, $U \models A_i$ para todo i

1. Se U é Satisfatível, $U \models A_i$ para todo i

$$U_1 = \{p, \neg p \vee q, q \wedge r\}$$

$$U_1 = \{V, V, V\}$$

1. Se U é Satisfatível, $U \models \{A_i\}$ para todo i

A_0



$U_1 = \{p, \neg p \vee q, q \wedge r\}$

$U_1 = \{V, V, V\}$

1. Se U é Satisfatível, $U \models \{A_i\}$ para todo i

$$\begin{array}{cc} A_0 & A_1 \\ \uparrow & \uparrow \\ U_1 = \{p, \neg p \vee q, q \wedge r\} \\ U_1 = \{V, V, V\} \end{array}$$

1. Se U é Satisfatível, $U \models \{A_i\}$ para todo i

$$\begin{array}{ccc} A_0 & A_1 & A_2 \\ \uparrow & \uparrow & \uparrow \\ U_1 = \{p, \neg p \vee q, q \wedge r\} \\ U_1 = \{V, V, V\} \end{array}$$

1. Se U é Satisfatível, $U \models \{A_i\}$ para todo i

$$\begin{array}{ccc}
 A_0 & A_1 & A_2 \\
 \uparrow & \uparrow & \uparrow \\
 U_1 = \{p, \neg p \vee q, q \wedge r\} & & \left[- \{A_i\} \right] \\
 U_1 = \{V, V, V\} & &
 \end{array}$$

1. Se U é Satisfatível, $U \models \{A_i\}$ para todo i

$$\begin{array}{ccc}
 A_0 & A_1 & A_2 \\
 \uparrow & \uparrow & \uparrow \\
 U_1 = \{p, \neg p \vee q, q \wedge r\} & \left[-\{A_i\} \right] & U_1 = \{p, \neg p \vee q, q \wedge r\} \\
 U_1 = \{V, V, V\} & & U_1 = \{V, \text{F}, V\}
 \end{array}$$

1. Se U é Satisfatível, $U \models \{A_i\}$ para todo i

$$\begin{array}{ccc}
 \begin{array}{c} A_0 \\ \uparrow \\ U_1 = \{p, \neg p \vee q, q \wedge r\} \\ U_1 = \{V, V, V\} \end{array} & \left[- \{A_i\} \right] & \begin{array}{c} A_0 \quad A_1 \\ \uparrow \quad \uparrow \\ U_1 = \{p, q \wedge r\} \\ U_1 = \{V, V\} \end{array}
 \end{array}$$

- 1. Se U é Satisfatível, $U - \{A_i\}$ para todo i**
- 2. Se U é Satisfatível e B é Válido, $U \cup \{B\}$ é SAT**

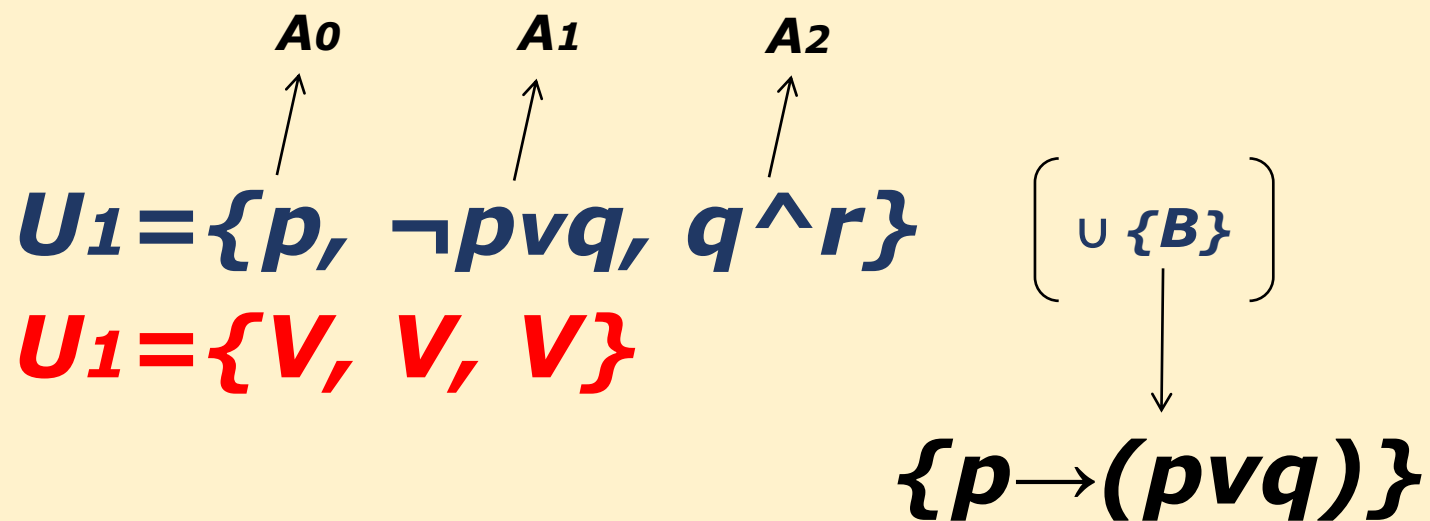
1. Se U é Satisfatível, $U - \{A_i\}$ para todo i
2. Se U é Satisfatível e B é Válido, $U \cup \{B\}$ é SAT

$$\begin{array}{ccc} A_0 & A_1 & A_2 \\ \uparrow & \uparrow & \uparrow \\ U_1 = \{p, \neg p \vee q, q \wedge r\} \\ U_1 = \{V, V, V\} \end{array}$$

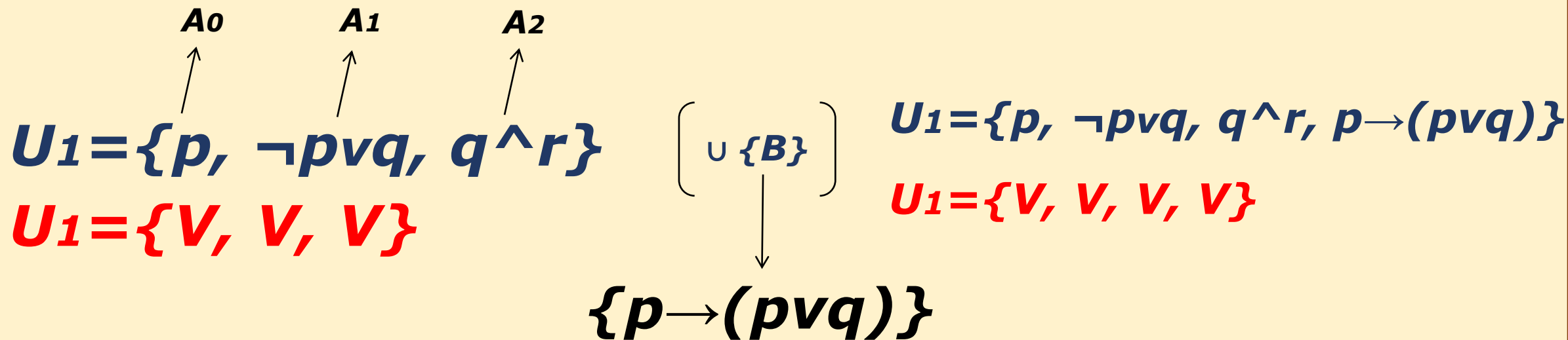
1. Se U é Satisfatível, $U \models \{A_i\}$ para todo i
2. Se U é Satisfatível e B é Válido, $U \cup \{B\}$ é SAT

$$\begin{array}{ccc}
 A_0 & A_1 & A_2 \\
 \uparrow & \uparrow & \uparrow \\
 U_1 = \{p, \neg p \vee q, q \wedge r\} & & \left(\cup \{B\} \right) \\
 U_1 = \{V, V, V\} & &
 \end{array}$$

1. Se U é Satisfatível, $U \models A_i$ para todo i
2. Se U é Satisfatível e B é Válido, $U \cup \{B\}$ é SAT



1. Se U é Satisfatível, $U \models \{A_i\}$ para todo i
2. Se U é Satisfatível e B é Válido, $U \cup \{B\}$ é SAT



1. Se U é Satisfatível, $U - \{A_i\}$ para todo i
2. Se U é Satisfatível e B é Válido, $U \cup \{B\}$ é SAT
3. Se U é **Insatisfatível**, então para $U \cup \{B\}$ é **UNSAT**

1. Se U é Satisfatível, $U - \{A_i\}$ para todo i
2. Se U é Satisfatível e B é Válido, $U \cup \{B\}$ é SAT
3. Se U é **Insatisfatível**, então para $U \cup \{B\}$ é **UNSAT**

$$U_2 = \{p, \neg p \vee q, \neg p\}$$

$$U_2 = \{V, V, F\}$$

1. Se U é Satisfatível, $U - \{A_i\}$ para todo i
2. Se U é Satisfatível e B é Válido, $U \cup \{B\}$ é SAT
3. Se U é **Insatisfatível**, então para $U \cup \{B\}$ é **UNSAT**

$$\begin{array}{lcl}
 U_2 = \{p, \neg p \vee q, \neg p\} & \left(\cup \{B\} \right) & \\
 U_2 = \{V, V, F\} & \downarrow & \\
 & \{p \rightarrow (p \vee q)\} &
 \end{array}$$

1. Se U é Satisfatível, $U - \{A_i\}$ para todo i
2. Se U é Satisfatível e B é Válido, $U \cup \{B\}$ é SAT
3. Se U é **Insatisfatível**, então para $U \cup \{B\}$ é **UNSAT**

$$\begin{array}{ccc}
 U_2 = \{p, \neg p \vee q, \neg p\} & \left(\cup \{B\} \right) & U_2 = \{p, \neg p \vee q, \neg p, p \rightarrow (p \vee q)\} \\
 U_2 = \{V, V, F\} & \downarrow & U_2 = \{V, V, F, V\} \\
 & \{p \rightarrow (p \vee q)\} &
 \end{array}$$

1. Se U é Satisfatível, $U - \{A_i\}$ para todo i
2. Se U é Satisfatível e B é Válido, $U \cup \{B\}$ é SAT
3. Se U é Insatisfatível, então para $U \cup \{B\}$ é UNSAT
4. Se U é Insatisfatível e A_i é Válido, $U - \{A_i\}$ é UNSAT

1. Se U é Satisfatível, $U - \{A_i\}$ para todo i
2. Se U é Satisfatível e B é Válido, $U \cup \{B\}$ é SAT
3. Se U é Insatisfatível, então para $U \cup \{B\}$ é UNSAT
4. Se U é Insatisfatível e A_i é Válido, $U - \{A_i\}$ é UNSAT

$$U_3 = \{p, \neg p \vee q, \neg p, p \rightarrow (p \vee q)\}$$

$$U_3 = \{V, V, F, V\}$$

1. Se U é Satisfatível, $U - \{A_i\}$ para todo i
2. Se U é Satisfatível e B é Válido, $U \cup \{B\}$ é SAT
3. Se U é Insatisfatível, então para $U \cup \{B\}$ é UNSAT
4. Se U é Insatisfatível e A_i é Válido, $U - \{A_i\}$ é UNSAT

$$\begin{array}{cccc} A_0 & A_1 & A_2 & A_3 \\ \uparrow & \uparrow & \uparrow & \uparrow \\ U_3 = \{p, \neg p \vee q, \neg p, p \rightarrow (p \vee q)\} \\ U_3 = \{V, V, F, V\} \end{array}$$

1. Se U é Satisfatível, $U - \{A_i\}$ para todo i
2. Se U é Satisfatível e B é Válido, $U \cup \{B\}$ é SAT
3. Se U é **Insatisfatível**, então para $U \cup \{B\}$ é **UNSAT**
4. Se U é **Insatisfatível** e A_i é Válido, $U - \{A_i\}$ é **UNSAT**

$$\begin{array}{cccc}
 A_0 & A_1 & A_2 & A_3 \\
 \uparrow & \uparrow & \uparrow & \uparrow \\
 U_3 = \{p, \neg p \vee q, \neg p, p \rightarrow (p \vee q)\} & \left(- \{A_i\} \right) \\
 U_3 = \{V, V, F, V\}
 \end{array}$$

1. Se U é Satisfatível, $U - \{A_i\}$ para todo i
2. Se U é Satisfatível e B é Válido, $U \cup \{B\}$ é SAT
3. Se U é **Insatisfatível**, então para $U \cup \{B\}$ é **UNSAT**
4. Se U é **Insatisfatível** e A_i é Válido, $U - \{A_i\}$ é **UNSAT**

$$\begin{array}{ccc}
 \begin{array}{cccc}
 A_0 & A_1 & A_2 & A_3 \\
 \uparrow & \uparrow & \uparrow & \uparrow \\
 U_3 = \{p, \neg p \vee q, \neg p, p \rightarrow (p \vee q)\} \\
 U_3 = \{V, V, F, V\}
 \end{array}
 & \left[- \{A_i\} \right] &
 \begin{array}{cccc}
 A_0 & A_1 & A_2 & A_3 \\
 \uparrow & \uparrow & \uparrow & \uparrow \\
 U_3 = \{p, \neg p \vee q, \neg p, p \rightarrow (p \vee q)\} \\
 U_3 = \{V, V, F, \text{?}\}
 \end{array}
 \end{array}$$

1. Se U é Satisfatível, $U - \{A_i\}$ para todo i
2. Se U é Satisfatível e B é Válido, $U \cup \{B\}$ é SAT
3. Se U é **Insatisfatível**, então para $U \cup \{B\}$ é **UNSAT**
4. Se U é **Insatisfatível** e A_i é Válido, $U - \{A_i\}$ é **UNSAT**

$$\begin{array}{ccc}
 A_0 & A_1 & A_2 & A_3 \\
 \uparrow & \uparrow & \uparrow & \uparrow \\
 U_3 = \{p, \neg p \vee q, \neg p, p \rightarrow (p \vee q)\} & \left[- \{A_i\} \right] & U_3 = \{p, \neg p \vee q, \neg p\} \\
 U_3 = \{V, V, F, V\} & & U_3 = \{V, V, F\}
 \end{array}$$

A top-down view of a wooden desk. On the right side, there is a black laptop with its keyboard visible. Below the laptop, a pair of blue-rimmed glasses lies on the desk. To the left of the glasses, a black pen is positioned diagonally. Further down and to the right, a black smartphone is lying flat. The background is the natural wood grain of the desk.

Lógica Computacional
by Professor Pantoja

Equivalência Lógica

Aula 8

$$\mathbf{fml \equiv fml}$$

$fml \equiv fml$

Indica que uma fórmula tem o mesmo resultado lógico que a outra

$fml \equiv fml$ 

*Indica que uma fórmula tem o mesmo
resultado lógico que a outra*

$fml \equiv fml$  **$fml \leftrightarrow fml$**

*Indica que uma fórmula tem o mesmo
resultado lógico que a outra*

$fml \equiv fml$

Indica que uma fórmula tem o mesmo resultado lógico que a outra

 **$fml \leftrightarrow fml$**

Sse, for uma tautologia ou válida.

$$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$$

$$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$$

p	q
F	F
F	V
V	F
V	V

$$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$$

p	q	p ∨ q
F	F	
F	V	
V	F	
V	V	

$$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$$

p	q	p ∨ q
F	F	F
F	V	V
V	F	V
V	V	V

$$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$$

p	q	p ∨ q	¬(p ∨ q)
F	F	F	
F	V	V	
V	F	V	
V	V	V	

$$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$$

p	q	p ∨ q	¬(p ∨ q)
F	F	F	V
F	V	V	F
V	F	V	F
V	V	V	F

$$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$$

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$
F	F	F	V	V	V
F	V	V	F	V	F
V	F	V	F	F	V
V	V	V	F	F	F

$$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$$

p	q	$p \vee q$	$\neg(p \vee q)$	$\neg p$	$\neg q$
F	F	F	V	V	V
F	V	V	F	V	F
V	F	V	F	F	V
V	V	V	F	F	F

$$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$$

p	q	p ∨ q	¬(p ∨ q)	¬p	¬q	¬p ∧ ¬q
F	F	F	V	V	V	
F	V	V	F	V	F	
V	F	V	F	F	V	
V	V	V	F	F	F	

$$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$$

p	q	p ∨ q	¬(p ∨ q)	¬p	¬q	¬p ∧ ¬q
F	F	F	V	V	V	V
F	V	V	F	V	F	F
V	F	V	F	F	V	F
V	V	V	F	F	F	F

$$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$$

p	q	p ∨ q	¬(p ∨ q)	¬p	¬q	¬p ∧ ¬q	¬(p ∨ q) ↔ ¬p ∧ ¬q
F	F	F	V	V	V	V	
F	V	V	F	V	F	F	
V	F	V	F	F	V	F	
V	V	V	F	F	F	F	

$$\neg(p \vee q) \leftrightarrow \neg p \wedge \neg q$$

p	q	p ∨ q	¬(p ∨ q)	¬p	¬q	¬p ∧ ¬q	¬(p ∨ q) ↔ ¬p ∧ ¬q
F	F	F	V	V	V	V	V
F	V	V	F	V	F	F	V
V	F	V	F	F	V	F	V
V	V	V	F	F	F	F	V

$$a \equiv \neg \neg a$$

$$a \equiv \neg \neg a$$

a	$\neg a$
F	
V	

$$a \equiv \neg \neg a$$

a	$\neg a$
F	V
V	F

$$a \equiv \neg \neg a$$

a	$\neg a$	$\neg \neg a$
F	V	
V	F	

$$a \equiv \neg \neg a$$

a	$\neg a$	$\neg \neg a$
F	V	F
V	F	V

$$a \equiv \neg\neg a$$

a	$\neg a$	$\neg\neg a$	$\neg\neg\neg a$
F	V	F	
V	F	V	

$$a \equiv \neg\neg a$$

a	$\neg a$	$\neg\neg a$	$\neg\neg\neg a$
F	V	F	V
V	F	V	F

$$a \equiv \neg\neg a$$

a	$\neg a$	$\neg\neg a$	$\neg\neg\neg a$
F	V	F	V
V	F	V	F

$$a \equiv \neg \neg a$$

a	$\neg a$	$\neg \neg a$	$\neg \neg \neg a$
F	V	F	V
V	F	V	F

$$p \equiv p \vee p$$

$$p \equiv p \vee p$$

p	p ∨ p
F	
V	

$$p \equiv p \vee p$$

p	p ∨ p
F	F
V	V

$$p \equiv p \vee p$$

$$p \equiv p \wedge p$$

p	p ∨ p
F	F
V	V

$$p \equiv p \vee p$$

p	$p \vee p$
F	F
V	V

$$p \equiv p \wedge p$$

p	$p \wedge p$
F	
V	

$$p \equiv p \vee p$$

p	$p \vee p$
F	F
V	V

$$p \equiv p \wedge p$$

p	$p \wedge p$
F	F
V	V

$$p \vee q \equiv q \vee p$$

$$p \vee q \equiv q \vee p$$

p	q	$p \vee q$
F	F	F
F	V	V
V	F	V
V	V	V

$$p \vee q \equiv q \vee p$$

$$p \wedge q \equiv q \wedge p$$

p	q	$p \vee q$
F	F	F
F	V	V
V	F	V
V	V	V

$$p \vee q \equiv q \vee p$$

p	q	$p \vee q$
F	F	F
F	V	V
V	F	V
V	V	V

$$p \wedge q \equiv q \wedge p$$

p	q	$p \wedge q$
F	F	F
F	V	F
V	F	F
V	V	V

$$p \vee (q \vee r) \equiv (p \vee q) \vee r$$

$$p \vee (q \vee r) \equiv (p \vee q) \vee r$$

p	q	r	$p \vee (q \vee r)$	$(p \vee q) \vee r$
F	F	F		
F	F	V		
F	V	F		
F	V	V		
V	F	F		
V	F	V		
V	V	F		
V	V	V		

$$p \vee (q \vee r) \equiv (p \vee q) \vee r$$

p	q	r	$p \vee (q \vee r)$	$(p \vee q) \vee r$
F	F	F	F	
F	F	V	V	
F	V	F	V	
F	V	V	V	
V	F	F	V	
V	F	V	V	
V	V	F	V	
V	V	V	V	

$$p \vee (q \vee r) \equiv (p \vee q) \vee r$$

p	q	r	$p \vee (q \vee r)$	$(p \vee q) \vee r$
F	F	F	F	F
F	F	V	V	V
F	V	F	V	V
F	V	V	V	V
V	F	F	V	V
V	F	V	V	V
V	V	F	V	V
V	V	V	V	V

$$p \vee (q \vee r) \equiv (p \vee q) \vee r$$

$$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$$

p	q	r	$p \vee (q \vee r)$	$(p \vee q) \vee r$
F	F	F	F	F
F	F	V	V	V
F	V	F	V	V
F	V	V	V	V
V	F	F	V	V
V	F	V	V	V
V	V	F	V	V
V	V	V	V	V

$$p \vee (q \vee r) \equiv (p \vee q) \vee r$$

p	q	r	$p \vee (q \vee r)$	$(p \vee q) \vee r$
F	F	F	F	F
F	F	V	V	V
F	V	F	V	V
F	V	V	V	V
V	F	F	V	V
V	F	V	V	V
V	V	F	V	V
V	V	V	V	V

$$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$$

p	q	r	$p \wedge (q \wedge r)$	$(p \wedge q) \wedge r$
F	F	F		
F	F	V		
F	V	F		
F	V	V		
V	F	F		
V	F	V		
V	V	F		
V	V	V		

$$p \vee (q \vee r) \equiv (p \vee q) \vee r$$

p	q	r	$p \vee (q \vee r)$	$(p \vee q) \vee r$
F	F	F	F	F
F	F	V	V	V
F	V	F	V	V
F	V	V	V	V
V	F	F	V	V
V	F	V	V	V
V	V	F	V	V
V	V	V	V	V

$$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$$

p	q	r	$p \wedge (q \wedge r)$	$(p \wedge q) \wedge r$
F	F	F	F	F
F	F	V	F	F
F	V	F	F	F
F	V	V	F	F
V	F	F	F	F
V	F	V	F	F
V	V	F	F	F
V	V	V	V	V

$$p \vee (q \vee r) \equiv (p \vee q) \vee r$$

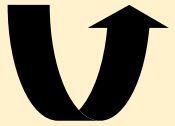
p	q	r	$p \vee (q \vee r)$	$(p \vee q) \vee r$
F	F	F	F	F
F	F	V	V	V
F	V	F	V	V
F	V	V	V	V
V	F	F	V	V
V	F	V	V	V
V	V	F	V	V
V	V	V	V	V

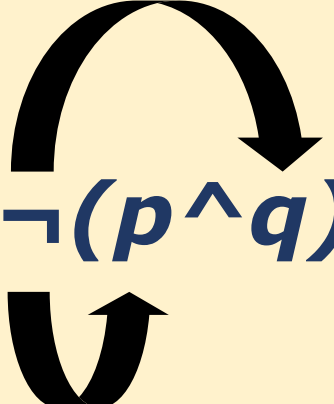
$$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$$

p	q	r	$p \wedge (q \wedge r)$	$(p \wedge q) \wedge r$
F	F	F	F	F
F	F	V	F	F
F	V	F	F	F
F	V	V	F	F
V	F	F	F	F
V	F	V	F	F
V	V	F	F	F
V	V	V	V	V

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

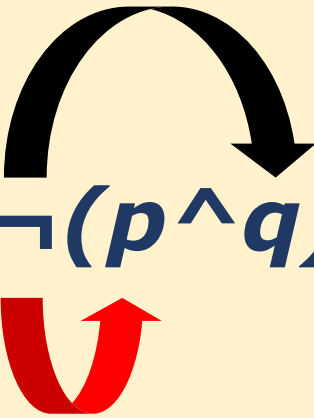
$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

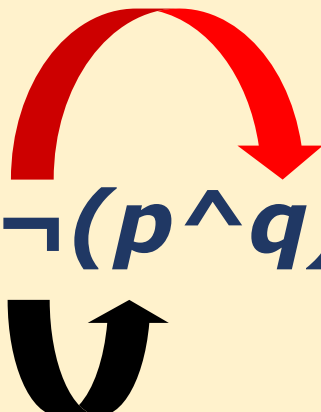


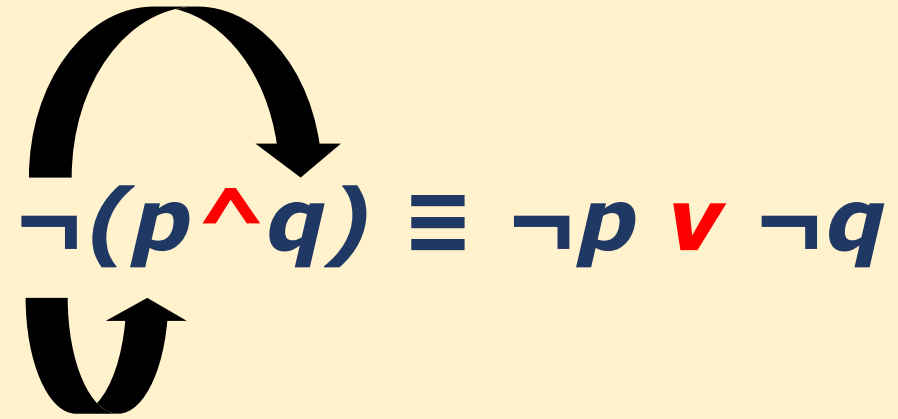


The diagram features two curved arrows forming a cycle. The top arrow starts at the expression $\neg(p \wedge q)$ and points to $\neg p \vee \neg q$. The bottom arrow starts at $\neg p \vee \neg q$ and points back to $\neg(p \wedge q)$, indicating that the two expressions are logically equivalent.

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$


$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$


$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$



The diagram illustrates De Morgan's Law with the equation $\neg(p \wedge q) \equiv \neg p \vee \neg q$. The expression is written in blue text, with the logical connectives $\wedge$ and $\vee$ highlighted in red. Two large, thick black curved arrows are positioned around the equation: one starts at the top left and points down to the right, and the other starts at the bottom left and points up to the right, framing the central text.

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(\neg p \wedge q) \equiv \neg \neg p \vee \neg q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee \neg \neg q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(\neg p \wedge \neg q) \equiv p \vee q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(\neg p \wedge \neg q) \equiv p \vee q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(\neg p \wedge \neg q) \equiv \neg \neg p \vee q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(\neg p \wedge \neg q) \equiv p \vee q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(\neg p \wedge \neg q) \equiv p \vee q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(\neg p \wedge \neg q) \equiv p \vee \neg\neg q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(\neg p \wedge \neg q) \equiv p \vee q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

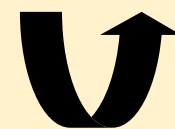
$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(\neg p \wedge \neg q) \equiv p \vee q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

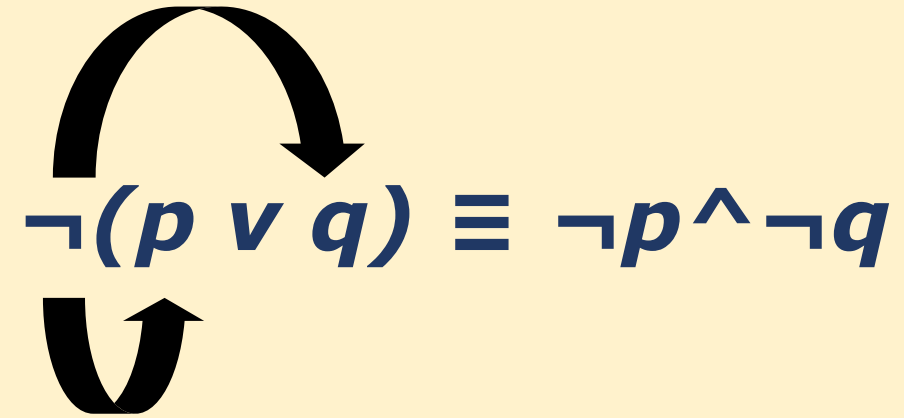


$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(\neg p \wedge \neg q) \equiv p \vee q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

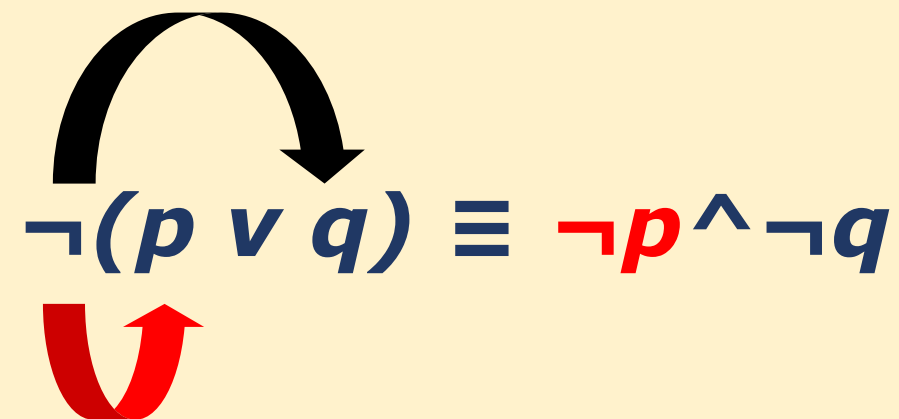

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(\neg p \wedge \neg q) \equiv p \vee q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

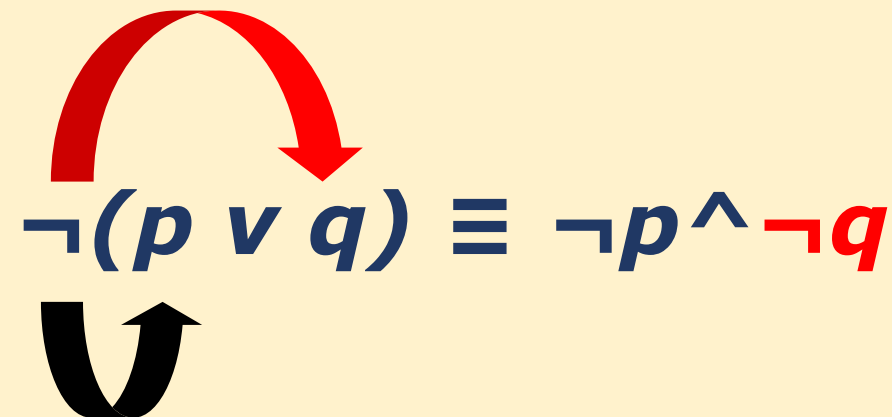

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(\neg p \wedge \neg q) \equiv p \vee q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

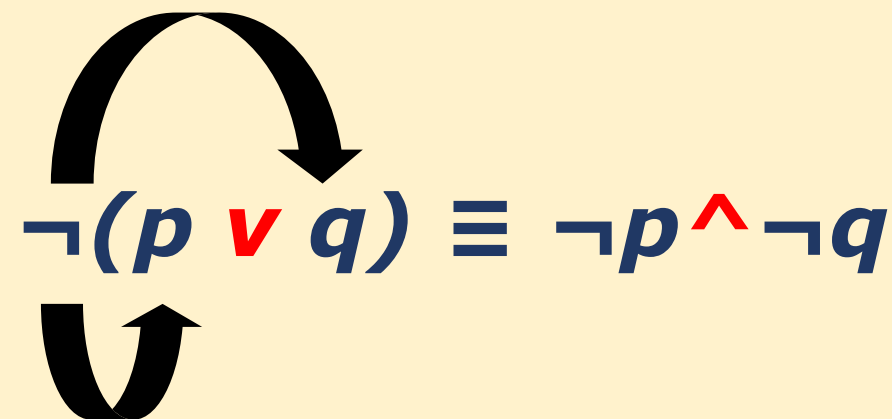

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(\neg p \wedge \neg q) \equiv p \vee q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$


$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(\neg p \wedge \neg q) \equiv p \vee q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(\neg p \vee q) \equiv p \wedge \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(\neg p \wedge \neg q) \equiv p \vee q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(\neg p \vee q) \equiv p \wedge \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(\neg p \wedge \neg q) \equiv p \vee q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(\neg p \vee q) \equiv \neg \neg p \wedge \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(\neg p \wedge \neg q) \equiv p \vee q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(\neg p \vee q) \equiv p \wedge \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(\neg p \wedge \neg q) \equiv p \vee q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(\neg p \vee q) \equiv p \wedge \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(\neg p \wedge \neg q) \equiv p \vee q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(\neg p \vee q) \equiv p \wedge \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(\neg p \wedge \neg q) \equiv p \vee q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(\neg p \vee q) \equiv p \wedge \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(p \vee \neg q) \equiv \neg p \wedge q$$

$$\neg(\neg p \wedge \neg q) \equiv p \vee q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(\neg p \vee q) \equiv p \wedge \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(\neg p \wedge \neg q) \equiv p \vee q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(\neg p \vee q) \equiv p \wedge \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(\neg p \wedge \neg q) \equiv p \vee q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(\neg p \vee q) \equiv p \wedge \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(p \vee \neg q) \equiv \neg p \wedge q$$

$$\neg(\neg p \wedge \neg q) \equiv p \vee q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(\neg p \vee q) \equiv p \wedge \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(p \vee \neg q) \equiv \neg p \wedge \neg \neg q$$

$$\neg(\neg p \wedge \neg q) \equiv p \vee q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(\neg p \vee q) \equiv p \wedge \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(p \vee \neg q) \equiv \neg p \wedge q$$

$$\neg(\neg p \wedge \neg q) \equiv p \vee q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(\neg p \vee q) \equiv p \wedge \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(p \vee \neg q) \equiv \neg p \wedge q$$

$$\neg(\neg p \wedge \neg q) \equiv p \vee q$$

$$\neg(\neg p \vee \neg q) \equiv p \wedge q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(\neg p \vee q) \equiv p \wedge \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(p \vee \neg q) \equiv \neg p \wedge q$$

$$\neg(\neg p \wedge \neg q) \equiv p \vee q$$

$$\neg(\neg p \vee \neg q) \equiv p \wedge q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(\neg p \vee q) \equiv p \wedge \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(p \vee \neg q) \equiv \neg p \wedge q$$

$$\neg(\neg p \wedge \neg q) \equiv p \vee q$$

$$\neg(\neg p \vee \neg q) \equiv \neg \neg p \wedge q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(\neg p \vee q) \equiv p \wedge \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(p \vee \neg q) \equiv \neg p \wedge q$$

$$\neg(\neg p \wedge \neg q) \equiv p \vee q$$

$$\neg(\neg p \vee \neg q) \equiv p \wedge q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(\neg p \vee q) \equiv p \wedge \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(p \vee \neg q) \equiv \neg p \wedge q$$

$$\neg(\neg p \wedge \neg q) \equiv p \vee q$$

$$\neg(\neg p \vee \neg q) \equiv p \wedge q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(\neg p \vee q) \equiv p \wedge \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(p \vee \neg q) \equiv \neg p \wedge q$$

$$\neg(\neg p \wedge \neg q) \equiv p \vee q$$

$$\neg(\neg p \vee \neg q) \equiv p \wedge q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(\neg p \vee q) \equiv p \wedge \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(p \vee \neg q) \equiv \neg p \wedge q$$

$$\neg(\neg p \wedge \neg q) \equiv p \vee q$$

$$\neg(\neg p \vee \neg q) \equiv p \wedge q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

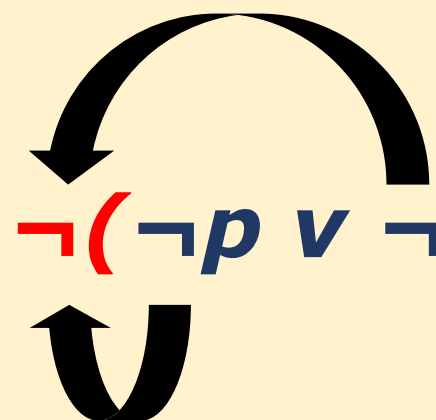
$$\neg(\neg p \vee q) \equiv p \wedge \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(p \vee \neg q) \equiv \neg p \wedge q$$

$$\neg(\neg p \wedge \neg q) \equiv p \vee q$$

$$\neg(\neg p \vee \neg q) \equiv p \wedge q$$

$$\neg(p \wedge q) \equiv \neg(\neg p \vee \neg q)$$


$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(\neg p \vee q) \equiv p \wedge \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(p \vee \neg q) \equiv \neg p \wedge q$$

$$\neg(\neg p \wedge \neg q) \equiv p \vee q$$

$$\neg(\neg p \vee \neg q) \equiv p \wedge q$$

$$\neg(p \wedge q) \equiv \neg(p \vee q)$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(\neg p \vee q) \equiv p \wedge \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(p \vee \neg q) \equiv \neg p \wedge q$$

$$\neg(\neg p \wedge \neg q) \equiv p \vee q$$

$$\neg(\neg p \vee \neg q) \equiv p \wedge q$$

$$\neg(p \wedge q) \equiv \neg(p \wedge q)$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(\neg p \vee q) \equiv p \wedge \neg q$$

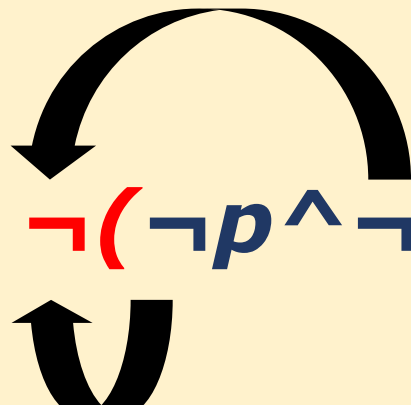
$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(p \vee \neg q) \equiv \neg p \wedge q$$

$$\neg(\neg p \wedge \neg q) \equiv p \vee q$$

$$\neg(\neg p \vee \neg q) \equiv p \wedge q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg(\neg p \wedge \neg q)$$


$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(\neg p \wedge \neg q) \equiv p \vee q$$

$$\neg(\neg p \vee q) \equiv p \wedge \neg q$$

$$\neg(p \vee \neg q) \equiv \neg p \wedge q$$

$$\neg(\neg p \vee \neg q) \equiv p \wedge q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg(p \wedge q)$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(\neg p \vee q) \equiv p \wedge \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(p \vee \neg q) \equiv \neg p \wedge q$$

$$\neg(\neg p \wedge \neg q) \equiv p \vee q$$

$$\neg(\neg p \vee \neg q) \equiv p \wedge q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg(p \vee q)$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(\neg p \vee q) \equiv p \wedge \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

$$\neg(p \vee \neg q) \equiv \neg p \wedge q$$

$$\neg(\neg p \wedge \neg q) \equiv p \vee q$$

$$\neg(\neg p \vee \neg q) \equiv p \wedge q$$

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

$$\neg(\neg p \wedge q) \equiv p \vee \neg q$$

$$\neg(\neg p \vee q) \equiv p \wedge \neg q$$

$$\neg(p \wedge \neg q) \equiv \neg p \vee q$$

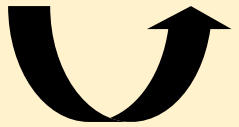
$$\neg(p \vee \neg q) \equiv \neg p \wedge q$$

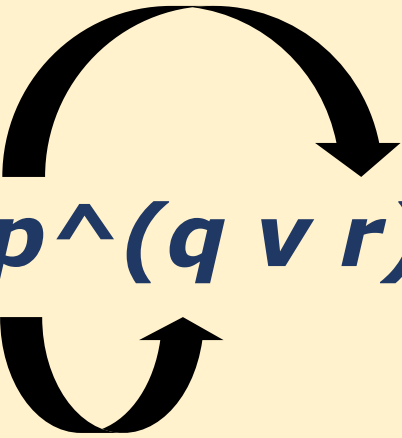
$$\neg(\neg p \wedge \neg q) \equiv p \vee q$$

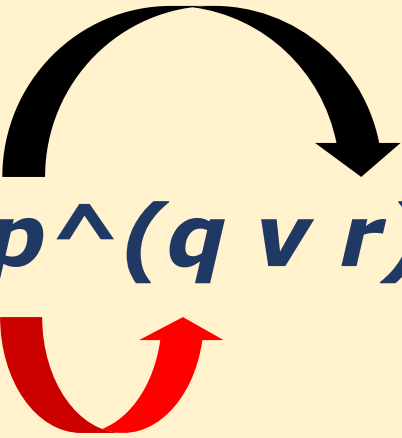
$$\neg(\neg p \vee \neg q) \equiv p \wedge q$$

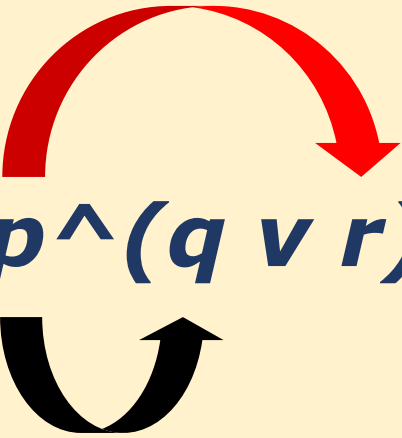
$$p \wedge (q \vee r) \equiv$$

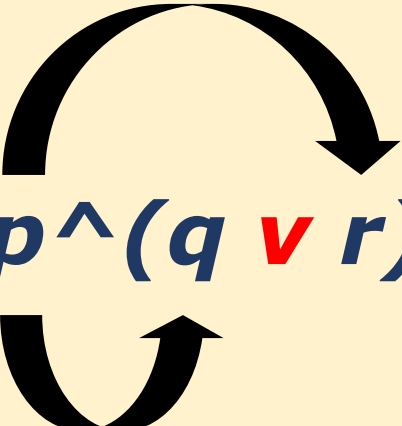
$$p \wedge (q \vee r) \equiv$$




$$p \wedge (q \vee r) \equiv$$


$$p \wedge (q \vee r) \equiv (p \wedge q)$$


$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$


$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

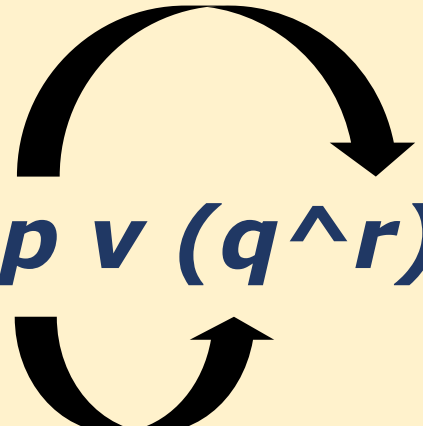
$$p \vee (q \wedge r) \equiv$$

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

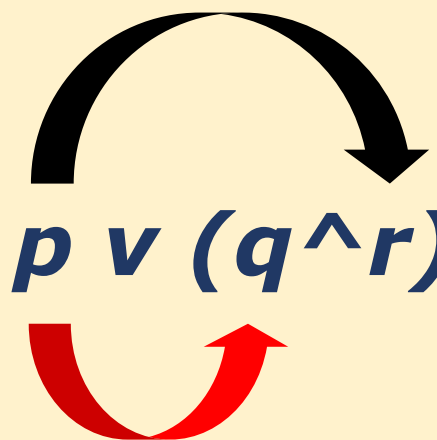
$$p \vee (q \wedge r) \equiv$$



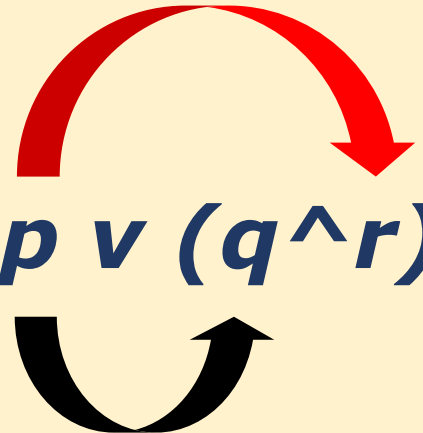
$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$


$$p \vee (q \wedge r) \equiv$$

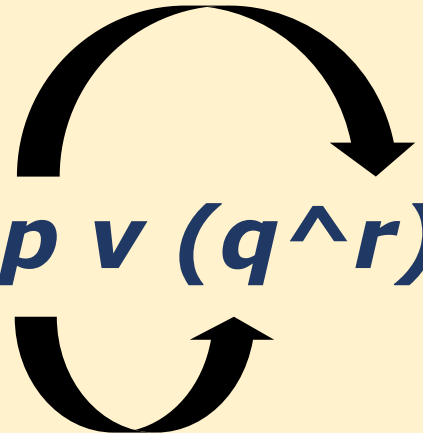
$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$


$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$


$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$


$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

$$p \rightarrow q \equiv \neg q \rightarrow \neg p$$

$$p \rightarrow q \equiv \neg q \rightarrow \neg p$$

p	q	$p \rightarrow q$
F	F	
F	V	
V	F	
V	V	

$$p \rightarrow q \equiv \neg q \rightarrow \neg p$$

p	q	$p \rightarrow q$
F	F	V
F	V	V
V	F	F
V	V	V

$$p \rightarrow q \equiv \neg q \rightarrow \neg p$$

p	q	$p \rightarrow q$	$\neg q$	$\neg p$
F	F	V		
F	V	V		
V	F	F		
V	V	V		

$$p \rightarrow q \equiv \boxed{\neg q} \rightarrow \neg p$$

p	q	$p \rightarrow q$	$\neg q$	$\neg p$
F	F	V	V	
F	V	V	F	
V	F	F	V	
V	V	V	F	

$$p \rightarrow q \equiv \neg q \rightarrow \boxed{\neg p}$$

p	q	$p \rightarrow q$	$\neg q$	$\neg p$
F	F	V	V	V
F	V	V	F	V
V	F	F	V	F
V	V	V	F	F

$$p \rightarrow q \equiv \boxed{\neg q \rightarrow \neg p}$$

p	q	$p \rightarrow q$	$\neg q$	$\neg p$	$\neg q \rightarrow \neg p$
F	F	V	V	V	
F	V	V	F	V	
V	F	F	V	F	
V	V	V	F	F	

$$p \rightarrow q \equiv \boxed{\neg q \rightarrow \neg p}$$

p	q	$p \rightarrow q$	$\neg q$	$\neg p$	$\neg q \rightarrow \neg p$
F	F	V	V	V	V
F	V	V	F	V	V
V	F	F	V	F	F
V	V	V	F	F	V

$$p \rightarrow q \equiv \neg p \vee q$$

$$p \rightarrow q \equiv \neg p \vee q$$

p	q	$p \rightarrow q$
F	F	
F	V	
V	F	
V	V	

$$p \rightarrow q \equiv \neg p \vee q$$

p	q	$p \rightarrow q$
F	F	V
F	V	V
V	F	F
V	V	V

$$p \rightarrow q \equiv \boxed{\neg p} \vee q$$

p	q	$p \rightarrow q$	$\neg p$
F	F	V	
F	V	V	
V	F	F	
V	V	V	

$$p \rightarrow q \equiv \boxed{\neg p} \vee q$$

p	q	$p \rightarrow q$	$\neg p$
F	F	V	V
F	V	V	V
V	F	F	F
V	V	V	F

$$p \rightarrow q \equiv \neg p \vee q$$

p	q	$p \rightarrow q$	$\neg p$	$\neg p \vee q$
F	F	V	V	
F	V	V	V	
V	F	F	F	
V	V	V	F	

$$p \rightarrow q \equiv \neg p \vee q$$

p	q	$p \rightarrow q$	$\neg p$	$\neg p \vee q$
F	F	V	V	V
F	V	V	V	V
V	F	F	F	F
V	V	V	F	V

A top-down view of a wooden desk. On the right side, there is a black laptop with its keyboard visible. Below the laptop, a pair of blue-rimmed glasses lies on the desk. To the left of the glasses, a black pen is positioned diagonally. Further down and to the right, a black smartphone is lying flat. The background is the natural wood grain of the desk.

Lógica Computacional
by Professor Pantoja

Tableaux Semântico

Aula 9

- ***Um conjunto de átomos é Satisfatível sse***

▪ **Um conjunto de átomos é Satisfatível sse**

i. não contém: literal e $\neg$ literal

Ex.: p e $\neg p$

▪ **Um conjunto de átomos é *Satisfatível* sse**

i. não contém: literal e $\neg$ literal

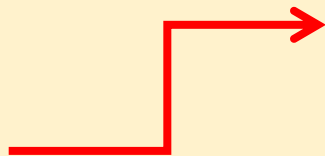
Ex.: p e $\neg p$

literais

▪ **Um conjunto de átomos é *Satisfatível* sse**

i. não contém: literal e $\neg$ literal

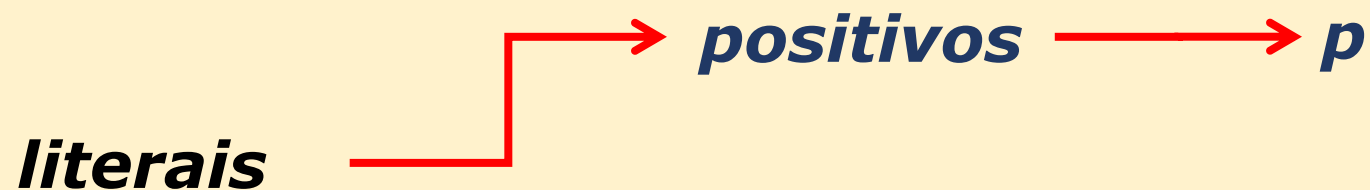
Ex.: p e $\neg p$

literais  ***positivos***

- **Um conjunto de átomos é Satisfatível sse**

i. não contém: *literal* e $\neg$ *literal*

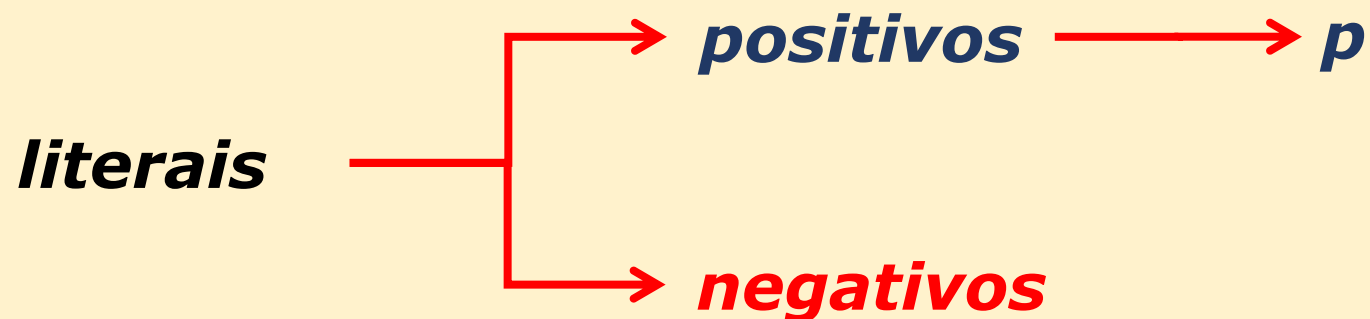
Ex.: p e $\neg p$



- Um conjunto de átomos é *Satisfatível* sse

i. não contém: *literal* e $\neg$ *literal*

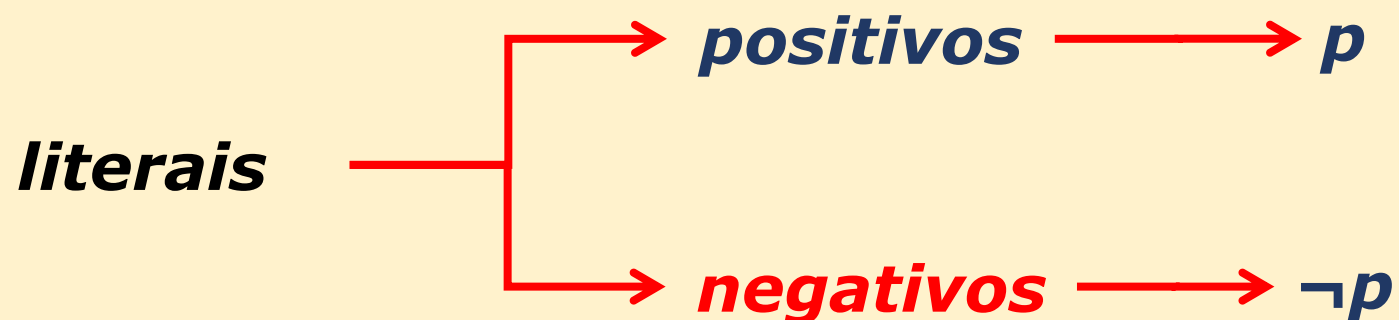
Ex.: p e $\neg p$



- Um conjunto de átomos é *Satisfatível* sse

i. não contém: *literal* e $\neg$ *literal*

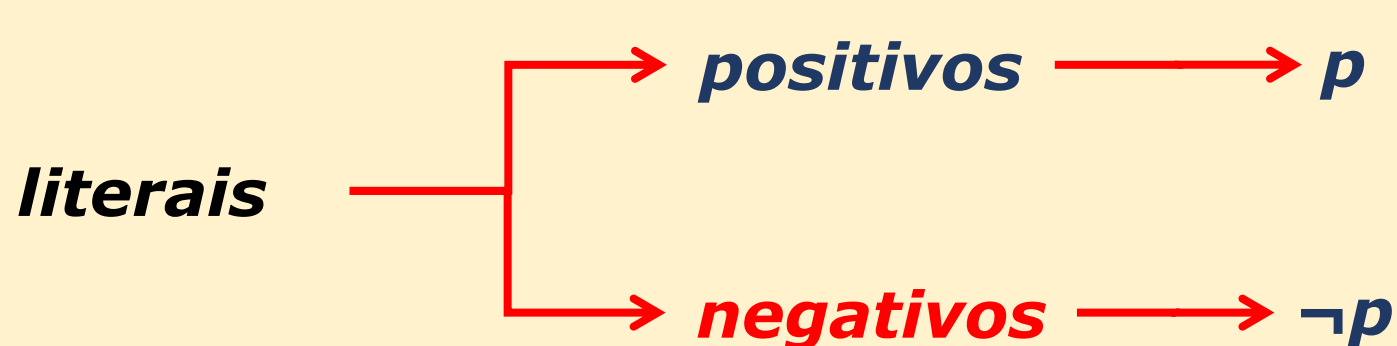
Ex.: p e $\neg p$



- Um conjunto de átomos é *Satisfatível* sse

i. não contém: *literal* e $\neg$ *literal*

Ex.: p e $\neg p$

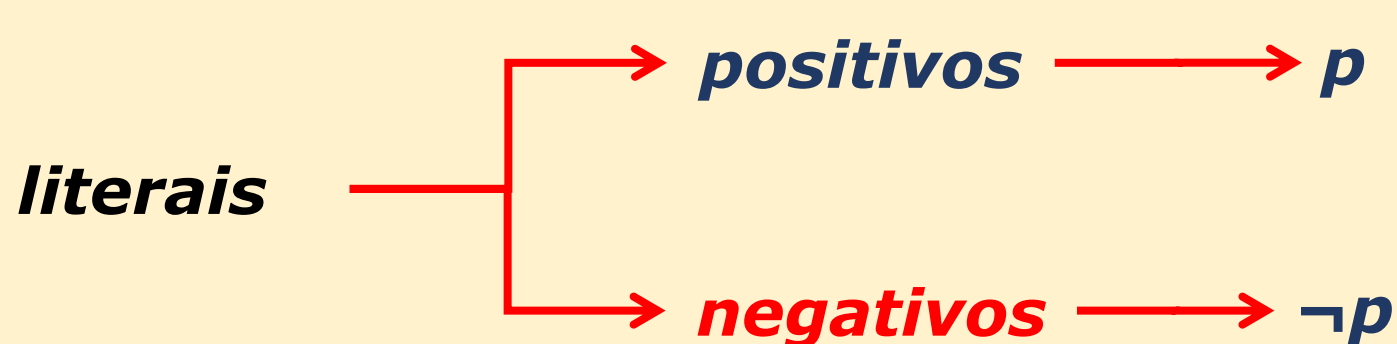


Não pode conter
um
par complementar

▪ Um conjunto de átomos é **Satisfatível** sse

i. não contém: **literal** e $\neg$ **literal**

Ex.: **p** e $\neg$ **p**



Não pode conter
um
par complementar

ii. A fml é **Satisfatível** se pelo menos
um conjunto for **Satisfatível**

α **$\alpha 1$** **$\alpha 2$** **α**

a	a1	a2
----------	-----------	-----------

a
↓
{a1, a2}

a	a1	a2
$\neg\neg A1$	A1	-

 $\neg\neg A1$

a	a1	a2
$\neg\neg A1$	A1	-

 $\neg\neg A1$  **$\{A1\}$**

a	a1	a2
$\neg\neg A1$	A1	-
$A1 \wedge A2$	A1	A2

 $A1 \wedge A2$

a	a1	a2
$\neg\neg A1$	A1	-
$A1 \wedge A2$	A1	A2

$A1 \wedge A2$

 $\{A1, A2\}$

a	a1	a2
$\neg\neg A1$	A1	-
$A1 \wedge A2$	A1	A2
$\neg(A1 \vee A2)$	$\neg A1$	$\neg A2$

 $\neg(A1 \vee A2)$

a	a1	a2
$\neg\neg A1$	A1	-
$A1 \wedge A2$	A1	A2
$\neg(A1 \vee A2)$	$\neg A1$	$\neg A2$

$\neg(A1 \vee A2)$



$\{\neg A1, \neg A2\}$

a	a1	a2
$\neg\neg A1$	A1	-
$A1 \wedge A2$	A1	A2
$\neg(A1 \vee A2)$	$\neg A1$	$\neg A2$
$\neg(A1 \rightarrow A2)$	A1	$\neg A2$

 $\neg(A1 \rightarrow A2)$

a	a1	a2
$\neg\neg A1$	$A1$	-
$A1 \wedge A2$	$A1$	$A2$
$\neg(A1 \vee A2)$	$\neg A1$	$\neg A2$
$\neg(A1 \rightarrow A2)$	$A1$	$\neg A2$

$\neg(A1 \rightarrow A2)$



$\{A1, \neg A2\}$

a	a1	a2
$\neg\neg A1$	A1	-
$A1 \wedge A2$	A1	A2
$\neg(A1 \vee A2)$	$\neg A1$	$\neg A2$
$\neg(A1 \rightarrow A2)$	A1	$\neg A2$
$\neg(A1 \uparrow A2)$	A1	A2

$\neg(A1 \uparrow A2)$

a	a1	a2
$\neg\neg A1$	A1	-
$A1 \wedge A2$	A1	A2
$\neg(A1 \vee A2)$	$\neg A1$	$\neg A2$
$\neg(A1 \rightarrow A2)$	A1	$\neg A2$
$\neg(A1 \uparrow A2)$	A1	A2

$$\neg(A1 \uparrow A2)$$


$$\{A1, A2\}$$

$A1 \downarrow A2$

α	$\alpha1$	$\alpha2$
$\neg\neg A1$	$A1$	-
$A1 \wedge A2$	$A1$	$A2$
$\neg(A1 \vee A2)$	$\neg A1$	$\neg A2$
$\neg(A1 \rightarrow A2)$	$A1$	$\neg A2$
$\neg(A1 \uparrow A2)$	$A1$	$A2$
$A1 \downarrow A2$	$\neg A1$	$\neg A2$

a	a1	a2
$\neg\neg A1$	A1	-
$A1 \wedge A2$	A1	A2
$\neg(A1 \vee A2)$	$\neg A1$	$\neg A2$
$\neg(A1 \rightarrow A2)$	A1	$\neg A2$
$\neg(A1 \uparrow A2)$	A1	A2
$A1 \downarrow A2$	$\neg A1$	$\neg A2$

$A1 \downarrow A2$



$\{\neg A1, \neg A2\}$

a	a1	a2
$\neg\neg A1$	A1	-
$A1 \wedge A2$	A1	A2
$\neg(A1 \vee A2)$	$\neg A1$	$\neg A2$
$\neg(A1 \rightarrow A2)$	A1	$\neg A2$
$\neg(A1 \uparrow A2)$	A1	A2
$A1 \downarrow A2$	$\neg A1$	$\neg A2$

$A1 \downarrow A2$

 $\{\neg A1, \neg A2\}$

a	a1	a2
$\neg\neg A1$	$A1$	-
$A1 \wedge A2$	$A1$	$A2$
$\neg(A1 \vee A2)$	$\neg A1$	$\neg A2$
$\neg(A1 \rightarrow A2)$	$A1$	$\neg A2$
$\neg(A1 \uparrow A2)$	$A1$	$A2$
$A1 \downarrow A2$	$\neg A1$	$\neg A2$
$A1 \leftrightarrow A2$	$A1 \rightarrow A2$	$A2 \rightarrow A1$

 $A1 \leftrightarrow A2$

a	a1	a2
$\neg\neg A1$	$A1$	-
$A1 \wedge A2$	$A1$	$A2$
$\neg(A1 \vee A2)$	$\neg A1$	$\neg A2$
$\neg(A1 \rightarrow A2)$	$A1$	$\neg A2$
$\neg(A1 \uparrow A2)$	$A1$	$A2$
$A1 \downarrow A2$	$\neg A1$	$\neg A2$
$A1 \leftrightarrow A2$	$A1 \rightarrow A2$	$A2 \rightarrow A1$

$$A1 \leftrightarrow A2$$



$$\{A1 \rightarrow A2, A2 \rightarrow A1\}$$

a	a1	a2
$\neg\neg A1$	$A1$	-
$A1 \wedge A2$	$A1$	$A2$
$\neg(A1 \vee A2)$	$\neg A1$	$\neg A2$
$\neg(A1 \rightarrow A2)$	$A1$	$\neg A2$
$\neg(A1 \uparrow A2)$	$A1$	$A2$
$A1 \downarrow A2$	$\neg A1$	$\neg A2$
$A1 \leftrightarrow A2$	$A1 \rightarrow A2$	$A2 \rightarrow A1$
$\neg(A1 \oplus A2)$	$A1 \rightarrow A2$	$A2 \rightarrow A1$

 $A1 \leftrightarrow A2$

a	a1	a2
$\neg\neg A1$	$A1$	-
$A1 \wedge A2$	$A1$	$A2$
$\neg(A1 \vee A2)$	$\neg A1$	$\neg A2$
$\neg(A1 \rightarrow A2)$	$A1$	$\neg A2$
$\neg(A1 \uparrow A2)$	$A1$	$A2$
$A1 \downarrow A2$	$\neg A1$	$\neg A2$
$A1 \leftrightarrow A2$	$A1 \rightarrow A2$	$A2 \rightarrow A1$
$\neg(A1 \oplus A2)$	$A1 \rightarrow A2$	$A2 \rightarrow A1$

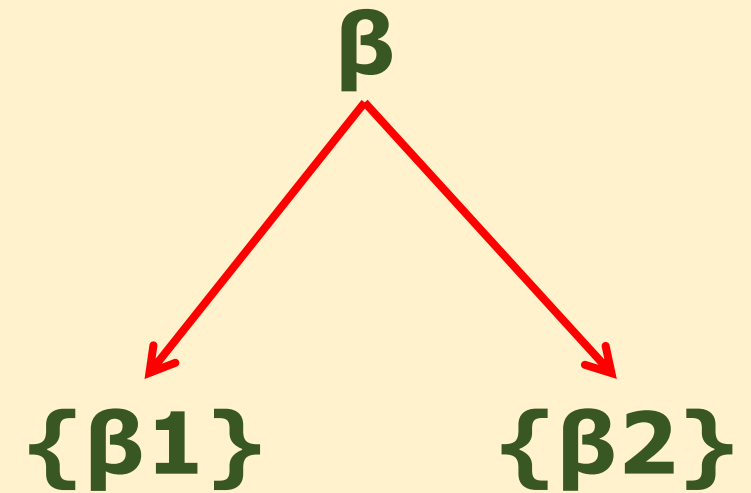
$$A1 \leftrightarrow A2$$



$$\{A1 \rightarrow A2, A2 \rightarrow A1\}$$

β $\beta 1$ $\beta 2$ β

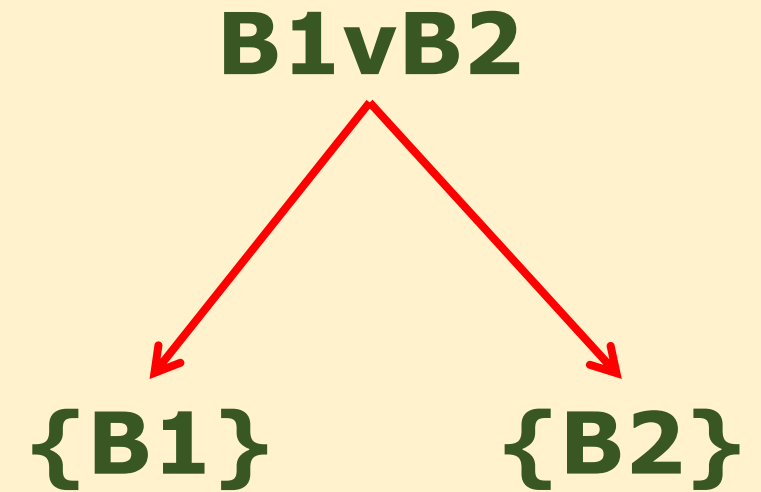
β	$\beta 1$	$\beta 2$
---------	-----------	-----------



β	$\beta 1$	$\beta 2$
$B1 \vee B2$	$B1$	$B2$

 $B1 \vee B2$

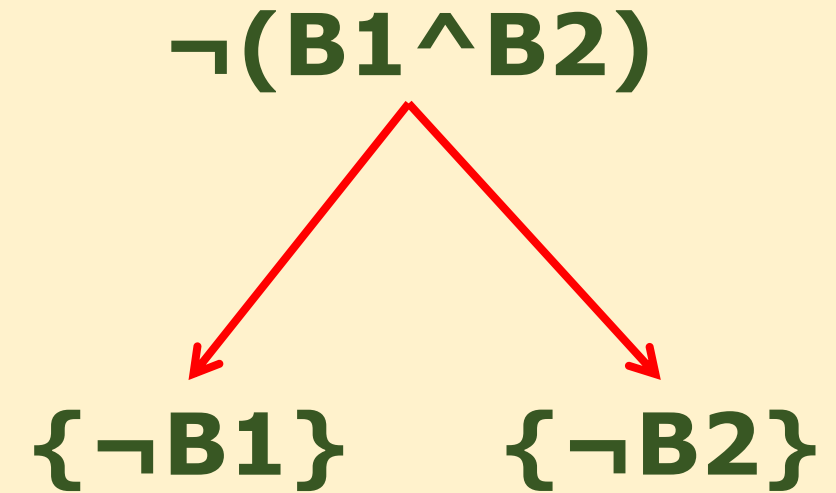
β	$\beta 1$	$\beta 2$
$B1 \vee B2$	$B1$	$B2$



β	$\beta 1$	$\beta 2$
$B1 \vee B2$	$B1$	$B2$
$\neg(B1 \wedge B2)$	$\neg B1$	$\neg B2$

$\neg(B1 \wedge B2)$

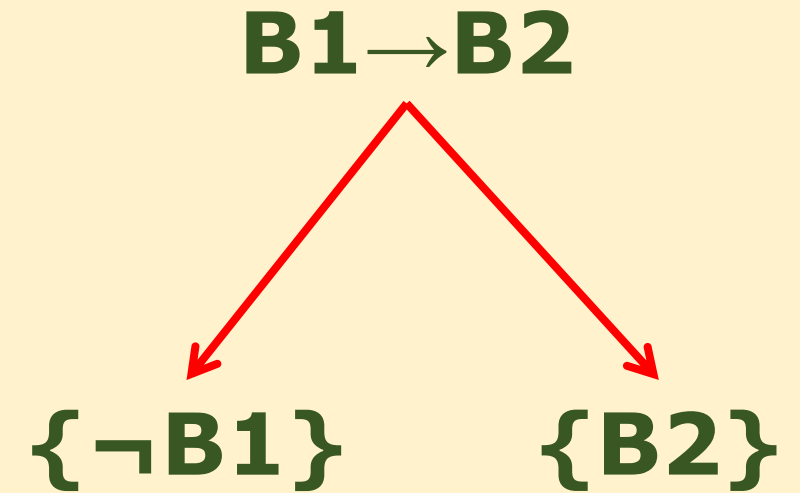
β	$\beta 1$	$\beta 2$
$B1 \vee B2$	$B1$	$B2$
$\neg(B1 \wedge B2)$	$\neg B1$	$\neg B2$



β	$\beta 1$	$\beta 2$
$B1 \vee B2$	$B1$	$B2$
$\neg(B1 \wedge B2)$	$\neg B1$	$\neg B2$
$B1 \rightarrow B2$	$\neg B1$	$B2$

 $B1 \rightarrow B2$

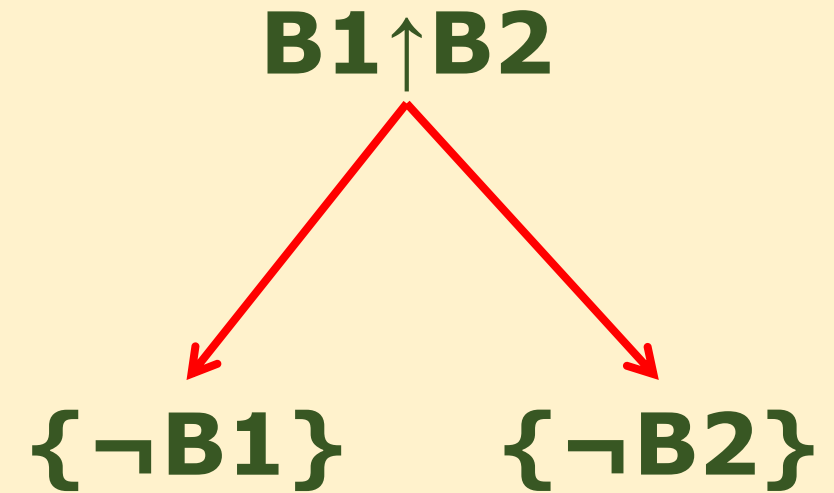
β	$\beta 1$	$\beta 2$
$B1 \vee B2$	$B1$	$B2$
$\neg(B1 \wedge B2)$	$\neg B1$	$\neg B2$
$B1 \rightarrow B2$	$\neg B1$	$B2$



β	$\beta 1$	$\beta 2$
$B1 \vee B2$	$B1$	$B2$
$\neg(B1 \wedge B2)$	$\neg B1$	$\neg B2$
$B1 \rightarrow B2$	$\neg B1$	$B2$
$B1 \uparrow B2$	$\neg B1$	$\neg B2$

 $B1 \uparrow B2$

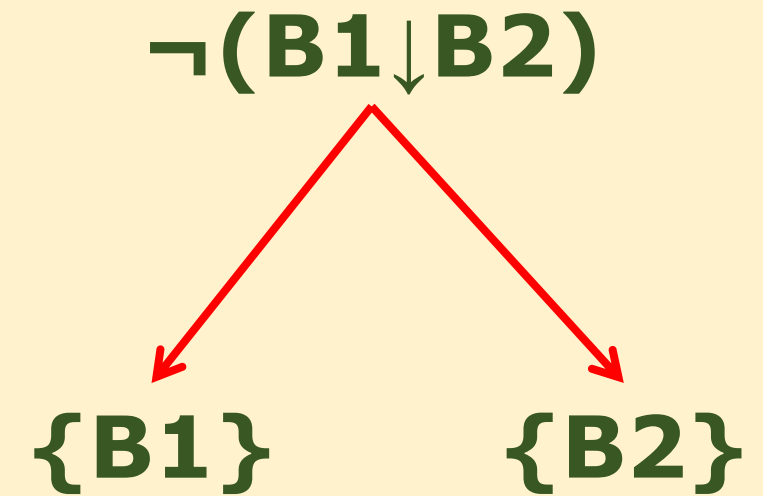
β	$\beta 1$	$\beta 2$
$B1 \vee B2$	$B1$	$B2$
$\neg(B1 \wedge B2)$	$\neg B1$	$\neg B2$
$B1 \rightarrow B2$	$\neg B1$	$B2$
$B1 \uparrow B2$	$\neg B1$	$\neg B2$



β	$\beta 1$	$\beta 2$
$B1 \vee B2$	$B1$	$B2$
$\neg(B1 \wedge B2)$	$\neg B1$	$\neg B2$
$B1 \rightarrow B2$	$\neg B1$	$B2$
$B1 \uparrow B2$	$\neg B1$	$\neg B2$
$\neg(B1 \downarrow B2)$	$B1$	$B2$

$\neg(B1 \downarrow B2)$

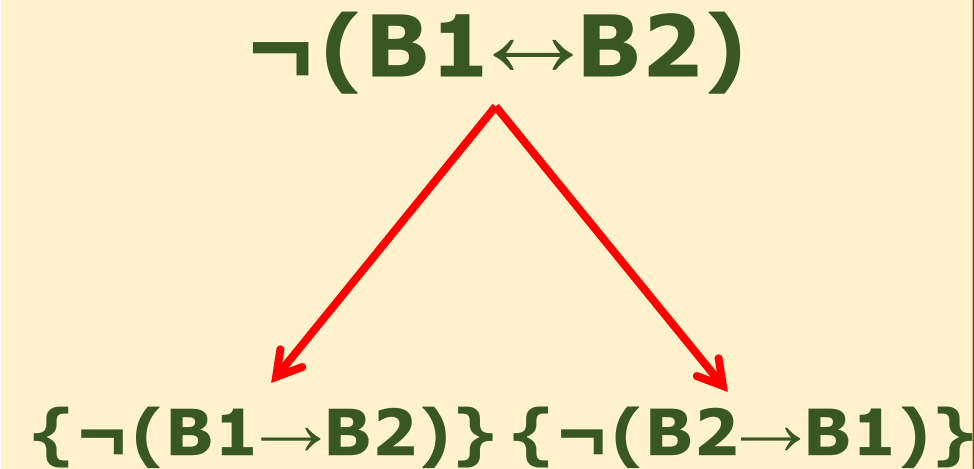
β	$\beta 1$	$\beta 2$
$B1 \vee B2$	$B1$	$B2$
$\neg(B1 \wedge B2)$	$\neg B1$	$\neg B2$
$B1 \rightarrow B2$	$\neg B1$	$B2$
$B1 \uparrow B2$	$\neg B1$	$\neg B2$
$\neg(B1 \downarrow B2)$	$B1$	$B2$



β	$\beta 1$	$\beta 2$
$B1 \vee B2$	$B1$	$B2$
$\neg(B1 \wedge B2)$	$\neg B1$	$\neg B2$
$B1 \rightarrow B2$	$\neg B1$	$B2$
$B1 \uparrow B2$	$\neg B1$	$\neg B2$
$\neg(B1 \downarrow B2)$	$B1$	$B2$
$\neg(B1 \leftrightarrow B2)$	$\neg(B1 \rightarrow B2)$	$\neg(B2 \rightarrow B1)$

$$\neg(B1 \leftrightarrow B2)$$

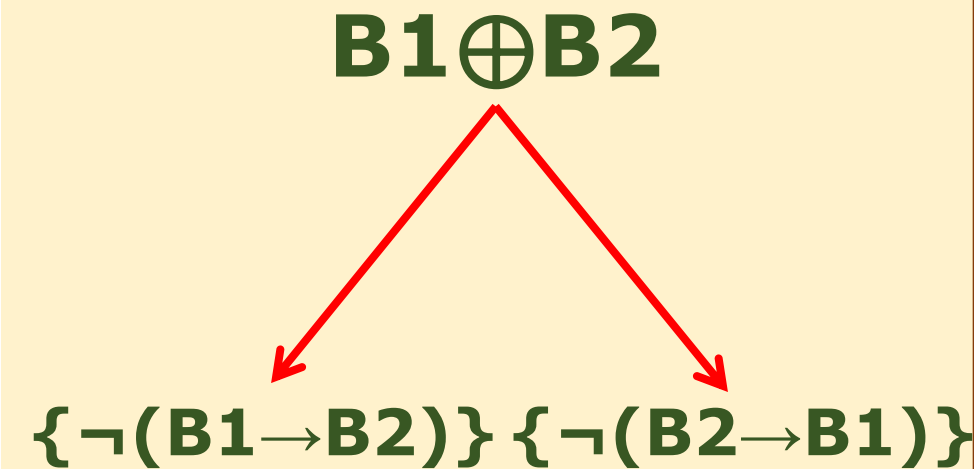
β	$\beta 1$	$\beta 2$
$B1 \vee B2$	$B1$	$B2$
$\neg(B1 \wedge B2)$	$\neg B1$	$\neg B2$
$B1 \rightarrow B2$	$\neg B1$	$B2$
$B1 \uparrow B2$	$\neg B1$	$\neg B2$
$\neg(B1 \downarrow B2)$	$B1$	$B2$
$\neg(B1 \leftrightarrow B2)$	$\neg(B1 \rightarrow B2)$	$\neg(B2 \rightarrow B1)$



β	$\beta 1$	$\beta 2$
$B1 \vee B2$	$B1$	$B2$
$\neg(B1 \wedge B2)$	$\neg B1$	$\neg B2$
$B1 \rightarrow B2$	$\neg B1$	$B2$
$B1 \uparrow B2$	$\neg B1$	$\neg B2$
$\neg(B1 \downarrow B2)$	$B1$	$B2$
$\neg(B1 \leftrightarrow B2)$	$\neg(B1 \rightarrow B2)$	$\neg(B2 \rightarrow B1)$
$B1 \oplus B2$	$\neg(B1 \rightarrow B2)$	$\neg(B2 \rightarrow B1)$

 $B1 \oplus B2$

β	$\beta 1$	$\beta 2$
$B1 \vee B2$	$B1$	$B2$
$\neg(B1 \wedge B2)$	$\neg B1$	$\neg B2$
$B1 \rightarrow B2$	$\neg B1$	$B2$
$B1 \uparrow B2$	$\neg B1$	$\neg B2$
$\neg(B1 \downarrow B2)$	$B1$	$B2$
$\neg(B1 \leftrightarrow B2)$	$\neg(B1 \rightarrow B2)$	$\neg(B2 \rightarrow B1)$
$B1 \oplus B2$	$\neg(B1 \rightarrow B2)$	$\neg(B2 \rightarrow B1)$



$$p \wedge (\neg q \vee \neg p)$$

$$p \boxed{\wedge} (\neg q \vee \neg p)$$


conector mais externo

$$p \wedge (\neg q \vee \neg p)$$

$$p \wedge (\neg q \vee \neg p)$$



$$\{p, (\neg q \vee \neg p)\}$$

$$p \wedge (\neg q \vee \neg p)$$



$$\{p, (\neg q \vee \neg p)\}$$

somente literal ←

$$p \wedge (\neg q \vee \neg p)$$



$$\{p, (\neg q \vee \neg p)\}$$

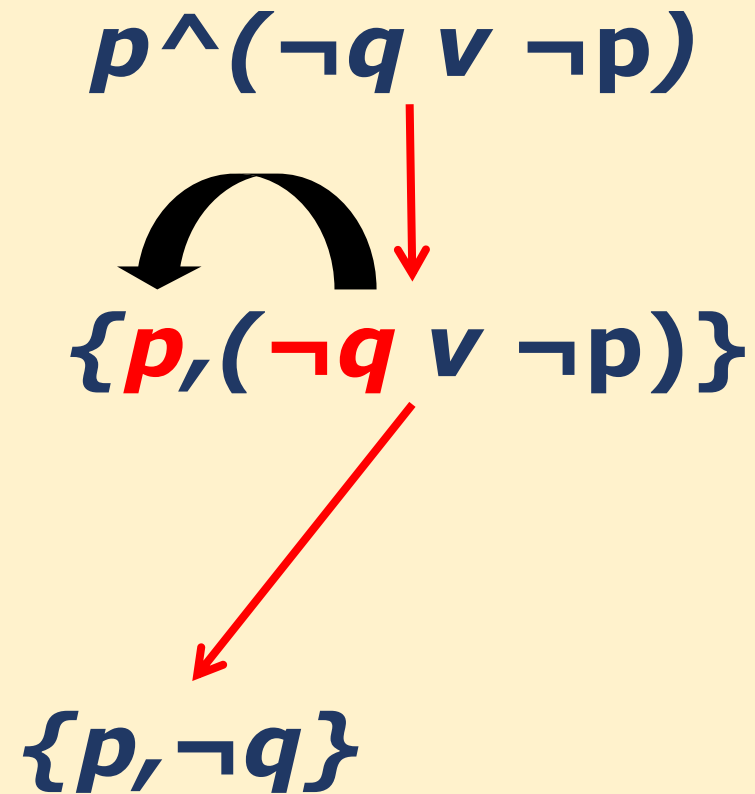
somente literal ←

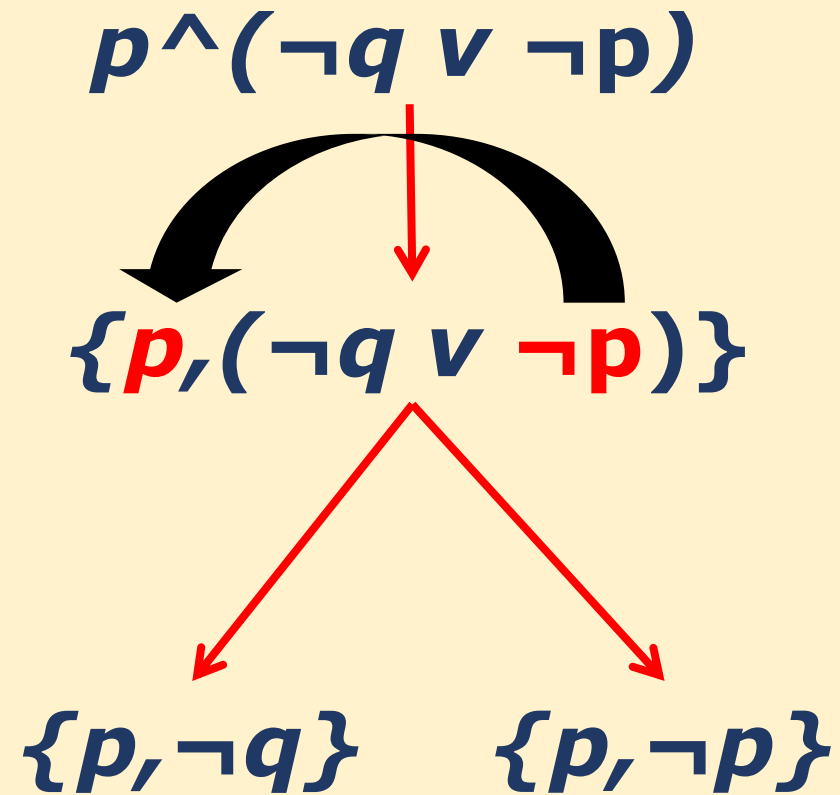
→ *possui conector*

$$p \wedge (\neg q \vee \neg p)$$

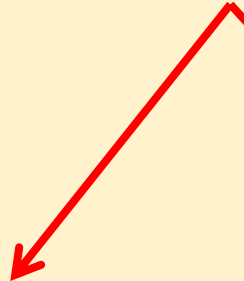
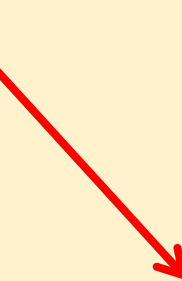


$$\{p, (\neg q \vee \neg p)\}$$

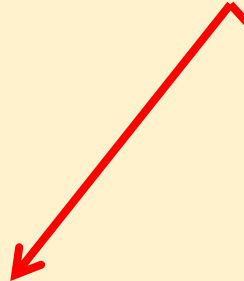
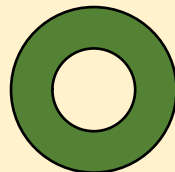


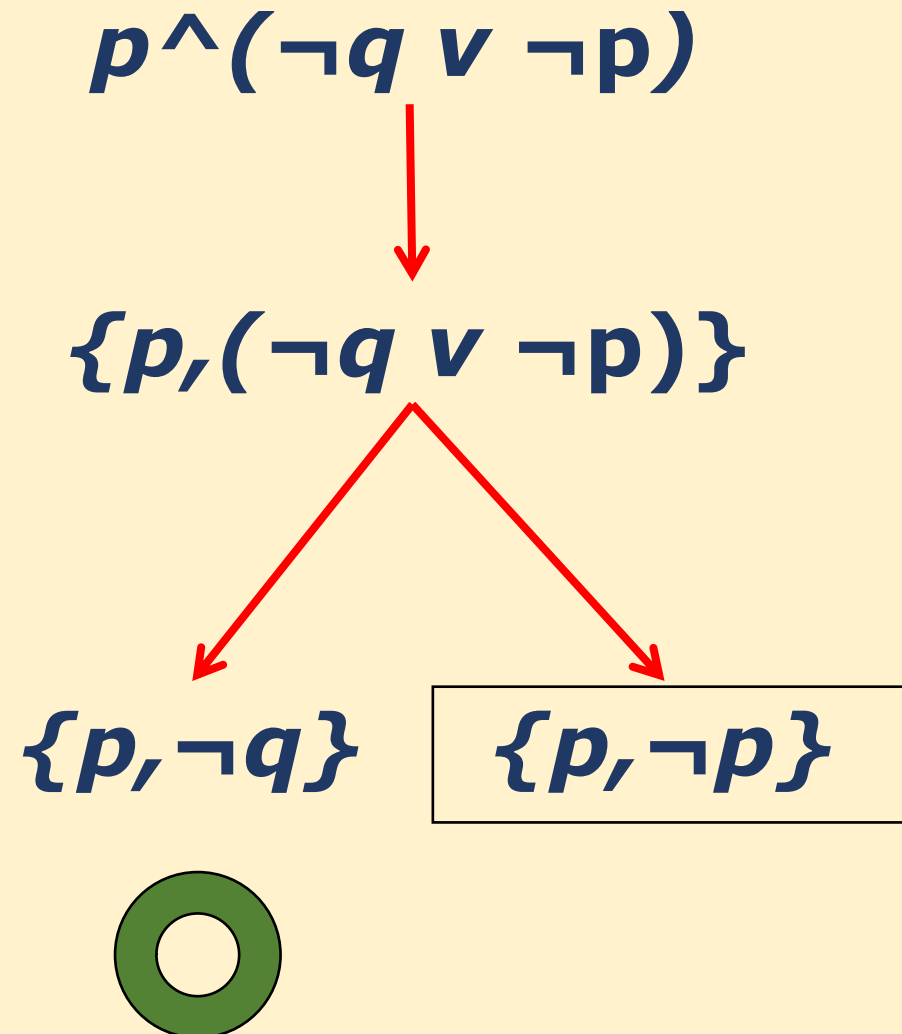


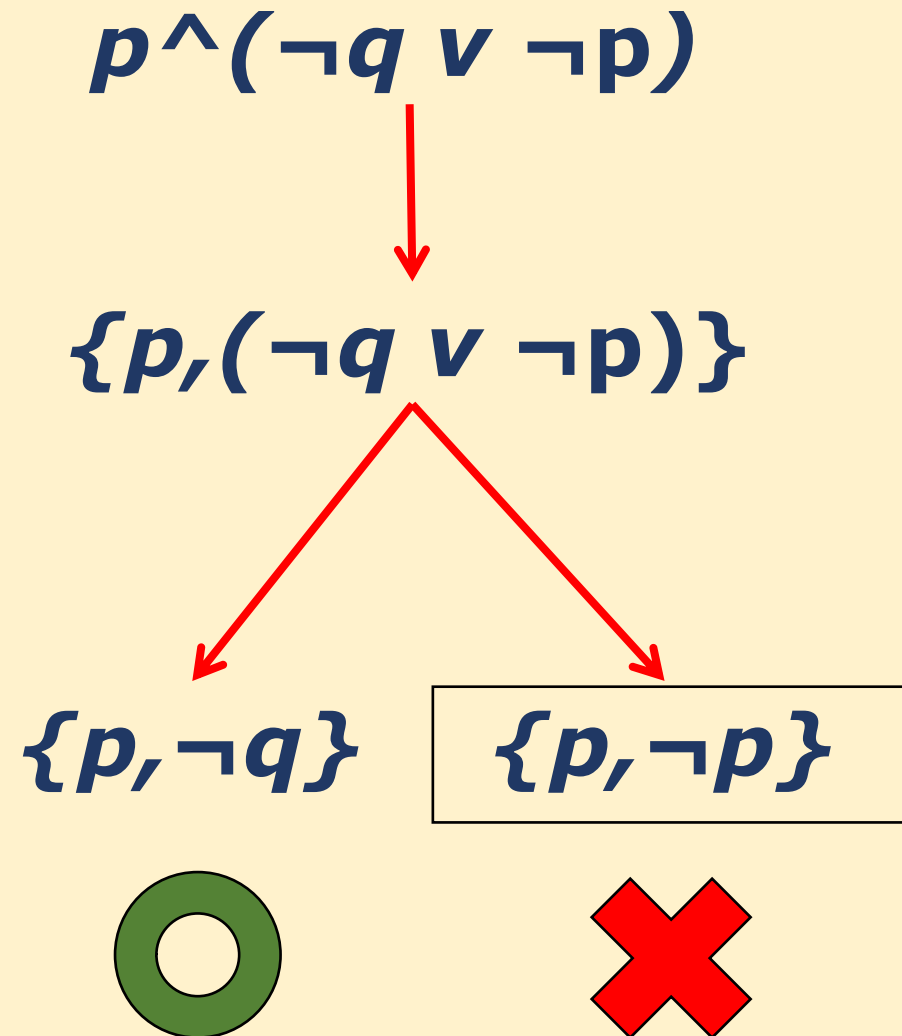
$$p \wedge (\neg q \vee \neg p)$$

$$\{p, (\neg q \vee \neg p)\}$$

$$\{p, \neg q\}$$

$$\{p, \neg p\}$$

$$p \wedge (\neg q \vee \neg p)$$

$$\{p, (\neg q \vee \neg p)\}$$

$$\{p, \neg q\}$$
$$\{p, \neg p\}$$






$$(p \vee q) \boxed{\wedge} (\neg q \wedge \neg p)$$



conector mais externo

$$(p \vee q) \wedge (\neg q \wedge \neg p)$$



conector mais externo

$$(p \vee q) \wedge (\neg q \wedge \neg p)$$



$$\{(p \vee q), (\neg q \wedge \neg p)\}$$

$$(p \vee q) \wedge (\neg q \wedge \neg p)$$



$$\{(p \boxed{\vee} q), (\neg q \boxed{\wedge} \neg p)\}$$

$$(p \vee q) \wedge (\neg q \wedge \neg p)$$



$$\{(p \vee q), (\neg q \wedge \neg p)\}$$

$$(p \vee q) \wedge (\neg q \wedge \neg p)$$



$$\{(p \vee q), (\neg q \wedge \neg p)\}$$



$$\{(p \vee q), \neg q, \neg p\}$$

$$(p \vee q) \wedge (\neg q \wedge \neg p)$$



$$\{(p \vee q), (\neg q \wedge \neg p)\}$$



$$\{(p \vee q), \neg q, \neg p\}$$

$$(p \vee q) \wedge (\neg q \wedge \neg p)$$



$$\{(p \vee q), (\neg q \wedge \neg p)\}$$



$$\{(p \vee q), \neg q, \neg p\}$$

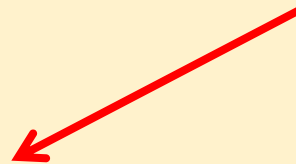
$$(p \vee q) \wedge (\neg q \wedge \neg p)$$



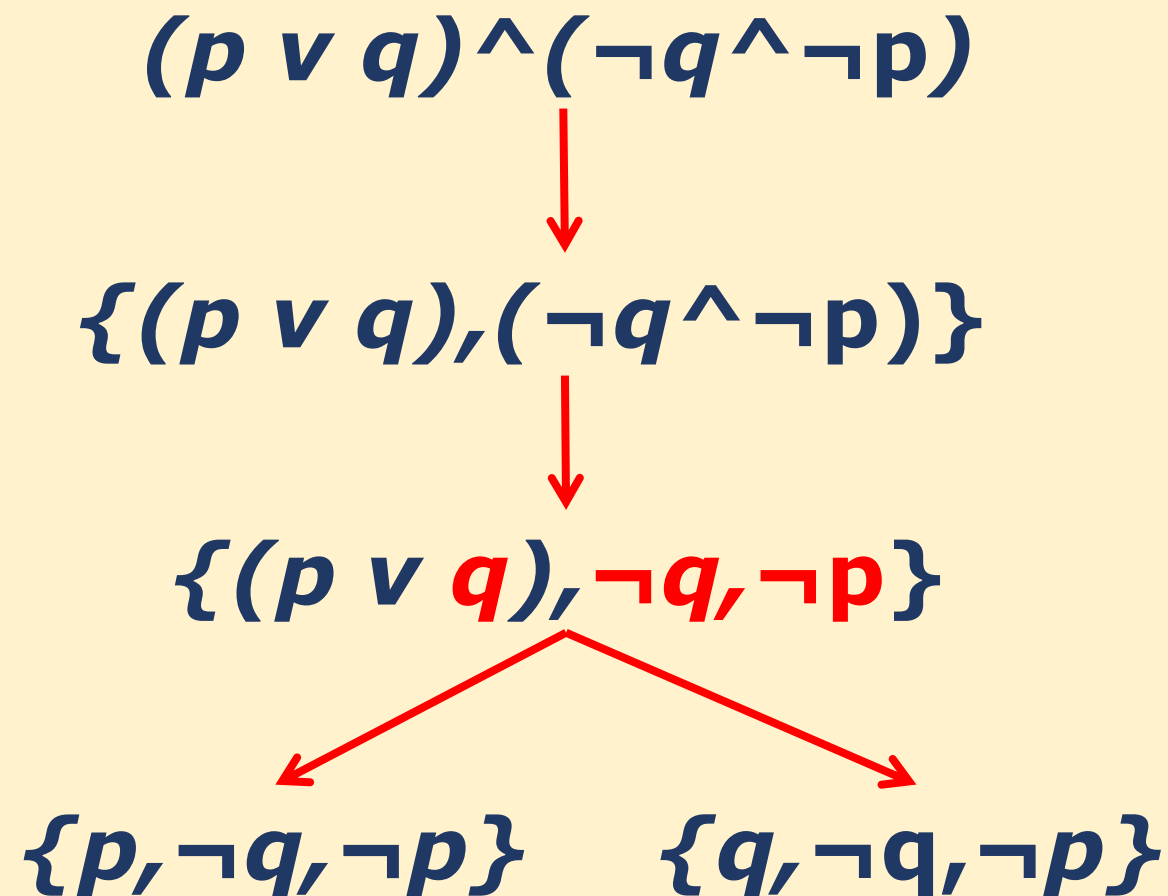
$$\{(p \vee q), (\neg q \wedge \neg p)\}$$

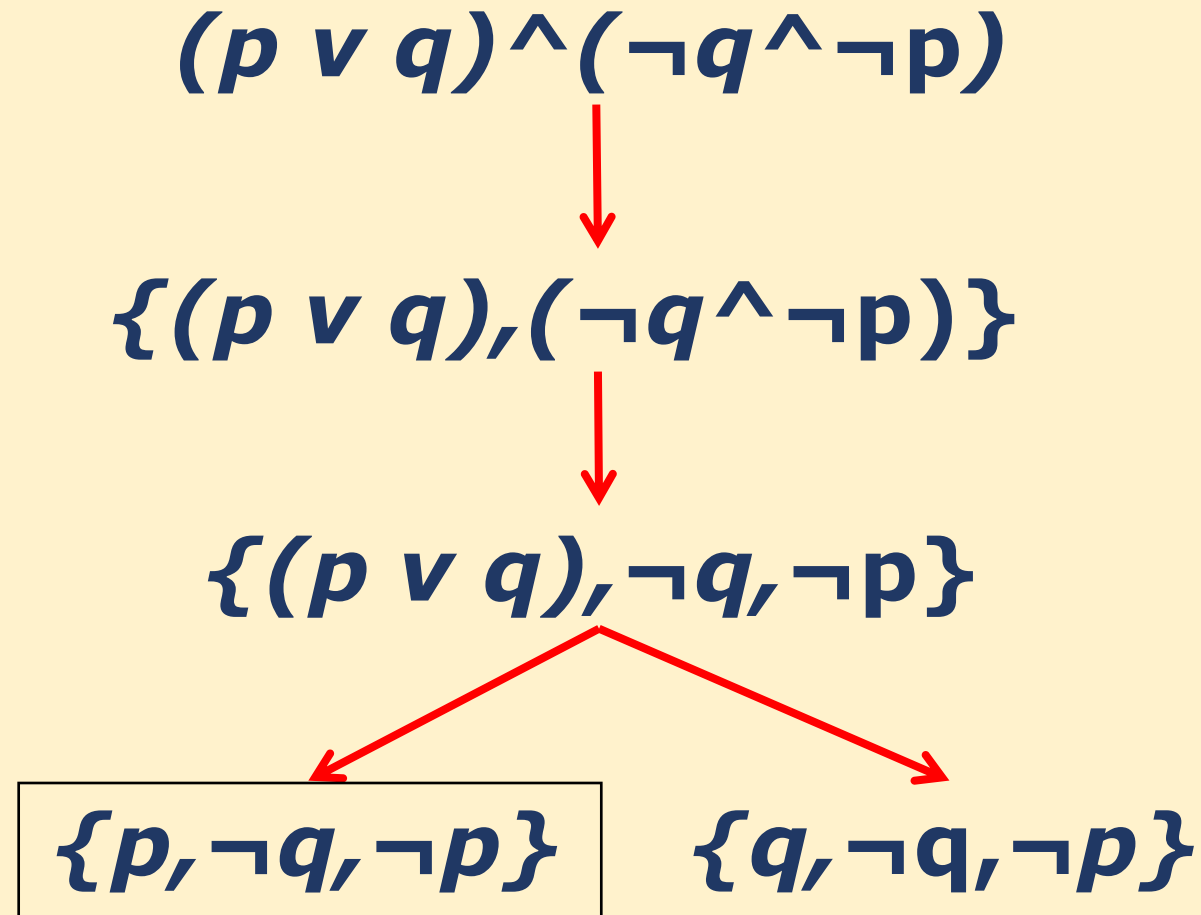


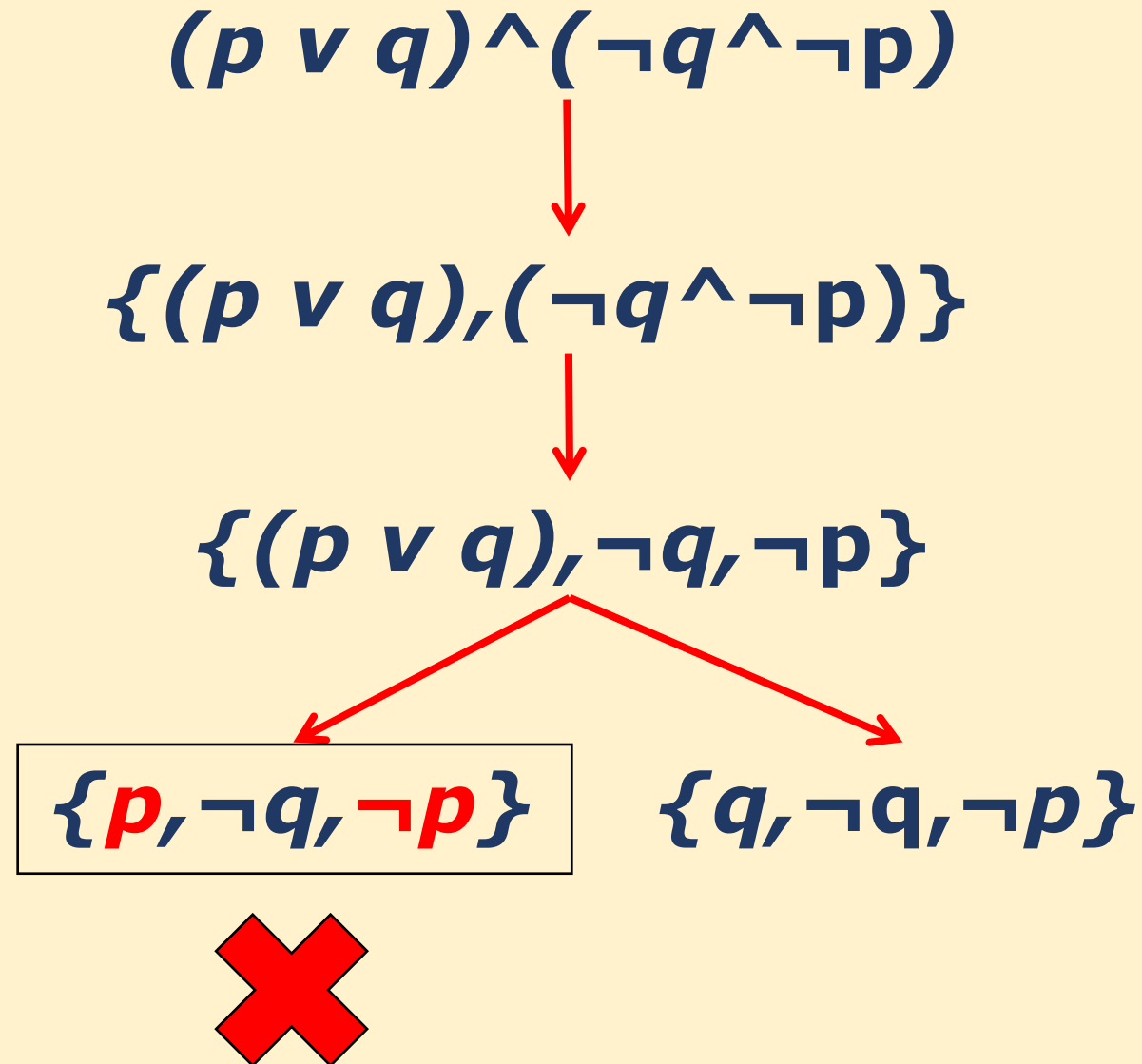
$$\{(p \vee q), \neg q, \neg p\}$$

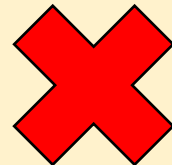
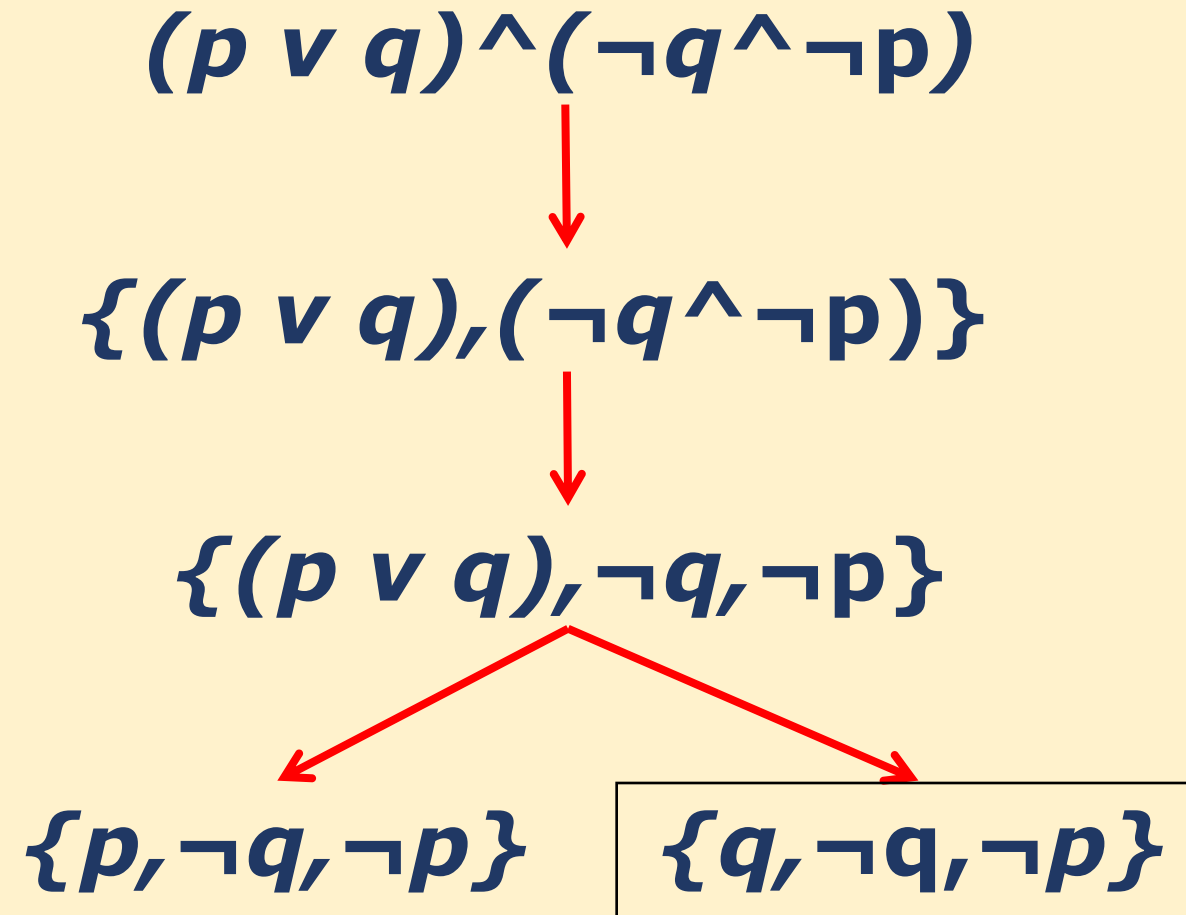


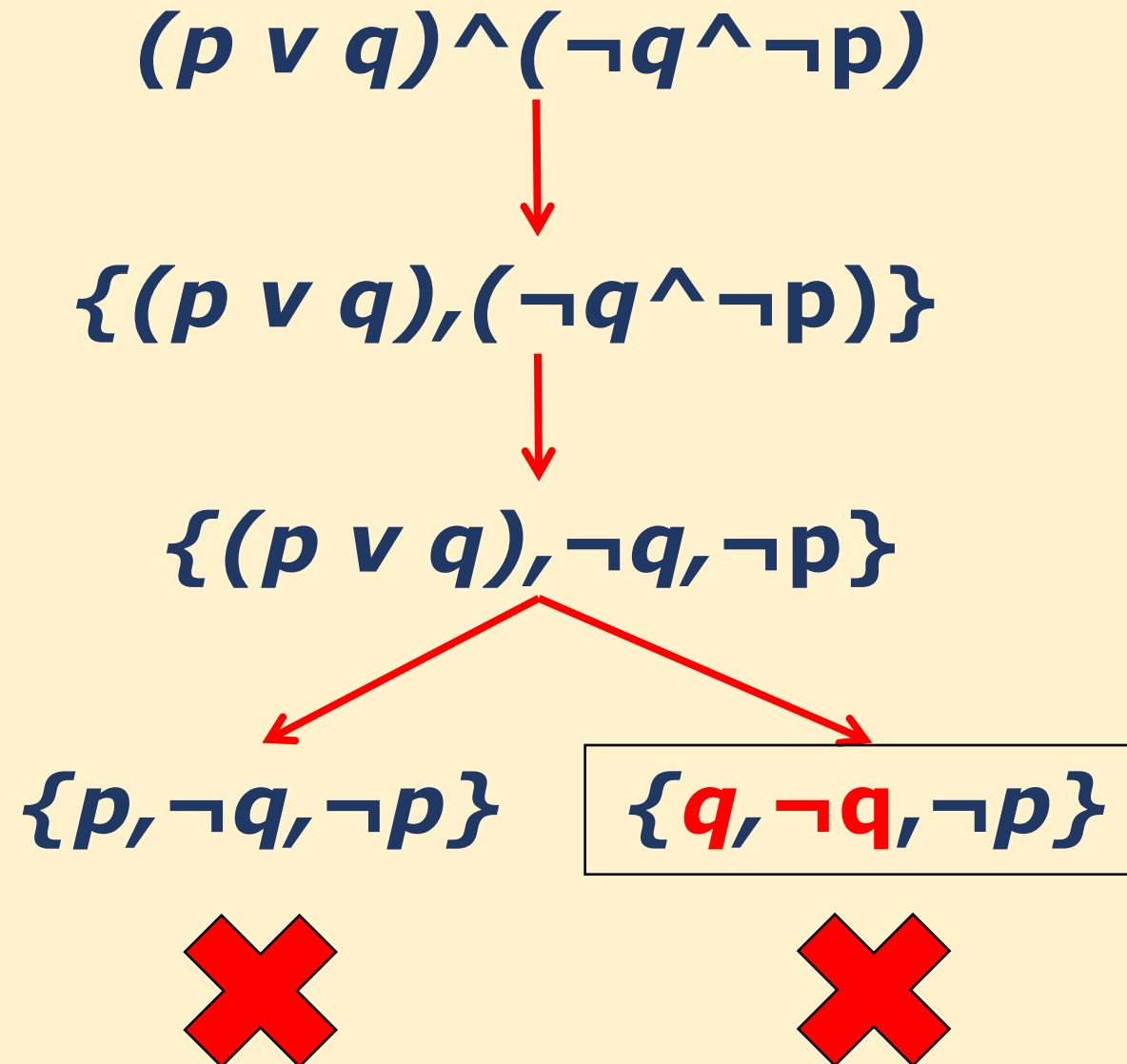
$$\{p, \neg q, \neg p\}$$











$$\neg(p \rightarrow (p \vee q))$$

$$\neg(p \rightarrow (p \vee q))$$



$$\{p, \neg(p \vee q)\}$$

$$\neg(p \rightarrow (p \vee q))$$



$$\{p, \neg(p \vee q)\}$$



$$\{p, \neg p, \neg q\}$$

$$\neg(p \rightarrow (p \vee q))$$



$$\{p, \neg(p \vee q)\}$$



$$\{\textcolor{red}{p}, \neg\textcolor{red}{p}, \neg q\}$$

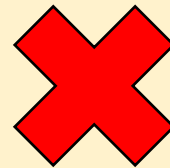
$$\neg(p \rightarrow (p \vee q))$$

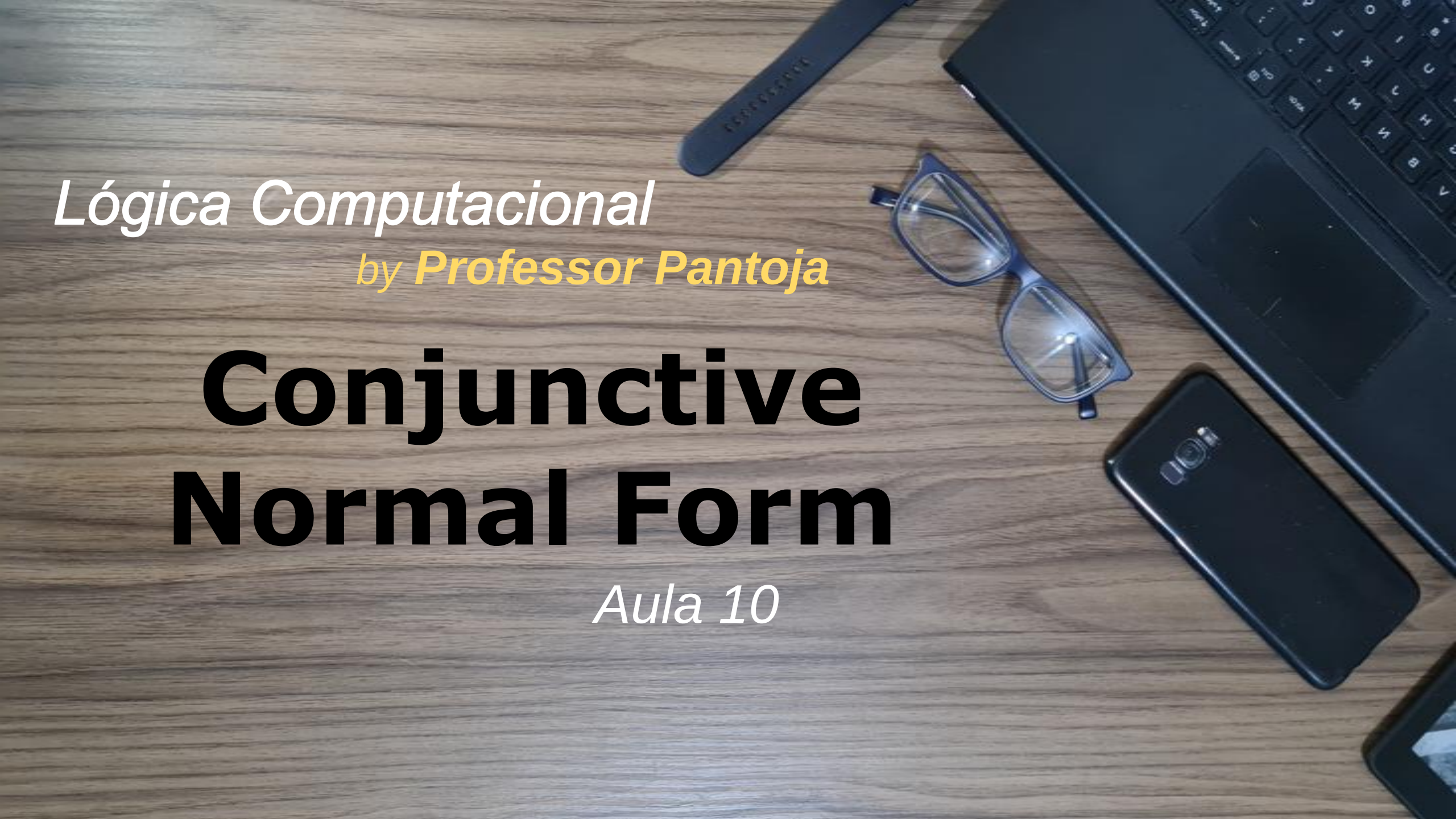


$$\{p, \neg(p \vee q)\}$$



$$\{p, \neg p, \neg q\}$$



A top-down view of a wooden desk. On the right side, there is a black laptop with its keyboard visible. Below the laptop, a pair of blue-rimmed glasses lies on the desk. To the left of the glasses, a black pen is positioned diagonally. Further down and to the right, a black smartphone is lying flat. The background is the natural wood grain of the desk.

Lógica Computacional
by Professor Pantoja

Conjunctive Normal Form

Aula 10

$$(\neg p \vee q \vee r) \wedge (\neg q \vee r) \wedge (\neg r)$$

$$(\neg p \vee q \vee r) \wedge (\neg q \vee r) \wedge (\neg r)$$

$$(\neg p \vee q \vee r) \wedge (\neg q \vee r) \wedge (\neg r)$$

i. $(\neg p \vee q \vee r) \wedge ((\neg p \wedge q) \vee r) \wedge (\neg r)$

$$(\neg p \vee q \vee r) \wedge (\neg q \vee r) \wedge (\neg r)$$

$$i. (\neg p \vee q \vee r) \wedge ((\neg p \wedge q) \vee r) \wedge (\neg r)$$

$$(\neg p \vee q \vee r) \wedge (\neg q \vee r) \wedge (\neg r)$$

i. $(\neg p \vee q \vee r) \wedge ((\neg p \wedge q) \vee r) \wedge (\neg r)$

ii. $(\neg p \vee q \vee r) \wedge \neg(\neg q \vee r) \wedge (\neg r)$

$$(\neg p \vee q \vee r) \wedge (\neg q \vee r) \wedge (\neg r)$$

i. $(\neg p \vee q \vee r) \wedge ((\neg p \wedge q) \vee r) \wedge (\neg r)$

ii. $(\neg p \vee q \vee r) \wedge \neg(\neg q \vee r) \wedge (\neg r)$

$$(\neg p \vee q \vee r) \wedge (\neg q \vee r) \wedge (\neg r)$$

i. $(\neg p \vee q \vee r) \wedge ((\neg p \wedge q) \vee r) \wedge (\neg r)$

ii. $(\neg p \vee q \vee r) \wedge \neg(\neg q \vee r) \wedge (\neg r)$

Toda e qualquer fórmula pode ser transformada em uma CNF equivalente

- 1. Eliminar todos os conectores exceto: *NEGAÇÃO*, *OU* e *E***

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$$A \leftrightarrow B \equiv (A \rightarrow B) \wedge (B \rightarrow A)$$

1. **Eliminar todos os conectores exceto: *NEGAÇÃO*, *OU* e *E***

$$A \leftrightarrow B \equiv (A \rightarrow B) \wedge (B \rightarrow A)$$

$$A \oplus B \equiv \neg(A \rightarrow B) \vee \neg(B \rightarrow A)$$

- 1. Eliminar todos os conectores exceto: *NEGAÇÃO*, *OU* e *E***

$$A \leftrightarrow B \equiv (A \rightarrow B) \wedge (B \rightarrow A)$$

$$A \oplus B \equiv \neg(A \rightarrow B) \vee \neg(B \rightarrow A)$$

$$A \rightarrow B \equiv \neg A \vee B$$

- 1. Eliminar todos os conectores exceto: *NEGAÇÃO*, *OU* e *E***

$$A \leftrightarrow B \equiv (A \rightarrow B) \wedge (B \rightarrow A)$$

$$A \oplus B \equiv \neg(A \rightarrow B) \vee \neg(B \rightarrow A)$$

$$A \rightarrow B \equiv \neg A \vee B$$

$$A \uparrow B \equiv \neg(A \wedge B)$$

- 1. Eliminar todos os conectores exceto: *NEGAÇÃO*, *OU* e *E***

$$A \leftrightarrow B \equiv (A \rightarrow B) \wedge (B \rightarrow A)$$

$$A \oplus B \equiv \neg(A \rightarrow B) \vee \neg(B \rightarrow A)$$

$$A \rightarrow B \equiv \neg A \vee B$$

$$A \uparrow B \equiv \neg(A \wedge B)$$

$$A \downarrow B \equiv \neg(A \vee B)$$

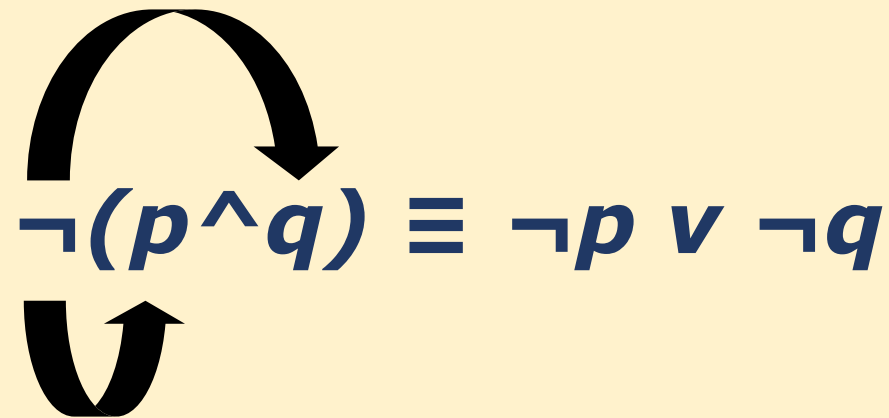
2. Internalizar a negação usando *De Morgan*

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

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$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$


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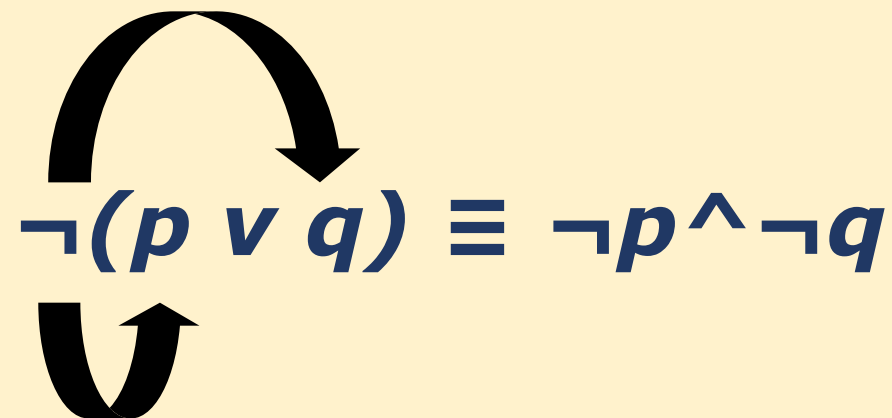
2. Internalizar a negação usando De Morgan

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$


2. Internalizar a negação usando *De Morgan*

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$


$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

3. Eliminar a Dupla Negação

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$$\neg\neg A \equiv A$$

$$\neg\neg\neg A \equiv \neg A$$

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$$\neg\neg A \equiv A$$

$$\neg\neg\neg A \equiv \neg A$$

4. Usar as Leis Distributivas

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$$\neg\neg A \equiv A$$

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4. Usar as Leis Distributivas

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

3. Eliminar a Dupla Negação

$$\neg\neg A \equiv A$$

$$\neg\neg\neg A \equiv \neg A$$

4. Usar as Leis Distributivas

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

$$(\neg p \rightarrow q) \rightarrow (p \rightarrow q)$$

$$\begin{aligned} &(\neg p \rightarrow q) \rightarrow (p \rightarrow q) \\ &(\neg \neg p \vee q) \rightarrow (p \rightarrow q) \end{aligned}$$

$$\begin{aligned} &(\neg p \rightarrow q) \rightarrow (p \rightarrow q) \\ &(\neg \neg p \vee q) \rightarrow (p \rightarrow q) \end{aligned}$$

$$(\neg p \rightarrow q) \rightarrow (p \rightarrow q)$$

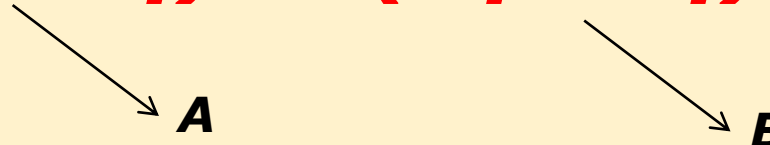
$$(\neg \neg p \vee q) \rightarrow (p \rightarrow q)$$

$$(\neg \neg p \vee q) \rightarrow (\neg p \vee q)$$

$$(\neg p \rightarrow q) \rightarrow (p \rightarrow q)$$

$$(\neg \neg p \vee q) \rightarrow (p \rightarrow q)$$

$$(\neg \neg p \vee q) \rightarrow (\neg p \vee q)$$



A

B

$$(\neg p \rightarrow q) \rightarrow (p \rightarrow q)$$

$$(\neg \neg p \vee q) \rightarrow (p \rightarrow q)$$

$$(\neg \neg p \vee q) \rightarrow (\neg p \vee q)$$

$$\neg(A) \vee (B)$$

$$(\neg p \rightarrow q) \rightarrow (p \rightarrow q)$$

$$(\neg \neg p \vee q) \rightarrow (p \rightarrow q)$$

$$(\neg \neg p \vee q) \rightarrow (\neg p \vee q)$$

$$\neg(\neg \neg p \vee q) \vee (\neg p \vee q)$$

$$(\neg p \rightarrow q) \rightarrow (p \rightarrow q)$$

$$(\neg \neg p \vee q) \rightarrow (p \rightarrow q)$$

$$(\neg \neg p \vee q) \rightarrow (\neg p \vee q)$$

$$\neg(\neg \neg p \vee q) \vee (\neg p \vee q)$$

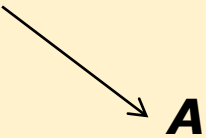
$$(\neg p \rightarrow q) \rightarrow (p \rightarrow q)$$

$$(\neg \neg p \vee q) \rightarrow (p \rightarrow q)$$

$$(\neg \neg p \vee q) \rightarrow (\neg p \vee q)$$

$$\neg(\neg \neg p \vee q) \vee (\neg p \vee q)$$

$$(\neg p \wedge \neg q) \vee (\neg p \vee q)$$

$$\begin{aligned} & (\neg p \rightarrow q) \rightarrow (p \rightarrow q) \\ & (\neg \neg p \vee q) \rightarrow (p \rightarrow q) \\ & (\neg \neg p \vee q) \rightarrow (\neg p \vee q) \\ & \neg(\neg \neg p \vee q) \vee (\neg p \vee q) \\ & (\neg p \wedge \neg q) \vee (\neg p \vee q) \end{aligned}$$


$$\begin{aligned}
 & (\neg p \rightarrow q) \rightarrow (p \rightarrow q) \\
 & (\neg \neg p \vee q) \rightarrow (p \rightarrow q) \\
 & (\neg \neg p \vee q) \rightarrow (\neg p \vee q) \\
 & \neg(\neg \neg p \vee q) \vee (\neg p \vee q) \\
 & (\neg p \wedge \neg q) \vee (\neg p \vee q) \longrightarrow A \\
 & (A \vee \neg p) \wedge (A \vee \neg q)
 \end{aligned}$$

$$(\neg p \rightarrow q) \rightarrow (p \rightarrow q)$$

$$(\neg \neg p \vee q) \rightarrow (p \rightarrow q)$$

$$(\neg \neg p \vee q) \rightarrow (\neg p \vee q)$$

$$\neg(\neg \neg p \vee q) \vee (\neg p \vee q)$$

$$(\neg p \wedge \neg q) \vee (\neg p \vee q) \longrightarrow A$$

$$((\neg p \vee q) \vee \neg p) \wedge ((\neg p \vee q) \vee \neg q)$$

$$(\neg p \rightarrow q) \rightarrow (p \rightarrow q)$$

$$(\neg \neg p \vee q) \rightarrow (p \rightarrow q)$$

$$(\neg \neg p \vee q) \rightarrow (\neg p \vee q)$$

$$\neg(\neg \neg p \vee q) \vee (\neg p \vee q)$$

$$(\neg p \wedge \neg q) \vee (\neg p \vee q)$$

$$(\neg p \vee q \vee \neg p) \wedge (\neg p \vee q \vee \neg q)$$

$$(\neg p \rightarrow q) \rightarrow (p \rightarrow q)$$

$$(\neg \neg p \vee q) \rightarrow (p \rightarrow q)$$

$$(\neg \neg p \vee q) \rightarrow (\neg p \vee q)$$

$$\neg(\neg \neg p \vee q) \vee (\neg p \vee q)$$

$$(\neg p \wedge \neg q) \vee (\neg p \vee q)$$

$$(\neg p \vee q \vee \neg p) \wedge (\neg p \vee q \vee \neg q)$$

A top-down view of a wooden desk. On the right side, there is a black laptop with its keyboard visible. Below the laptop, a pair of blue-rimmed glasses lies on the desk. To the left of the glasses is a black pen. Further down and to the right is a black smartphone. The text is overlaid on the left side of the image.

Lógica Computacional
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Forma Clausal

Aula 11

- **Uma Cláusula é um conjunto de literais**

- Uma Cláusula é um conjunto de literais
 - **disjunção**

- Uma Cláusula é um conjunto de literais
 - **disjunção**
- Cláusula Unitária contém apenas um literal

- Uma Cláusula é um conjunto de literais
 - **disjunção**
- Cláusula Unitária contém apenas um literal
- Uma Fórmula Clausal é um conjunto de Cláusulas

- Uma Cláusula é um conjunto de literais
 - **disjunção**
- Cláusula Unitária contém apenas um literal
- Uma Fórmula Clausal é um conjunto de Cláusulas
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 - um Conjunto Vazio de Fórmulas é **Válido**.

$$(p \vee r) \wedge (\neg q \vee \neg p \vee q) \wedge (p \vee \neg p \vee q \vee p \vee \neg p) \wedge (r \vee p)$$

$$(p \vee r) \wedge (\neg q \vee \neg p \vee q) \wedge (p \vee \neg p \vee q \vee p \vee \neg p) \wedge (r \vee p)$$



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$$\{\{p, r\}, \{\neg q, \neg p, q\}, \{p, \neg p, q\}\}$$

$$(p \vee r) \wedge (\neg q \vee \neg p \vee q) \wedge (p \vee \neg p \vee q \vee p \vee \neg p) \wedge (r \vee p)$$



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$$\{pr\}, \{\neg q \neg p q\}, \{p \neg p q\}$$

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$$\{\{p, r\}, \{\neg q, \neg p, q\}, \{p, \neg p, q\}\}$$



$$\{pr\}, \{\neg q \neg p q\}, \{p \neg p q\}$$



$$\{pr, \overline{q}\overline{p}q, p\overline{p}q\}$$

i. $n = 1$

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- ***criar dois átomos $p1$ e $p2$***

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(I1)

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i. $n = 1$

- *criar dois átomos $p1$ e $p2$*

$(l1)$



$(l1 \vee p1 \vee p2) \wedge (l1 \vee \neg p1 \vee p2) \wedge (l1 \vee p1 \vee \neg p2) \wedge (l1 \vee \neg p1 \vee \neg p2)$

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V



V



V



V

i. $n = 1$

- criar dois átomos $p1$ e $p2$

$(I1)$



$(I1 \vee p1 \vee p2) \wedge (I1 \vee \neg p1 \vee p2) \wedge (I1 \vee p1 \vee \neg p2) \wedge (I1 \vee \neg p1 \vee \neg p2)$



F



F



F



F

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F



F



F



F



F



F

ii. $n = 2$

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- ***criar um átomo $p1$***

ii. $n = 2$

- **criar um átomo $p1$**

$(l1 \vee l2)$

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$(l1 \vee l2 \vee p1) \wedge (l1 \vee l2 \vee \neg p1)$

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$(l1 \vee l2)$



$(l1 \vee l2 \vee p1) \wedge (l1 \vee l2 \vee \neg p1)$



V



F



V



F

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$(l1 \vee l2 \vee p1) \wedge (l1 \vee l2 \vee \neg p1)$



V



F



V



V



F



F

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$(l1 \vee l2 \vee p1) \wedge (l1 \vee l2 \vee \neg p1)$



F



F



F



F

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$(l1 \vee l2 \vee p1) \wedge (l1 \vee l2 \vee \neg p1)$



F



F



V



F



F

ii. $n = 2$

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$(l1 \vee l2)$



$(l1 \vee l2 \vee p1) \wedge (l1 \vee l2 \vee \neg p1)$



F



F



V



F



F



F

iii. $n = 3$

iii. $n = 3$

- ***não faz nada***

iii. $n = 3$

- ***não faz nada***

iv. $n > 3$

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- ***não faz nada***

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iv. criar $n-3$ átomos $p_1, p_2, \dots, p_{n-3}$

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$(l_1 \vee l_2 \vee l_3 \vee \dots \vee l_n)$

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- iv. criar $n-3$ átomos $p_1, p_2, \dots, p_{n-3}$***

$(l_1 \vee l_2 \vee l_3 \vee \dots \vee l_n)$



iii. $n = 3$

- *não faz nada*

iv. $n > 3$

iv. criar $n-3$ átomos $p_1, p_2, \dots, p_{n-3}$

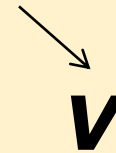
$$(l_1 \vee l_2 \vee l_3 \vee \dots \vee l_n)$$



$$(l_1 \vee l_2 \vee p_1) \wedge (\neg p_1 \vee l_3 \vee p_2) \wedge \dots \wedge (\neg p_{n-3} \vee l_{n-1} \vee l_n)$$

$$(l1 \vee l2 \vee l3 \vee l4 \vee l5)$$

$$(l1 \vee l2 \vee p1) \wedge (\neg p1 \vee l3 \vee p2) \wedge (\neg p2 \vee l4 \vee l5)$$

$$(l1 \vee l2 \vee l3 \vee l4 \vee \underline{l5})$$


v

$$(l1 \vee l2 \vee p1) \wedge (\neg p1 \vee l3 \vee p2) \wedge (\neg p2 \vee l4 \vee l5)$$

$$(I1 \vee I2 \vee I3 \vee I4 \vee \underline{I5}) = V$$

$\downarrow$
 V

$$(I1 \vee I2 \vee p1) \wedge (\neg p1 \vee I3 \vee p2) \wedge (\neg p2 \vee I4 \vee I5)$$

$$(I1 \vee I2 \vee I3 \vee I4 \vee \underline{I5}) = V$$

$\swarrow \quad \swarrow \quad \swarrow \quad \swarrow \quad \swarrow$
 $F \quad F \quad F \quad F \quad V$

$$(I1 \vee I2 \vee p1) \wedge (\neg p1 \vee I3 \vee p2) \wedge (\neg p2 \vee I4 \vee I5)$$

$$(l1 \vee l2 \vee l3 \vee l4 \vee \underline{l5})$$

$$\begin{array}{cc} \mathbf{F} & \mathbf{F} \\ \uparrow & \uparrow \\ (l1 \vee l2 \vee p1) \wedge (\neg p1 \vee l3 \vee p2) \wedge (\neg p2 \vee l4 \vee l5) \end{array}$$

$$(l1 \vee l2 \vee l3 \vee l4 \vee \underline{l5})$$

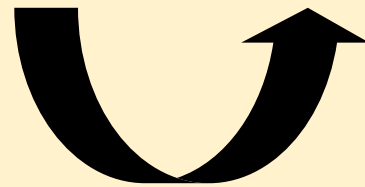
$$\begin{array}{ccc} \textbf{F} & \textbf{F} & \textbf{V} \\ \uparrow & \uparrow & \uparrow \\ (l1 \vee l2 \vee \textcolor{red}{p1}) \wedge (\neg \textcolor{red}{p1} \vee l3 \vee \textcolor{red}{p2}) \wedge (\neg \textcolor{red}{p2} \vee l4 \vee l5) \end{array}$$

$$(l1 \vee l2 \vee l3 \vee l4 \vee \underline{l5})$$

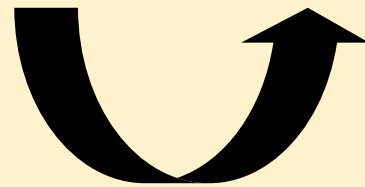
$$\begin{array}{ccccccc} \overset{\text{F}}{\uparrow} & \overset{\text{F}}{\uparrow} & \overset{\text{V}}{\uparrow} & & \overset{\text{V}}{\uparrow} & & \overset{\text{V}}{\uparrow} \\ (l1 \vee l2 \vee p1) \wedge (\neg p1 \vee l3 \vee p2) \wedge (\neg p2 \vee l4 \vee l5) \end{array}$$

$(l1 \vee l2 \vee l3 \vee l4 \vee \underline{l5})$

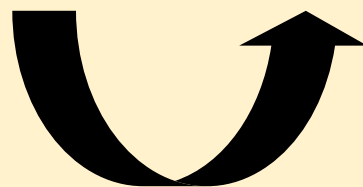
$\begin{array}{cccc} \textbf{F} & \textbf{F} & \textbf{V} & \textbf{F} \\ \uparrow & \uparrow & \uparrow & \uparrow \\ (l1 \vee l2 \vee \textcolor{red}{p1}) \wedge (\neg \textcolor{red}{p1} \vee l3 \vee \textcolor{red}{p2}) \wedge (\neg \textcolor{red}{p2} \vee l4 \vee l5) \end{array}$



$$(l1 \vee l2 \vee l3 \vee l4 \vee \underline{l5})$$

$$\begin{array}{ccccc} \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ F & F & V & F & F \\ (l1 \vee l2 \vee p1) \wedge (\neg p1 \vee l3 \vee p2) \wedge (\neg p2 \vee l4 \vee l5) \end{array}$$


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$$\begin{array}{ccccccc} \uparrow & \uparrow & \uparrow & & \uparrow & \uparrow & \uparrow \\ F & F & V & & F & F & V \\ (l1 \vee l2 \vee p1) \wedge (\neg p1 \vee l3 \vee p2) \wedge (\neg p2 \vee l4 \vee l5) \end{array}$$


$$(l1 \vee l2 \vee l3 \vee l4 \vee \underline{l5})$$

$$\begin{array}{ccccccc}
 \begin{array}{c} \textbf{F} \\ \uparrow \end{array} & \begin{array}{c} \textbf{F} \\ \uparrow \end{array} & \begin{array}{c} \textbf{V} \\ \uparrow \end{array} & & \begin{array}{c} \textbf{F} \\ \uparrow \end{array} & \begin{array}{c} \textbf{F} \\ \uparrow \end{array} & \begin{array}{c} \textbf{V} \\ \uparrow \end{array} & & \begin{array}{c} \textbf{F} \\ \uparrow \end{array} \\
 (l1 \vee l2 \vee \textcolor{red}{p1}) \wedge (\neg \textcolor{red}{p1} \vee l3 \vee \textcolor{red}{p2}) \wedge (\neg \textcolor{red}{p2} \vee l4 \vee l5)
 \end{array}$$

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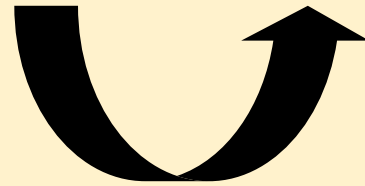
$$\begin{array}{ccccccc}
 \begin{array}{c} \textbf{F} \\ \uparrow \\ (l1 \vee l2 \vee p1) \end{array} & \begin{array}{c} \textbf{F} \\ \uparrow \\ l2 \end{array} & \begin{array}{c} \textbf{V} \\ \uparrow \\ p1 \end{array} & \wedge & \begin{array}{c} \textbf{F} \\ \uparrow \\ (\neg p1 \vee l3 \vee p2) \end{array} & \begin{array}{c} \textbf{F} \\ \uparrow \\ \neg p1 \end{array} & \begin{array}{c} \textbf{V} \\ \uparrow \\ l3 \end{array} & \wedge & \begin{array}{c} \textbf{F} \\ \uparrow \\ (\neg p2 \vee l4 \vee l5) \end{array} & \begin{array}{c} \textbf{F} \\ \uparrow \\ \neg p2 \end{array} & \begin{array}{c} \textbf{F} \\ \uparrow \\ l4 \end{array} & \begin{array}{c} \textbf{V} \\ \uparrow \\ l5 \end{array}
 \end{array}$$

$$(\underline{I1} \vee I2 \vee I3 \vee I4 \vee I5)$$

$$\begin{array}{cc} \mathbf{V} & \mathbf{F} \\ \uparrow & \uparrow \\ (I1 \vee I2 \vee \mathbf{p1}) \wedge (\neg \mathbf{p1} \vee I3 \vee \mathbf{p2}) \wedge (\neg \mathbf{p2} \vee I4 \vee I5) \end{array}$$

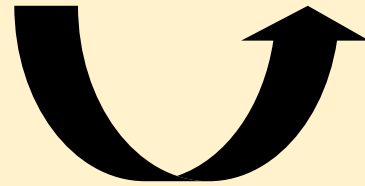
$(\underline{I1} \vee I2 \vee I3 \vee I4 \vee I5)$

$\begin{matrix} \uparrow & \uparrow & \uparrow \\ \mathbf{V} & \mathbf{F} & \mathbf{F} \end{matrix}$
 $(I1 \vee I2 \vee p1) \wedge (\neg p1 \vee I3 \vee p2) \wedge (\neg p2 \vee I4 \vee I5)$



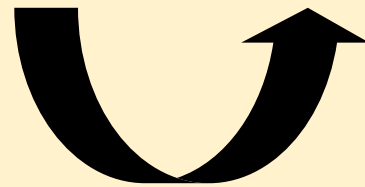
$(\underline{I1} \vee I2 \vee I3 \vee I4 \vee I5)$

$\begin{matrix} V & F & F & V \\ \uparrow & \uparrow & \uparrow & \uparrow \end{matrix}$
 $(I1 \vee I2 \vee p1) \wedge (\neg p1 \vee I3 \vee p2) \wedge (\neg p2 \vee I4 \vee I5)$

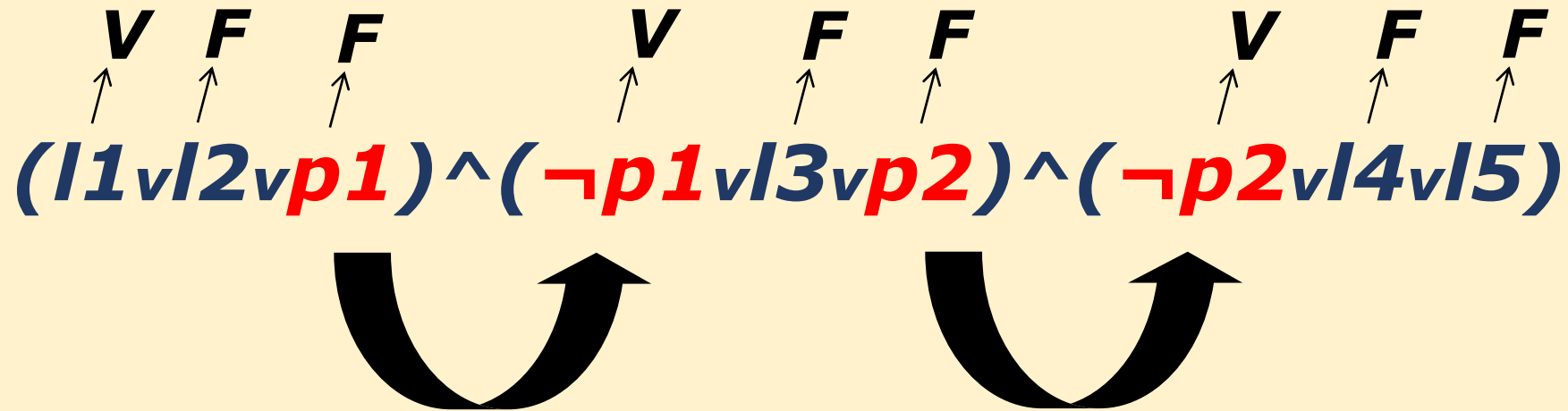


$(\underline{I1} \vee I2 \vee I3 \vee I4 \vee I5)$

$\begin{matrix} \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ V & F & F & V & F & F & F & F \end{matrix}$
 $(I1 \vee I2 \vee p1) \wedge (\neg p1 \vee I3 \vee p2) \wedge (\neg p2 \vee I4 \vee I5)$



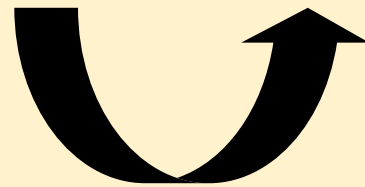
$(\underline{I1} \vee I2 \vee I3 \vee I4 \vee I5)$



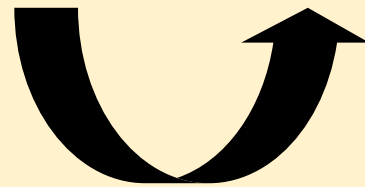
$$(l1 \vee l2 \vee \underline{l3} \vee l4 \vee l5)$$

$$\begin{array}{cc} \mathbf{F} & \mathbf{F} \\ \uparrow & \uparrow \\ (l1 \vee l2 \vee \mathbf{p1}) \wedge (\neg \mathbf{p1} \vee l3 \vee \mathbf{p2}) \wedge (\neg \mathbf{p2} \vee l4 \vee l5) \end{array}$$

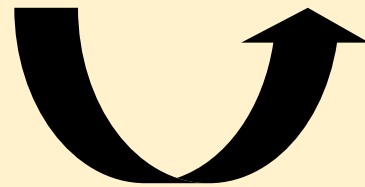
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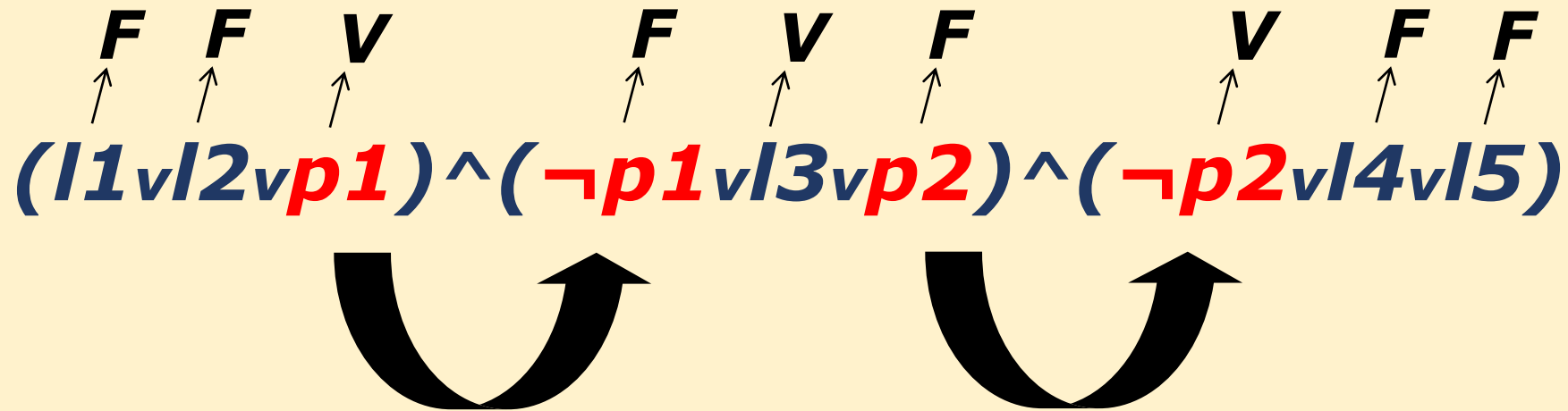
$$\begin{array}{ccccccc} & \uparrow & & \uparrow & & \uparrow & & \uparrow \\ & \mathbf{F} & & \mathbf{F} & & \mathbf{V} & & \mathbf{F} \\ (l1 \vee l2 \vee p1) \wedge (\neg p1 \vee l3 \vee p2) \wedge (\neg p2 \vee l4 \vee l5) \end{array}$$


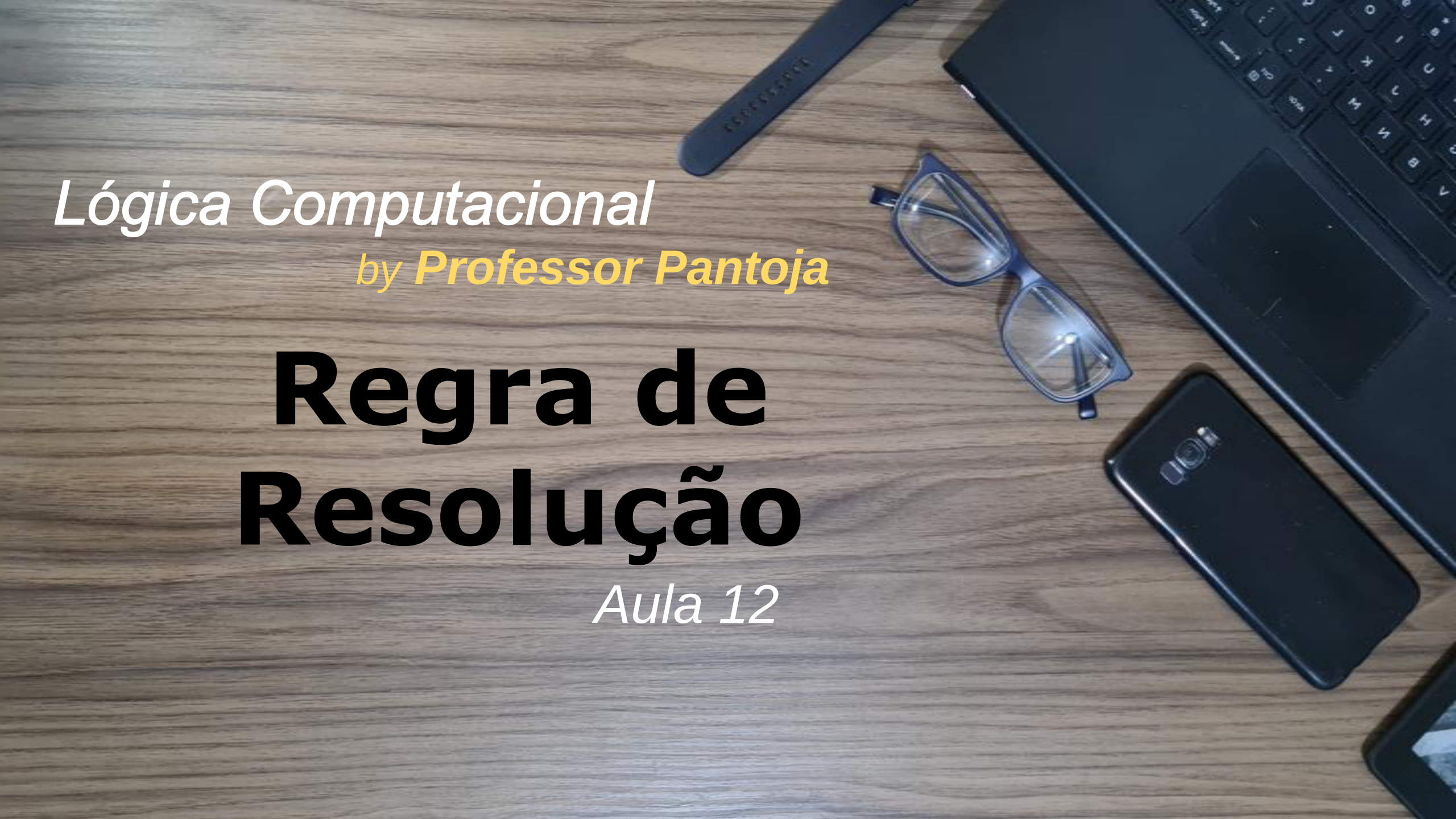
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$$\begin{array}{ccccc} \begin{array}{c} \textbf{F} \\ \uparrow \end{array} & \begin{array}{c} \textbf{F} \\ \uparrow \end{array} & \begin{array}{c} \textbf{V} \\ \uparrow \end{array} & \begin{array}{c} \textbf{F} \\ \uparrow \end{array} & \begin{array}{c} \textbf{V} \\ \uparrow \end{array} \\ (l1 \vee l2 \vee \textcolor{red}{p1}) \wedge (\neg \textcolor{red}{p1} \vee l3 \vee \textcolor{red}{p2}) \wedge (\neg \textcolor{red}{p2} \vee l4 \vee l5) \end{array}$$


$$(l1 \vee l2 \vee \underline{l3} \vee l4 \vee l5)$$

$$\begin{array}{ccccccc} \uparrow & \uparrow & \uparrow & & \uparrow & \uparrow & & \uparrow & \uparrow \\ F & F & V & & F & V & & F & F \\ (l1 \vee l2 \vee p1) \wedge (\neg p1 \vee l3 \vee p2) \wedge (\neg p2 \vee l4 \vee l5) \end{array}$$


$$(l1 \vee l2 \vee \underline{l3} \vee l4 \vee l5)$$


A top-down view of a wooden desk. On the right side, there is a black laptop with its keyboard visible. Below the laptop, a pair of blue-rimmed glasses lies on the desk. To the left of the glasses, a black pen is positioned diagonally. Further down and to the right, a black smartphone is lying flat. The background is the natural wood grain of the desk.

Lógica Computacional
by Professor Pantoja

Regra de Resolução

Aula 12

Sejam C1 e C2 Cláusulas tal que

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$$l \in C1, l^c \in C2$$

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Sejam C1 e C2 Cláusulas tal que

$$l \in C1, l^c \in C2 \quad \left(\begin{array}{c} \textit{Cláusulas com} \\ \textit{pares} \\ \textit{complementares} \end{array} \right)$$

A Resolução é:

Sejam $C1$ e $C2$ Cláusulas tal que

$$l \in C1, l^c \in C2 \quad \left(\begin{array}{c} \text{Cláusulas com} \\ \text{pares} \\ \text{complementares} \end{array} \right)$$

A Resolução é:

$$Res(C_1, C_2) = \{C_1 - l\} \cup \{C_2 - l^c\}$$

$$\{ab\bar{c}, bc\bar{e}\}$$

$$\{ab\bar{c}, bc\bar{e}\}$$

$$\{ab\bar{c}, bc\bar{e}\}$$

$$\{ab\bar{c} - \{\bar{c}\}, bc\bar{e} - \{c\}\}$$

$$\{ab\bar{c}, bc\bar{e}\}$$



$$\{ab\bar{c} - \{\bar{c}\}, bc\bar{e} - \{c\}\}$$



$$\{\{ab\} \cup \{b\bar{e}\}\}$$

$$\{ab\bar{c}, bc\bar{e}\}$$



$$\{ab\bar{c} - \{\bar{c}\}, bc\bar{e} - \{c\}\}$$



$$\{\{ab\} \cup \{b\bar{e}\}\}$$



$$\{\{abb\bar{e}\}\}$$

$$\{ab\bar{c}, bc\bar{e}\}$$



$$\{ab\bar{c} - \{\bar{c}\}, bc\bar{e} - \{c\}\}$$



$$\{\{ab\} \cup \{b\bar{e}\}\}$$



$$\{ab\bar{e}\}$$

Quando duas cláusulas possuem mais do que 2 pares complementares

Quando duas cláusulas possuem mais do que 2 pares complementares

$$\{l_1, l_2\} \cup C_1$$

$$\{l_1^c, l_2^c\} \cup C_2$$

Quando duas cláusulas possuem mais do que 2 pares complementares

$$\{l_1, l_2\} \cup C_1 \qquad \{l_1^c, l_2^c\} \cup C_2$$

$$\{l_2, l_2^c\} \cup C_1 \cup C_2$$

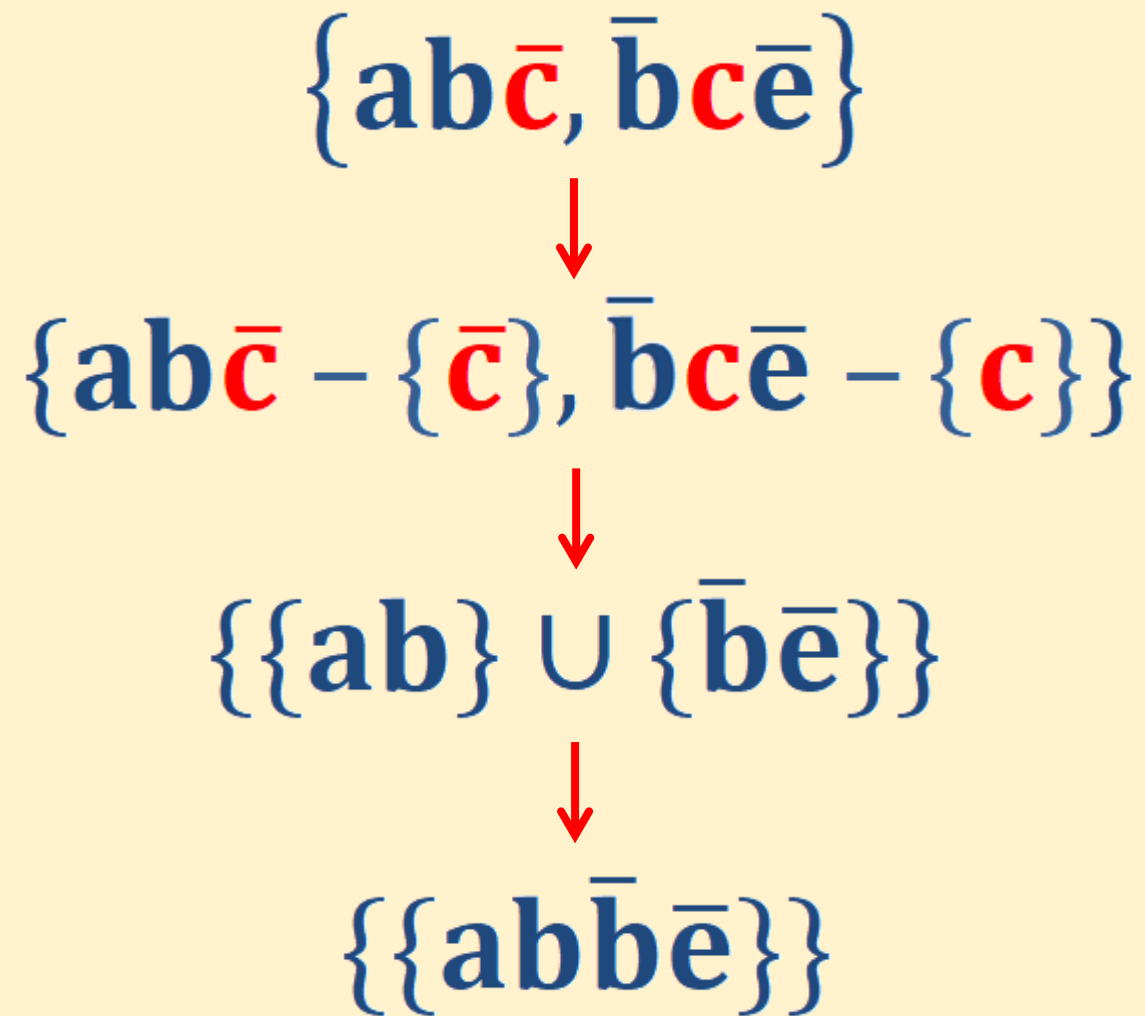
$$\{ab\bar{c}, \bar{b}c\bar{e}\}$$

$$\{ab\bar{c}, \bar{b}c\bar{e}\}$$

↓

$$\{ab\bar{c} - \{\bar{c}\}, \bar{b}c\bar{e} - \{c\}\}$$

$$\begin{array}{c} \{ab\bar{c}, \bar{b}c\bar{e}\} \\ \downarrow \\ \{ab\bar{c} - \{\bar{c}\}, \bar{b}c\bar{e} - \{c\}\} \\ \downarrow \\ \{\{ab\} \cup \{\bar{b}\bar{e}\}\} \end{array}$$



$$\begin{aligned} & \{ab\bar{c}, \bar{b}c\bar{e}\} \\ & \quad \downarrow \\ & \{ab\bar{c} - \{\bar{c}\}, \bar{b}c\bar{e} - \{c\}\} \\ & \quad \downarrow \\ & \{\{ab\} \cup \{\bar{b}\bar{e}\}\} \\ & \quad \downarrow \\ & \{\{ab\bar{b}\bar{e}\}\} \equiv \top \end{aligned}$$

Entrada: conjunto de cláusulas **S**

Saída: S é **SAT** ou **UNSAT**

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Saída: S é **SAT** ou **UNSAT**

1. $S_0 = S$

Entrada: conjunto de cláusulas **S**

Saída: S é **SAT** ou **UNSAT**

1. $S_0 = S$

2. Repetir os passos para obter S_{i+1} de S_i :

Entrada: conjunto de cláusulas S

Saída: S é **SAT** ou **UNSAT**

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1. Selecionar um par contraditório que ainda não tenha sido escolhido antes

Entrada: conjunto de cláusulas S

Saída: S é **SAT** ou **UNSAT**

1. $S_0 = S$

2. Repetir os passos para obter S_{i+1} de S_i :

1. Selecionar um par contraditório que ainda não tenha sido escolhido antes

2. Computar $C = \text{Res}(C_1, C_2)$

3. Se C não é uma Cláusula Trivial

3. Se C não é uma Cláusula Trivial

$$S_{i+1} = S_i \cup \{C\}$$

3. Se C não é uma Cláusula Trivial

$$S_{i+1} = S_i \cup \{C\}$$

senão

3. Se C não é uma Cláusula Trivial

$$S_{i+1} = S_i \cup \{C\}$$

senão

$$S_{i+1} = S_i$$

3. Se C não é uma Cláusula Trivial

$$S_{i+1} = S_i \cup \{C\}$$

senão

$$S_{i+1} = S_i$$

4. Termina Se:

3. Se C não é uma Cláusula Trivial

$$S_{i+1} = S_i \cup \{C\}$$

senão

$$S_{i+1} = S_i$$

4. Termina Se:

a) $C = \square$

3. Se C não é uma Cláusula Trivial

$$S_{i+1} = S_i \cup \{C\}$$

senão

$$S_{i+1} = S_i$$

4. Termina Se:

a) $C = \square$

b) Todos os pares complementares foram resolvidos

$$S_0 = \{\overline{p}qr, \overline{p}q, p, \overline{r}\}$$

$$S_0 = \{\overline{p}q\overline{r}, \overline{p}q, p, \overline{r}\}$$

$$S_0 = \{\overline{p}q\overline{r}, \overline{p}q, p, \overline{r}\} \rightarrow \{\overline{p}r\}$$

$$S_0 = \{\overline{p}q\overline{r}, \overline{p}q, p, \overline{r}\} \rightarrow \{\overline{p}r\}$$

$$S_1 = \{\overline{p}q\overline{r}, \overline{p}q, p, \overline{r}, \overline{p}r\}$$

$$S_0 = \{\overline{p}q\overline{r}, \overline{p}q, p, \overline{r}\} \rightarrow \{\overline{p}r\}$$

$$S_1 = \{\overline{p}q\overline{r}, \overline{p}q, p, \overline{r}, \overline{p}r\}$$

$$S_0 = \{\overline{p}qr, \overline{p}q, p, \overline{r}\} \rightarrow \{\overline{p}r\}$$

$$S_1 = \{\overline{p}qr, \overline{p}q, p, \overline{r}, \overline{p}r\} \rightarrow \{r\}$$

$$S_0 = \{\overline{p}q\overline{r}, \overline{p}q, p, \overline{r}\} \rightarrow \{\overline{p}r\}$$

$$S_1 = \{\overline{p}q\overline{r}, \overline{p}q, p, \overline{r}, \overline{p}r\} \rightarrow \{r\}$$

$$S_2 = \{\overline{p}q\overline{r}, \overline{p}q, p, \overline{r}, \overline{p}r, r\}$$

$$S_0 = \{\overline{p}q\mathbf{r}, \overline{p}q, \mathbf{p}, \bar{r}\} \rightarrow \{\overline{p}\mathbf{r}\}$$

$$S_1 = \{\overline{p}q\mathbf{r}, \overline{p}q, \mathbf{p}, \bar{r}, \overline{p}\mathbf{r}\} \rightarrow \{\mathbf{r}\}$$

$$S_2 = \{\overline{p}q\mathbf{r}, \overline{p}q, \mathbf{p}, \bar{r}, \overline{p}\mathbf{r}, \mathbf{r}\}$$

$$S_0 = \{\overline{p}q\overline{r}, \overline{p}q, p, \overline{r}\} \rightarrow \{\overline{p}r\}$$

$$S_1 = \{\overline{p}q\overline{r}, \overline{p}q, p, \overline{r}, \overline{p}r\} \rightarrow \{r\}$$

$$S_2 = \{\overline{p}q\overline{r}, \overline{p}q, p, \overline{r}, \overline{p}r, r\} \rightarrow \{\square\}$$

$$S_0 = \{\overline{p}q\overline{r}, \overline{p}q, p, \overline{r}\} \rightarrow \{\overline{p}r\}$$

$$S_1 = \{\overline{p}q\overline{r}, \overline{p}q, p, \overline{r}, \overline{p}r\} \rightarrow \{r\}$$

$$S_2 = \{\overline{p}q\overline{r}, \overline{p}q, p, \overline{r}, \overline{p}r, r\} \rightarrow \{\square\}$$

$$S_3 = \{\overline{p}q\overline{r}, \overline{p}q, p, \overline{r}, \overline{p}r, r, \square\}$$

A top-down view of a wooden desk. On the right side, there is a black laptop with its keyboard visible. Below the laptop, a pair of blue-rimmed glasses lies on the desk. To the left of the glasses is a black pen. Further down and to the right is a black smartphone. The text is overlaid on the left side of the image.

Lógica Computacional
by Professor Pantoja

Algoritmo Davis-Putnam

Aula 13

- **Literais Puros (Pure Literals)**

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 - **É um literal que aparece em:**

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 - **É um literal que aparece em:**
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- **Literais Puros (Pure Literals)**
 - **É um literal que aparece em:**
 - a. **pelo menos em uma cláusula em S**
 - b. **seu complemento não aparece em nenhuma cláusula**

$$S' = S - \{\text{quem tem } l\}$$

$$S = \{pqr, p\bar{q}, \bar{p}q\}$$

$$S = \{pq\bar{r}, p\bar{q}, \bar{p}q\}$$

$$S = \{pq\bar{r}, p\bar{q}, \bar{p}q\}$$

$$S = \{\mathbf{p}\mathbf{q}\bar{r}, p\bar{q}, \bar{p}q\}$$

$$S = \{pq\bar{r}, p\bar{q}, \bar{p}q\}$$

$$S = \{\mathbf{pq}\bar{r}, p\bar{q}, \bar{p}q\}$$

$$S' = \{p\bar{q}, \bar{p}q\}$$

- **Cláusulas Unitárias (Unit Clauses)**

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$$\{l\} \in S$$

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$$\{l\} \in S$$

$$S' = S - \{\text{quem tem } l\}$$

- **Cláusulas Unitárias (Unit Clauses)**

$$\{l\} \in S$$

$$S' = S - \{\text{quem tem } l\}$$

$$- \{\text{removendo } l^c \text{ das demais}\}$$

$$S = \{r, pq\bar{r}, p\bar{q}, \bar{q}s\}$$

$$S = \{\mathbf{r}, p\mathbf{q}\bar{\mathbf{r}}, p\bar{\mathbf{q}}, \bar{\mathbf{q}}s\}$$

$$S = \{\mathbf{r}, pq\mathbf{\bar{r}}, p\bar{q}, \bar{q}s\}$$

$$S = \{pq\mathbf{\bar{r}}, p\bar{q}, \bar{q}s\}$$

$$S = \{\mathbf{r}, pq\mathbf{\bar{r}}, p\bar{q}, \bar{q}s\}$$

$$S = \{pq\mathbf{\bar{r}}, p\bar{q}, \bar{q}s\}$$

$$S' = \{pq, p\bar{q}, \bar{q}s\}$$

Entrada: fórmula em Forma Clausal

Saída: S é SAT ou UNSAT

Entrada: fórmula em Forma Clausal

Saída: S é SAT ou UNSAT

- 1. Aplique repetidamente, mas a III só é usada caso não seja aplicável I e II.***

Entrada: fórmula em Forma Clausal

Saída: S é SAT ou UNSAT

1. Aplique repetidamente, mas a III só é usada caso não seja aplicável I e II.

I. Regra do Literal Unitário

Entrada: fórmula em Forma Clausal

Saída: S é **SAT** ou **UNSAT**

1. Aplique repetidamente, mas a III só é usada caso não seja aplicável I e II.

I. Regra do Literal Unitário

II. Regra do Literal Puro

Entrada: fórmula em Forma Clausal

Saída: S é **SAT** ou **UNSAT**

1. Aplique repetidamente, mas a III só é usada caso não seja aplicável I e II.

I. Regra do Literal Unitário

II. Regra do Literal Puro

III. Regra:

a) escolha um átomo p e execute a Resolução

Entrada: fórmula em Forma Clausal

Saída: S é **SAT** ou **UNSAT**

1. Aplique repetidamente, mas a III só é usada caso não seja aplicável I e II.

I. Regra do Literal Unitário

II. Regra do Literal Puro

III. Regra:

a) escolha um átomo p e execute a Resolução

b) adicione os resolventes a S

Entrada: fórmula em Forma Clausal

Saída: S é SAT ou UNSAT

1. Aplique repetidamente, mas a III só é usada caso não seja aplicável I e II.

I. Regra do Literal Unitário

II. Regra do Literal Puro

III. Regra:

a) escolha um átomo p e execute a Resolução

b) adicione os resolventes a S

c) elimine todas as cláusulas contendo p e $\bar{p}$

2. Termina Se:

2. Termina Se:

a) a cláusula vazia $\{\square\}$ for produzida - UNSAT

2. Termina Se:

- a) a cláusula vazia $\{\square\}$ for produzida - UNSAT***
- b) as regras não são mais aplicáveis - SAT***

$$S = \{p, \bar{p}q, \bar{q}r, \bar{r}st\}$$

$$S = \{p, \bar{p}q, \bar{q}r, \bar{r}st\}$$

$$S' = \{\textcolor{red}{p}, \textcolor{red}{\bar{p}}q, \bar{q}r, \bar{r}st\}$$

$$S = \{p, \bar{p}q, \bar{q}r, \bar{r}st\}$$

$$S' = \{q, \bar{q}r, \bar{r}st\}$$

$$S = \{p, \bar{p}q, \bar{q}r, \bar{r}st\}$$

$$S' = \{q, \bar{q}r, \bar{r}st\}$$

$$S' = \{\textcolor{red}{q}, \textcolor{red}{\bar{q}}r, \bar{r}st\}$$

$$S = \{p, \bar{p}q, \bar{q}r, \bar{r}st\}$$

$$S' = \{q, \bar{q}r, \bar{r}st\}$$

$$S' = \{r, \bar{r}st\}$$

$$S = \{p, \bar{p}q, \bar{q}r, \bar{r}st\}$$

$$S' = \{q, \bar{q}r, \bar{r}st\}$$

$$S' = \{r, \bar{r}st\}$$

$$S' = \{\mathbf{r}, \bar{\mathbf{r}}st\}$$

$$S = \{p, \bar{p}q, \bar{q}r, \bar{r}st\}$$

$$S' = \{q, \bar{q}r, \bar{r}st\}$$

$$S' = \{r, \bar{r}st\}$$

$$S' = \{st\}$$

$$S = \{p, \bar{p}q, \bar{q}r, \bar{r}st\}$$

$$S' = \{q, \bar{q}r, \bar{r}st\}$$

$$S' = \{r, \bar{r}st\}$$

$$S' = \{st\}$$

A aplicação repetida da regra do literal unitário até ela não ser mais aplicável é conhecida como

unit propagation ou Boolean constraint propagation.

$$S = \{r, pq\bar{r}, p\bar{q}, \bar{q}r\}$$

$$S = \{\mathbf{r}, pq\bar{\mathbf{r}}, p\bar{q}, \bar{q}\bar{\mathbf{r}}\}$$



L. Unitário

$$S = \{\mathbf{r}, pq\bar{\mathbf{r}}, p\bar{q}, \bar{q}\mathbf{r}\}$$



L. Unitário

$$S' = \{pq, p\bar{q}, \bar{q}\}$$

$$S = \{\mathbf{r}, p\mathbf{q}\bar{\mathbf{r}}, p\bar{\mathbf{q}}, \bar{\mathbf{q}}\bar{\mathbf{r}}\}$$

→ **L. Unitário**

$$S' = \{p\mathbf{q}, p\bar{\mathbf{q}}, \bar{\mathbf{q}}\}$$

→ **L. Unitário**

$$S = \{\mathbf{r}, pq\mathbf{\bar{r}}, p\bar{q}, \bar{q}\mathbf{\bar{r}}\}$$

→ **L. Unitário**

$$S' = \{\mathbf{p}\mathbf{q}, \mathbf{p}\bar{q}, \bar{q}\}$$

→ **L. Unitário**

$$S' = \{\mathbf{p}\}$$

$$S = \{\overline{p}q\overline{r}, \overline{p}q, p, \overline{r}\}$$

$$S = \{\overline{p}q\mathbf{r}, \overline{p}q, p, \mathbf{\bar{r}}\}$$



L. Unitário

$$S = \{\overline{p}q\mathbf{r}, \overline{p}q, p, \mathbf{\bar{r}}\}$$



L. Unitário

$$S' = \{\overline{p}q, \overline{p}q, p\}$$

$$S = \{\overline{p}q\mathbf{r}, \overline{p}q, p, \mathbf{\bar{r}}\}$$

→ L. Unitário

$$S' = \{\mathbf{\bar{p}}\overline{q}, \overline{p}q, \mathbf{p}\}$$

→ L. Unitário

$$S = \{\overline{p}q\mathbf{r}, \overline{p}q, p, \mathbf{\bar{r}}\}$$

→ L. Unitário

$$S' = \{\mathbf{\bar{p}}\overline{q}, \overline{p}q, \mathbf{p}\}$$

→ L. Unitário

$$S' = \{\overline{q}, q\}$$

$$S = \{\overline{p}q\mathbf{r}, \overline{p}q, p, \mathbf{\bar{r}}\}$$

→ L. Unitário

$$S' = \{\mathbf{\bar{p}}\overline{q}, \overline{p}q, \mathbf{p}\}$$

→ L. Unitário

$$S' = \{\overline{q}, q\}$$

→ $\{\square\}$

$$S = \{p\bar{q}, pq, \bar{p}r, \bar{p}\bar{r}, sr\}$$

$$S = \{p\bar{q}, pq, \bar{p}r, \bar{p}\bar{r}, \textcolor{red}{sr}\} \rightarrow \textbf{L. Puro}$$

$$S = \{p\bar{q}, pq, \bar{p}r, \bar{p}\bar{r}, \textcolor{red}{sr}\} \rightarrow \text{L. Puro}$$

$$S' = \{p\bar{q}, pq, \bar{p}r, \bar{p}\bar{r}\}$$

$$S = \{p\bar{q}, pq, \bar{p}r, \bar{p}\bar{r}, \textcolor{red}{sr}\}$$

→ L. Puro

$$S' = \{\textcolor{red}{p}\bar{q}, \textcolor{red}{p}q, \bar{p}r, \bar{p}\bar{r}\}$$

→ Resolução

$$S = \{p\bar{q}, pq, \bar{p}r, \bar{p}\bar{r}, \textcolor{red}{sr}\}$$

→ L. Puro

$$S' = \{\textcolor{red}{p}\bar{q}, \textcolor{red}{p}q, \bar{p}r, \bar{p}\bar{r}\}$$

→ Resolução

$$S' = \{p, \bar{p}r, \bar{p}\bar{r}\}$$

$$S = \{p\bar{q}, pq, \bar{p}r, \bar{p}\bar{r}, \textcolor{red}{sr}\}$$

→ L. Puro

$$S' = \{\textcolor{red}{p}\bar{q}, \textcolor{red}{p}q, \bar{p}r, \bar{p}\bar{r}\}$$

→ Resolução

$$S' = \{\textcolor{red}{p}, \bar{p}r, \bar{p}\bar{r}\}$$

→ L. Unitário

$$S = \{p\bar{q}, pq, \bar{p}r, \bar{p}\bar{r}, \textcolor{red}{sr}\}$$

→ L. Puro

$$S' = \{\textcolor{red}{p}\bar{q}, \textcolor{red}{p}q, \bar{p}r, \bar{p}\bar{r}\}$$

→ Resolução

$$S' = \{\textcolor{red}{p}, \bar{p}\textcolor{red}{r}, \bar{p}\bar{\textcolor{red}{r}}\}$$

→ L. Unitário

$$S' = \{r, \bar{r}\}$$

$$S = \{p\bar{q}, pq, \bar{p}r, \bar{p}\bar{r}, \textcolor{red}{sr}\}$$

→ L. Puro

$$S' = \{\textcolor{red}{p}\bar{q}, \textcolor{red}{p}q, \bar{p}r, \bar{p}\bar{r}\}$$

→ Resolução

$$S' = \{\textcolor{red}{p}, \bar{p}r, \textcolor{red}{p}\bar{r}\}$$

→ L. Unitário

$$S' = \{r, \bar{r}\}$$

→ $\{\square\}$

A top-down view of a wooden desk. On the right side, there is a black laptop with its keyboard visible. Below the laptop, a pair of blue-rimmed glasses lies on the desk. To the left of the glasses, a black pen is positioned diagonally. Further down and to the right, a black smartphone is lying flat. The text is overlaid on the left side of the image.

Lógica Computacional
by Professor Pantoja

Algoritmo DPLL

Aula 14

Entrada: fórmula em Forma Clausal

Saída: S é SAT e retorna uma interpretação parcial ou **UNSAT**

Entrada: fórmula em Forma Clausal

Saída: S é SAT e retorna uma interpretação parcial ou **UNSAT**

I. Função Recursiva DPLL(S,I)

Entrada: fórmula em Forma Clausal

Saída: S é SAT e retorna uma interpretação parcial ou **UNSAT**

I. Função Recursiva DPLL(S,I)

i. Crie um Subconjunto S' aplicando repetidamente a Propagação Unitária (Unit Propagation).

Entrada: fórmula em Forma Clausal

Saída: S é SAT e retorna uma interpretação parcial ou **UNSAT**

I. Função Recursiva DPLL(S,I)

- i. Crie um Subconjunto S' aplicando repetidamente a Propagação Unitária (Unit Propagation).***
- ii. Crie I' adicionando todas as Interpretações assumidas no processo.***

Entrada: fórmula em Forma Clausal

Saída: S é SAT e retorna uma interpretação parcial ou **UNSAT**

I. Função Recursiva DPLL(S,I)

- i. Crie um Subconjunto S' aplicando repetidamente a Propagação Unitária (Unit Propagation).***
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II. Avalie S' a partir de I'

Entrada: fórmula em Forma Clausal

Saída: S é SAT e retorna uma interpretação parcial ou **UNSAT**

I. Função Recursiva DPLL(S,I)

- i. Crie um Subconjunto S' aplicando repetidamente a Propagação Unitária (Unit Propagation).***
- ii. Crie I' adicionando todas as Interpretações assumidas no processo.***

II. Avalie S' a partir de I'

- i. Se S' contém Cláusula Conflitante, retorne **UNSAT**.***

Entrada: fórmula em Forma Clausal

Saída: S é **SAT** e retorna uma interpretação parcial ou **UNSAT**

I. Função Recursiva DPLL(S,I)

- i. Crie um Subconjunto S' aplicando repetidamente a Propagação Unitária (Unit Propagation).***
- ii. Crie I' adicionando todas as Interpretações assumidas no processo.***

II. Avalie S' a partir de I'

- i. Se S' contém Cláusula Conflitante, retorne **UNSAT**.***
- ii. Se S' é **SAT** retorne I' .***

Entrada: fórmula em Forma Clausal

Saída: S é **SAT** e retorna uma interpretação parcial ou **UNSAT**

I. Função Recursiva DPLL(S,I)

- i. Crie um Subconjunto S' aplicando repetidamente a Propagação Unitária (Unit Propagation).***
- ii. Crie I' adicionando todas as Interpretações assumidas no processo.***

II. Avalie S' a partir de I'

- i. Se S' contém Cláusula Conflitante, retorne **UNSAT**.***
- ii. Se S' é **SAT** retorne I' .***
- iii. Caso Contrário, continuar.***

II. Selecionar um átomo p em S'

II. Selecionar um átomo p em S'

i. Escolha um valor V ou F .

II. Selecionar um átomo p em S'

i. Escolha um valor V ou F .

ii. $I1$ é a interpretação I' com a valoração escolhida.

II. Selecionar um átomo p em S'

i. Escolha um valor V ou F .

ii. $I1$ é a interpretação I' com a valoração escolhida.

III. Resultado = DPLL($S', I1$)

II. Selecionar um átomo p em S'

i. Escolha um valor V ou F .

ii. $I1$ é a interpretação I' com a valoração escolhida.

III. $Resultado = DPLL(S', I1)$

i. Se o $Resultado$ não for **UNSAT, retornar $Resultado$.**

II. Selecionar um átomo p em S'

i. Escolha um valor V ou F .

ii. $I1$ é a interpretação I' com a valoração escolhida.

III. $Resultado = DPLL(S', I1)$

i. Se o $Resultado$ não for **UNSAT, retornar $Resultado$.**

ii. Caso Contrário, continuar.

II. Selecionar um átomo p em S'

i. Escolha um valor V ou F .

ii. $I1$ é a interpretação I' com a valoração escolhida.

III. $Resultado = DPLL(S', I1)$

i. Se o $Resultado$ não for **UNSAT, retornar $Resultado$.**

ii. Caso Contrário, continuar.

IV. $I2$ é a interpretação I' com valoração complementar de p .

II. Selecionar um átomo p em S'

i. Escolha um valor V ou F .

ii. $I1$ é a interpretação I' com a valoração escolhida.

III. $Resultado = DPLL(S', I1)$

i. Se o $Resultado$ não for **UNSAT, retornar $Resultado$.**

ii. Caso Contrário, continuar.

IV. $I2$ é a interpretação I' com valoração complementar de p .

V. $Resultado = DPLL(S', I2)$

II. Selecionar um átomo p em S'

i. Escolha um valor V ou F .

ii. $I1$ é a interpretação I' com a valoração escolhida.

III. $Resultado = DPLL(S', I1)$

i. Se o $Resultado$ não for **UNSAT, retornar $Resultado$.**

ii. Caso Contrário, continuar.

IV. $I2$ é a interpretação I' com valoração complementar de p .

V. $Resultado = DPLL(S', I2)$

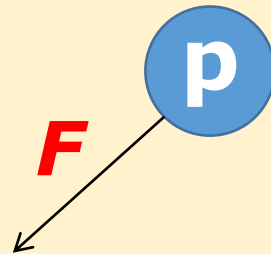
i. retornar $Resultado$.

$\{\bar{p}qr, prs, pr\bar{s}, p\bar{r}s, p\bar{r}\bar{s}, \bar{q}rs, \bar{p}q\bar{r}, \bar{p}q\bar{r}\}$

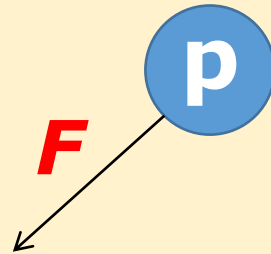
$\{\bar{p}qr, prs, pr\bar{s}, p\bar{r}s, p\bar{r}\bar{s}, \bar{q}rs, \bar{p}q\bar{r}, \bar{p}q\bar{r}\}$



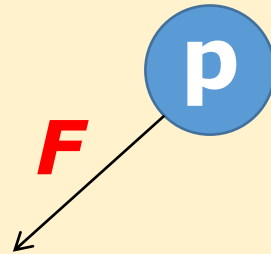
$\{\bar{p}qr, prs, pr\bar{s}, p\bar{r}s, p\bar{r}\bar{s}, \bar{q}rs, \bar{p}q\bar{r}, \bar{p}q\bar{r}\}$



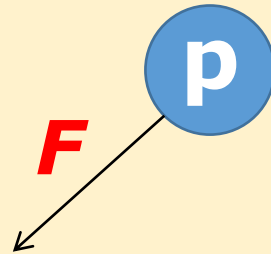
$\{\bar{p}qr, \boxed{p}rs, \boxed{p}r\bar{s}, \boxed{p}r\bar{s}, \boxed{p}r\bar{s}, \bar{q}rs, \bar{p}q\bar{r}, \bar{p}q\bar{r}\}$



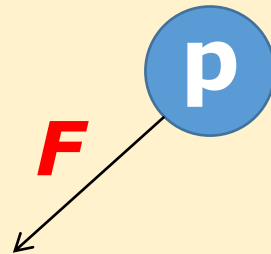
$\{\bar{p}qr, rs, r\bar{s}, \bar{r}s, \bar{r}\bar{s}, \bar{q}rs, \bar{p}q\bar{r}, \bar{p}q\bar{r}\}$



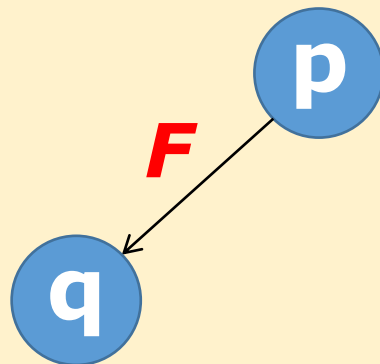
$\{pqr, rs, r\bar{s}, \bar{r}s, \bar{r}\bar{s}, \bar{q}rs, \bar{p}q\bar{r}, \bar{p}q\bar{r}\}$



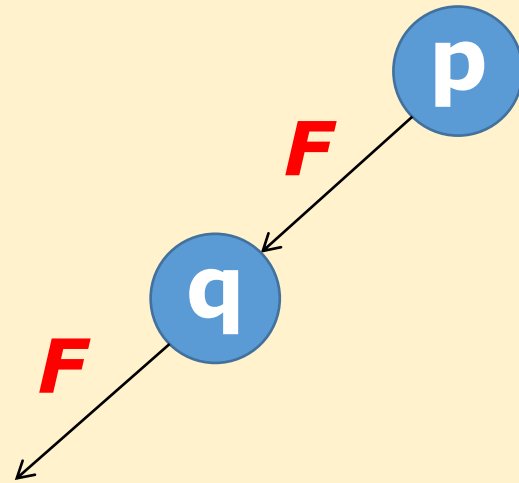
{ $rs, r\bar{s}, \bar{r}s, \bar{r}\bar{s}, \overline{qrs}$ }



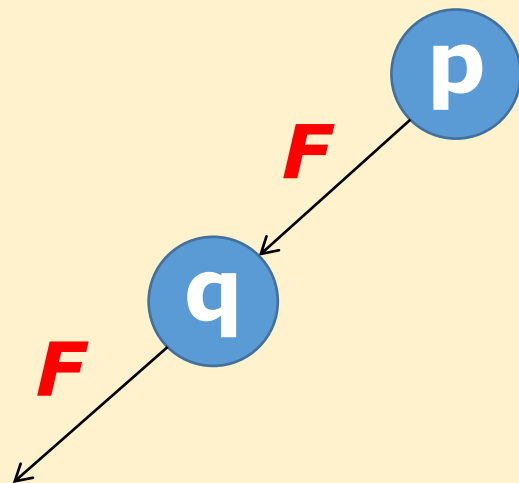
{ $rs, r\bar{s}, \bar{r}s, \bar{r}\bar{s}, \overline{qrs}$ }



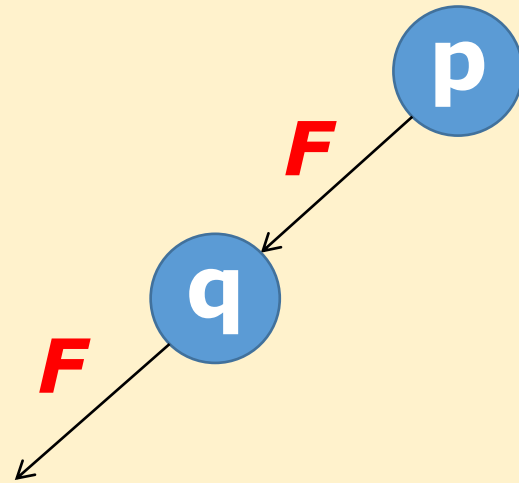
{ $rs, r\bar{s}, \bar{r}s, \bar{r}\bar{s}, \overline{qrs}$ }



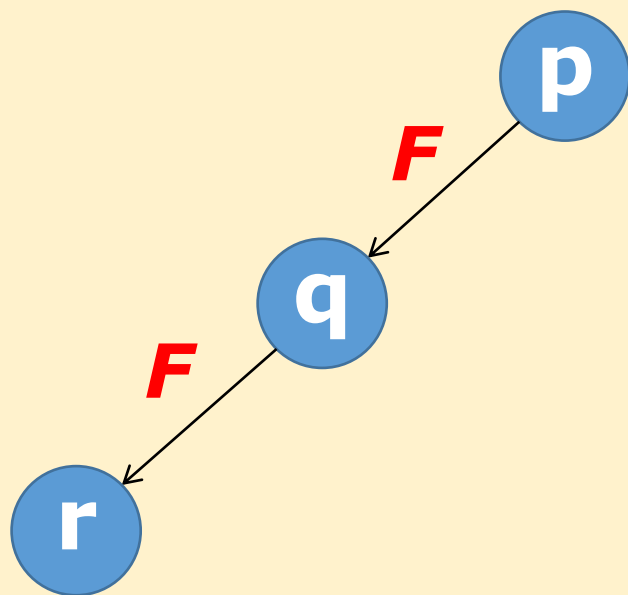
{ $rs, r\bar{s}, \bar{r}s, \bar{r}\bar{s}, \boxed{\bar{q}rs}$ }



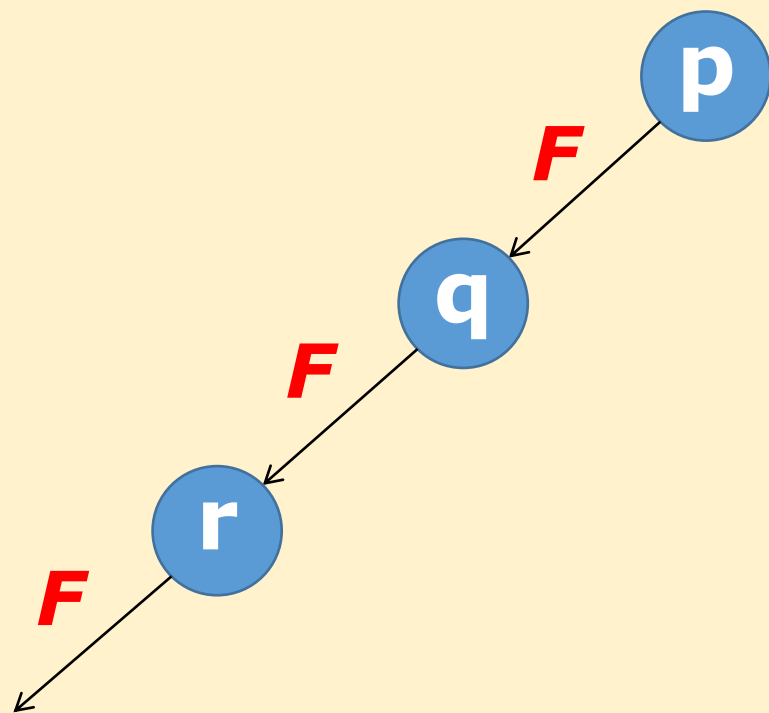
{ $rs, r\bar{s}, \bar{r}s, \bar{r}\bar{s},$ }



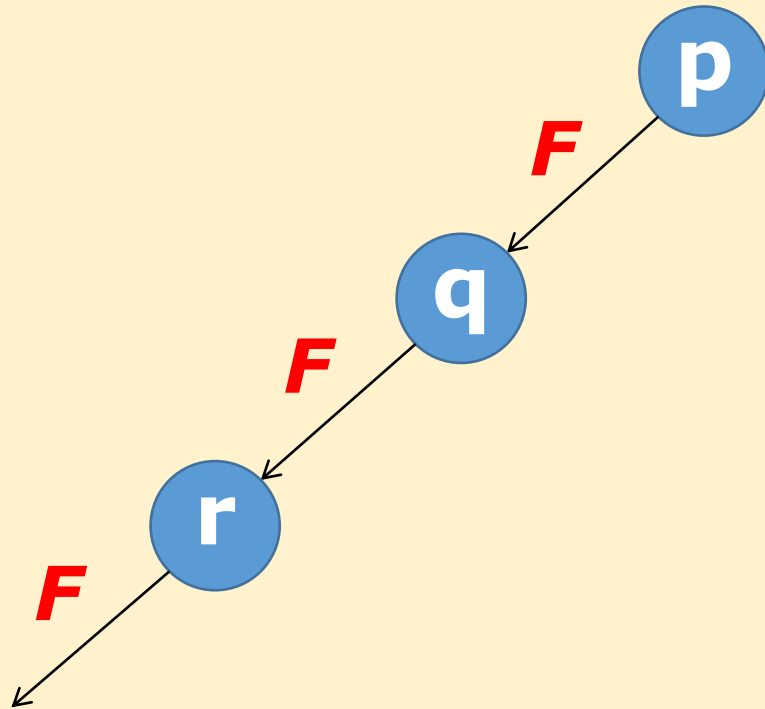
{ $rs, r\bar{s}, \bar{r}s, \bar{r}\bar{s},$ }



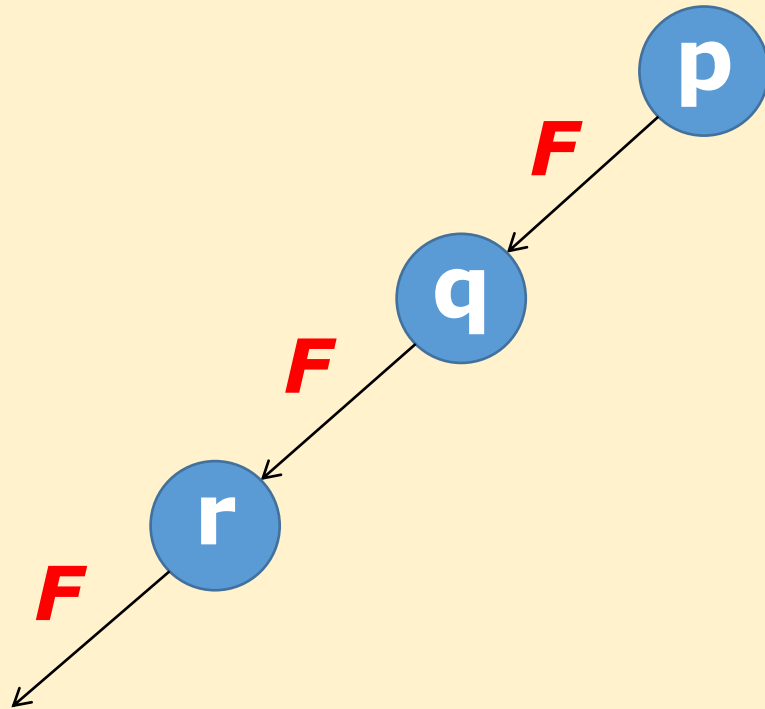
{ $rs, r\bar{s}, \bar{r}s, \bar{r}\bar{s},$ }



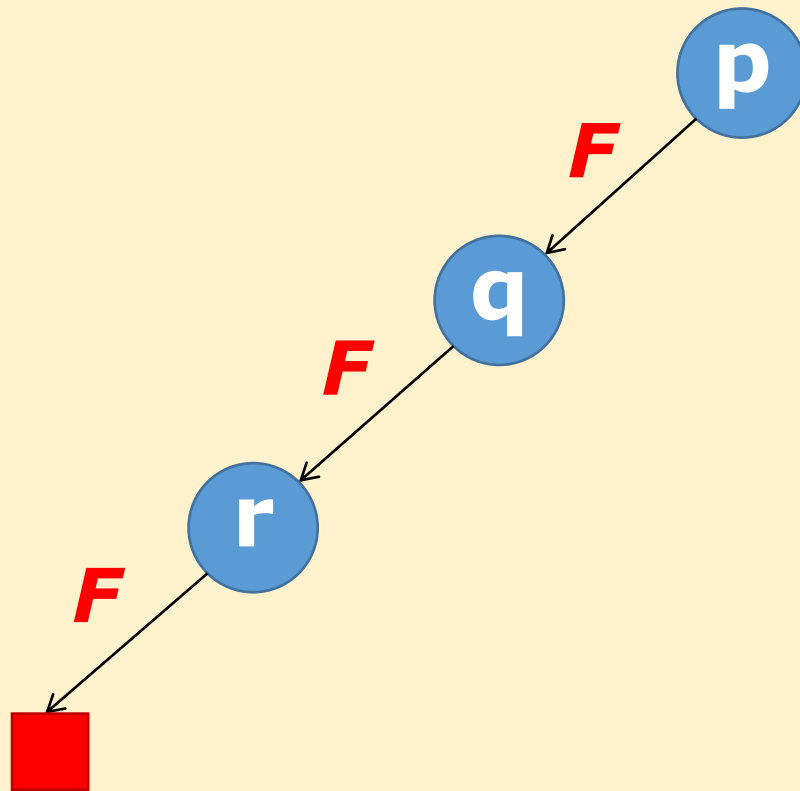
{ $\boxed{rs}$, $\boxed{r\bar{s}}$, $\bar{r}s$, $\bar{r}\bar{s}$, }



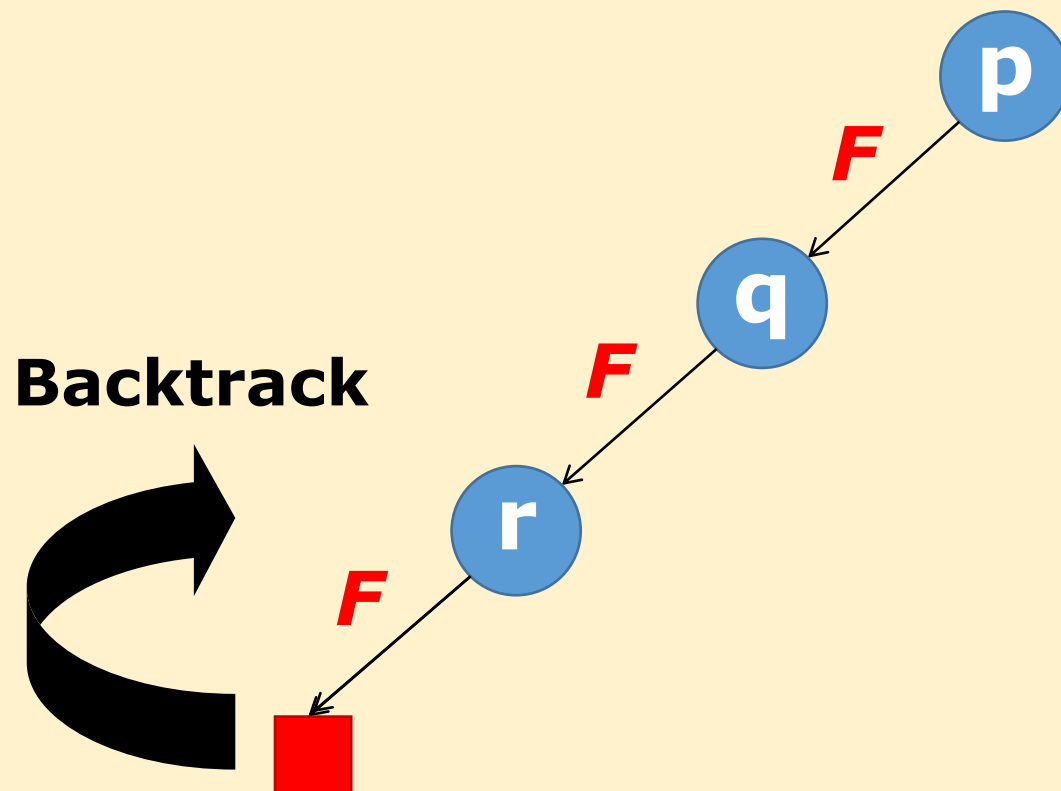
{ $s, \bar{s}, \bar{r}s, \bar{r}\bar{s},$ }



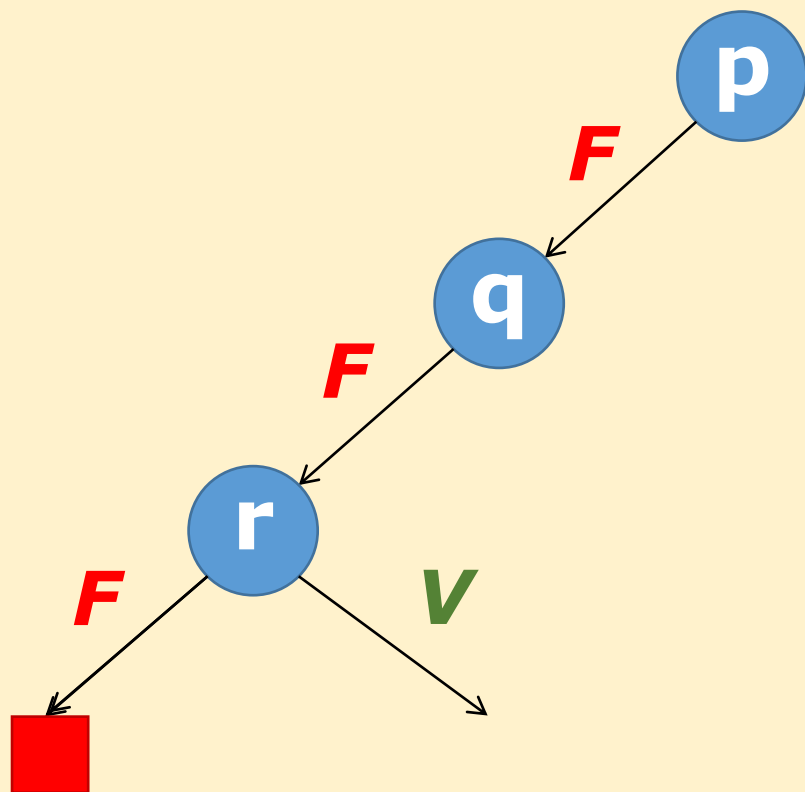
{ $s, \bar{s}, \bar{r}s, \overline{rs},$ }



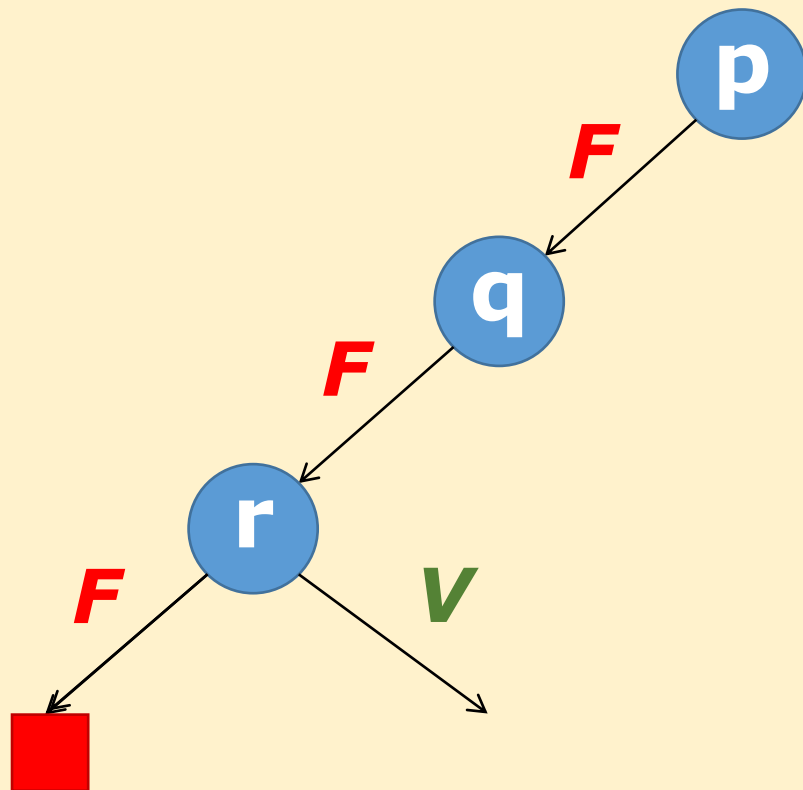
{ $rs, r\bar{s}, \bar{r}s, \bar{r}\bar{s},$ }



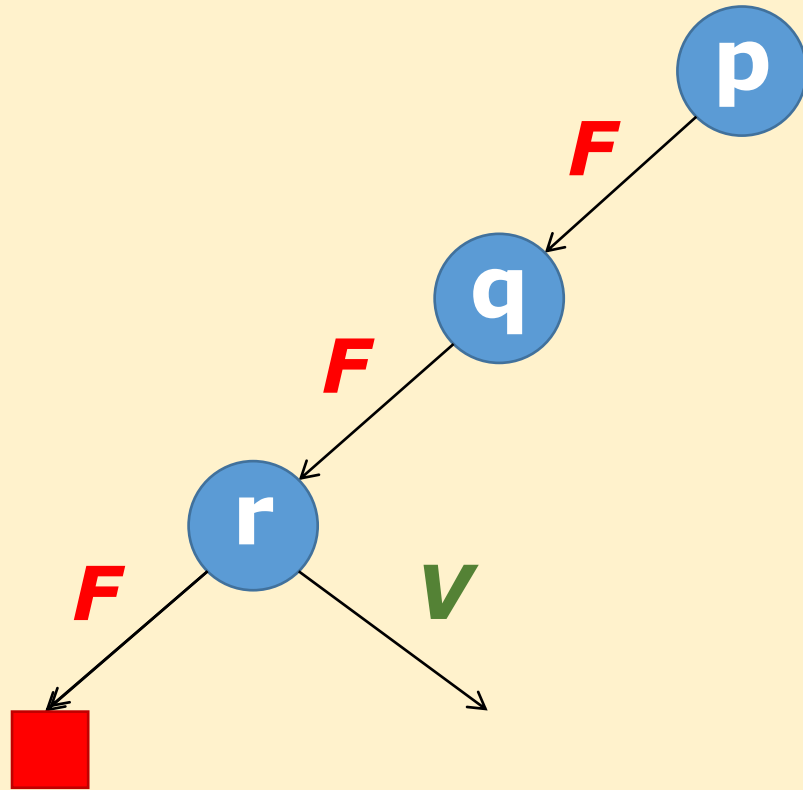
{ $rs, r\bar{s}, \bar{r}s, \bar{r}\bar{s},$ }



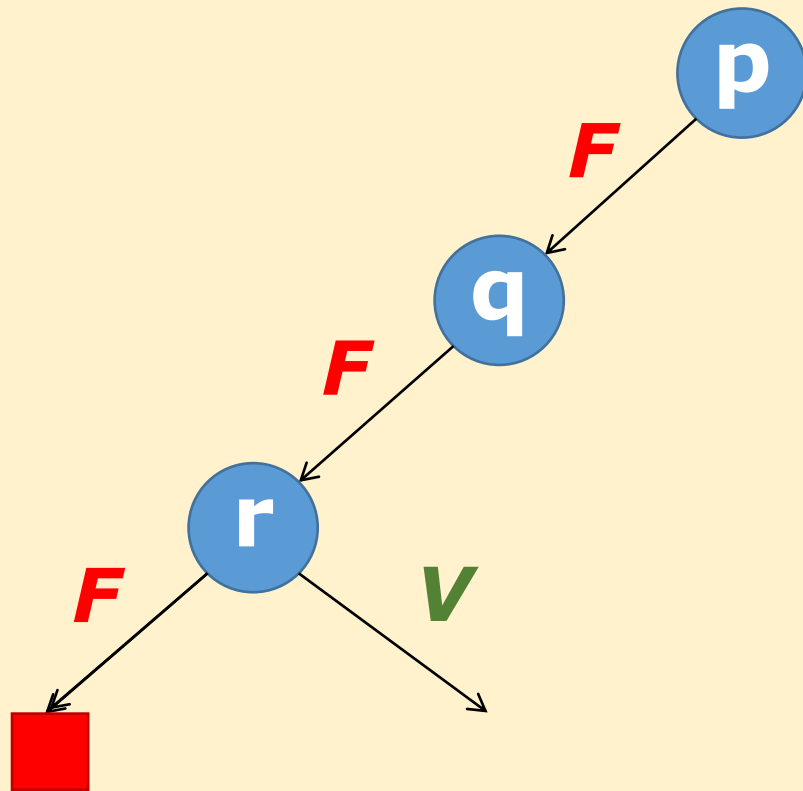
{ **rs, r $\bar{s}$,** $\bar{r}s$, $\bar{r}\bar{s}$, }



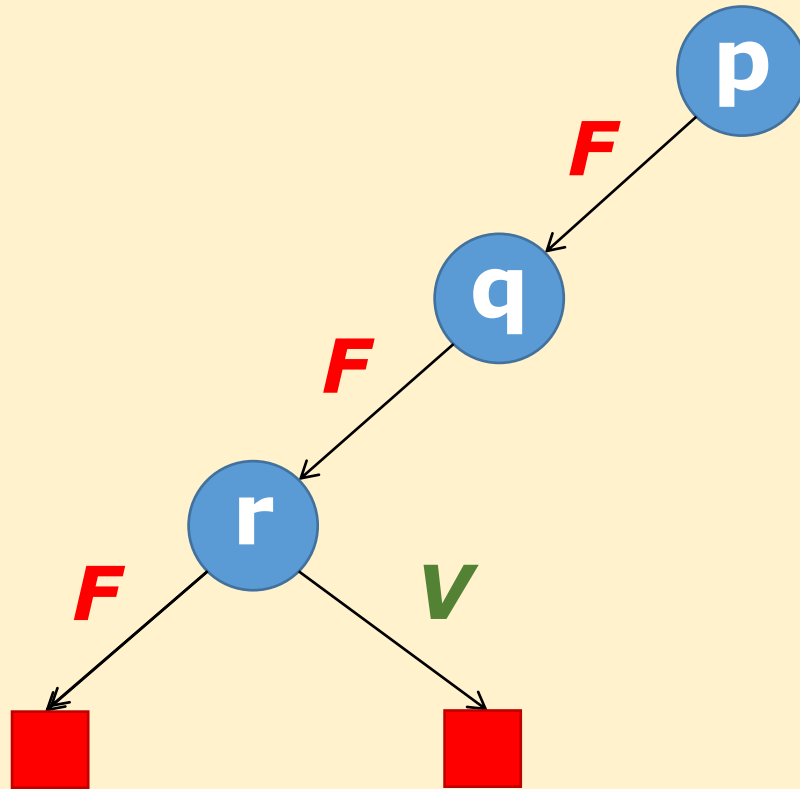
{ $\bar{r}s$, $\overline{r}s$, }



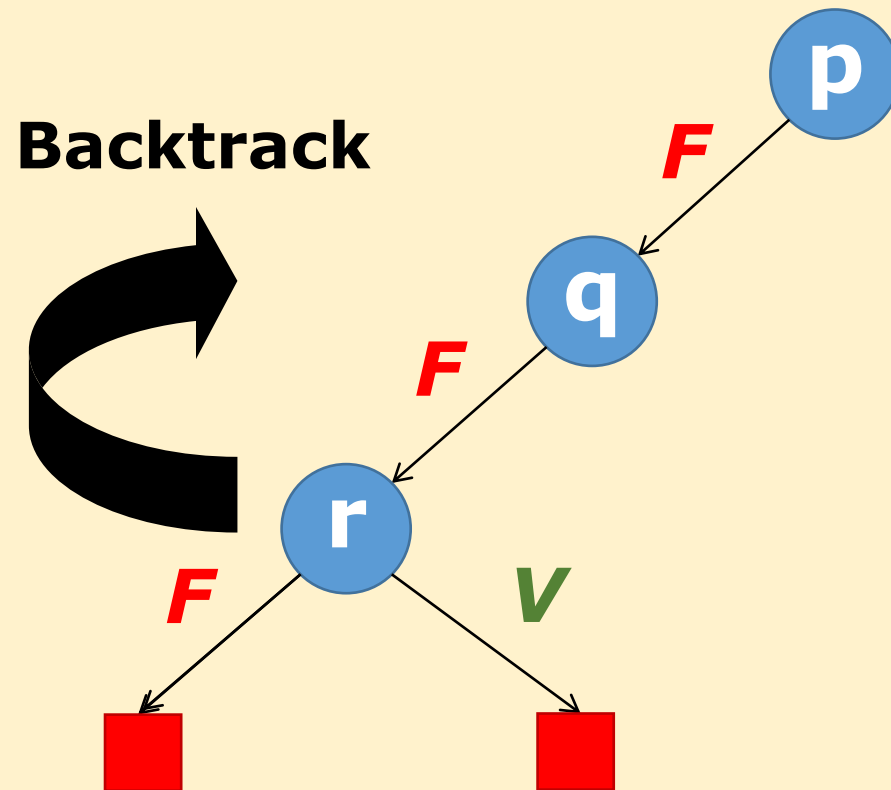
{ $\boxed{\bar{r}s}$, $\boxed{\bar{r}\bar{s}}$, }



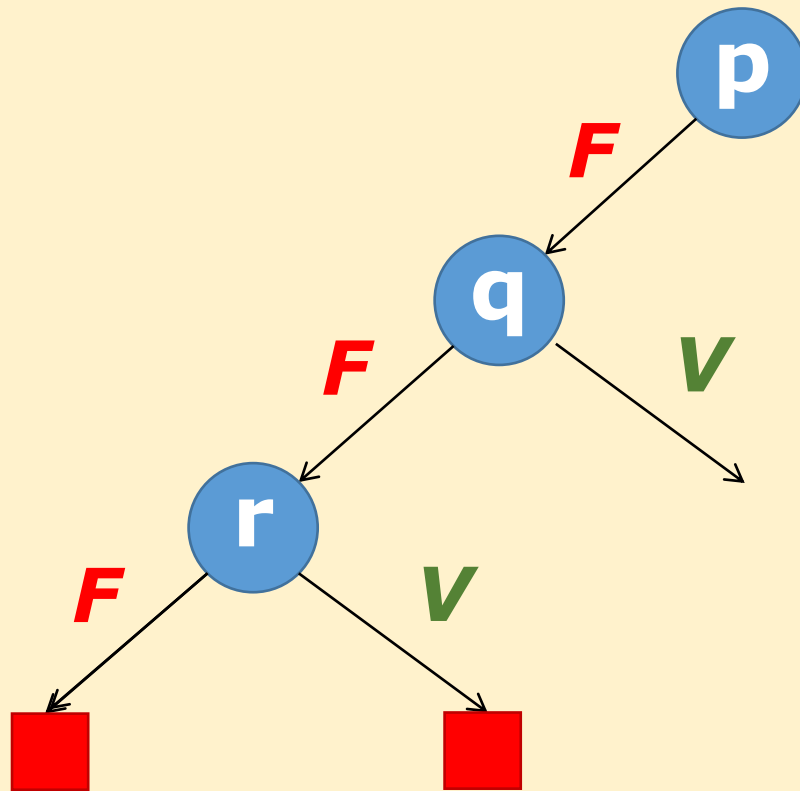
{ s, $\bar{s}$, }



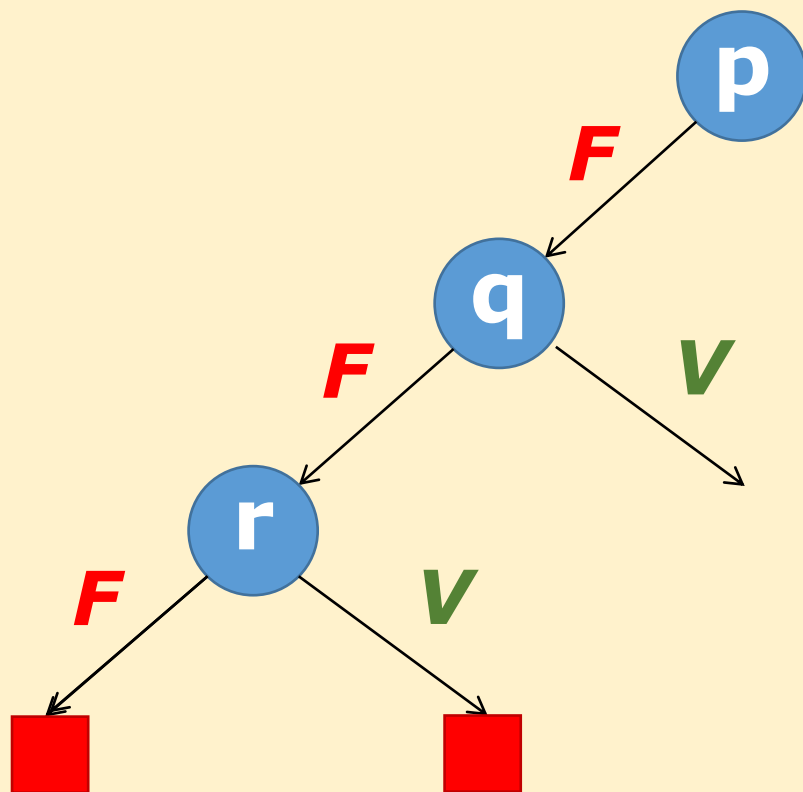
$\{ \quad rs, \ r\bar{s}, \ \bar{r}s, \ \bar{r}\bar{s}, \ \overline{qrs} \quad \}$



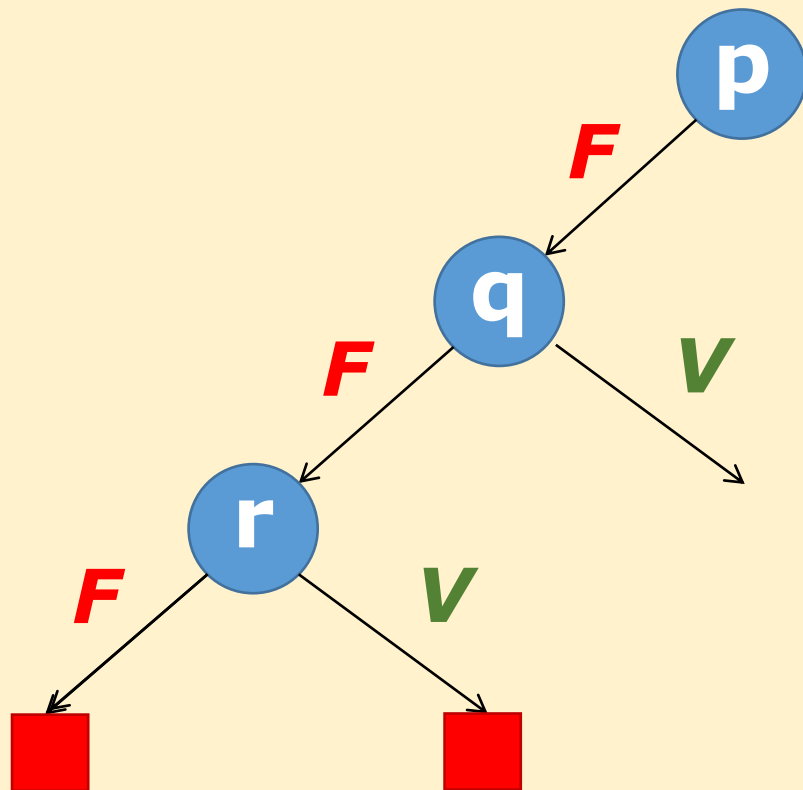
{ $rs, r\bar{s}, \bar{r}s, \bar{r}\bar{s}, \overline{qrs}$ }



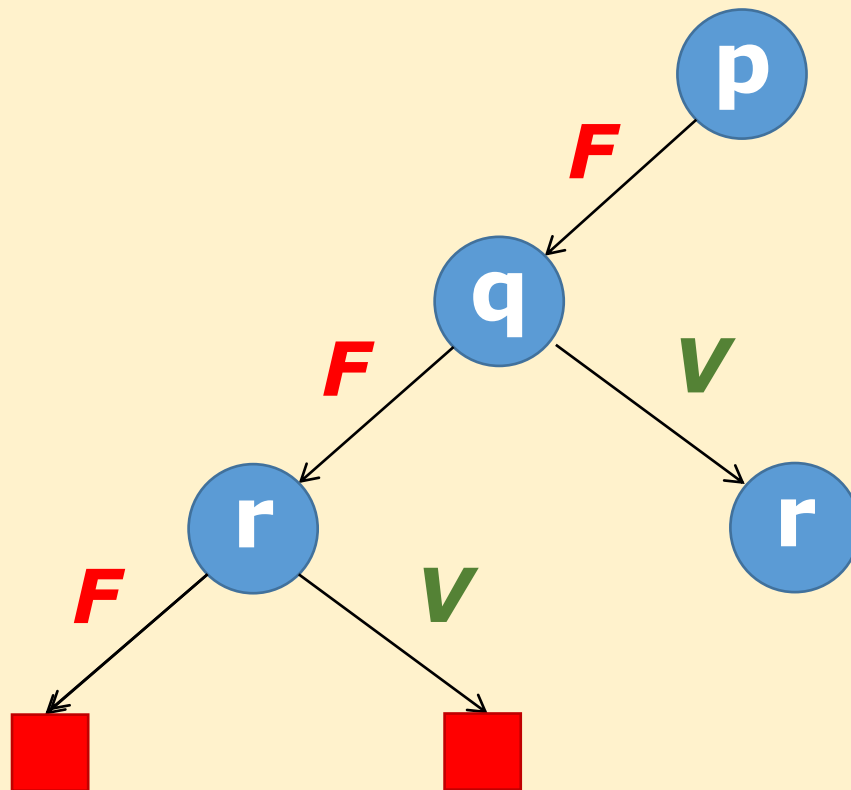
{ $rs, r\bar{s}, \bar{r}s, \bar{r}\bar{s}, \boxed{\bar{q}rs}$ }



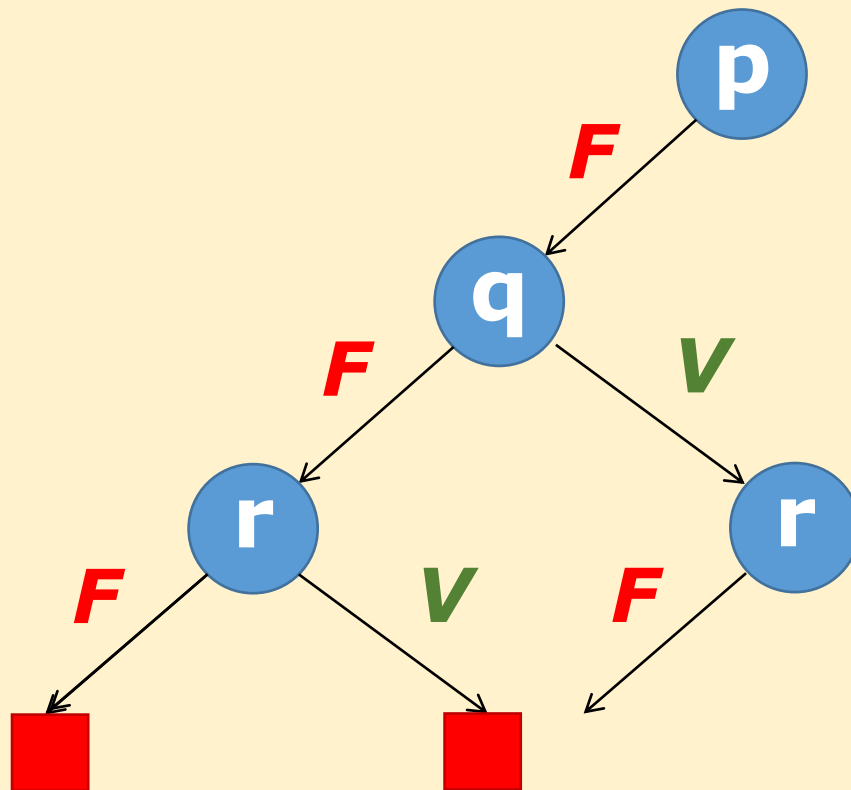
$\{ \quad rs, \ r\bar{s}, \ \bar{r}s, \ \bar{r}\bar{s}, \ \bar{r}s \quad \}$



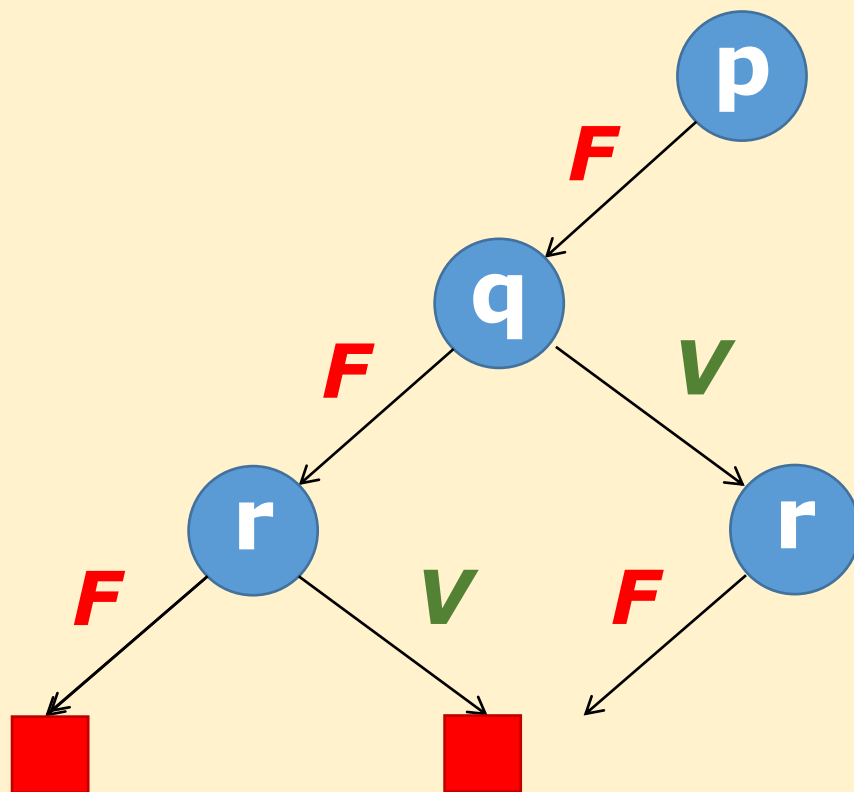
{ $rs, r\bar{s}, \bar{r}s, \bar{r}\bar{s}, \bar{r}s$ }



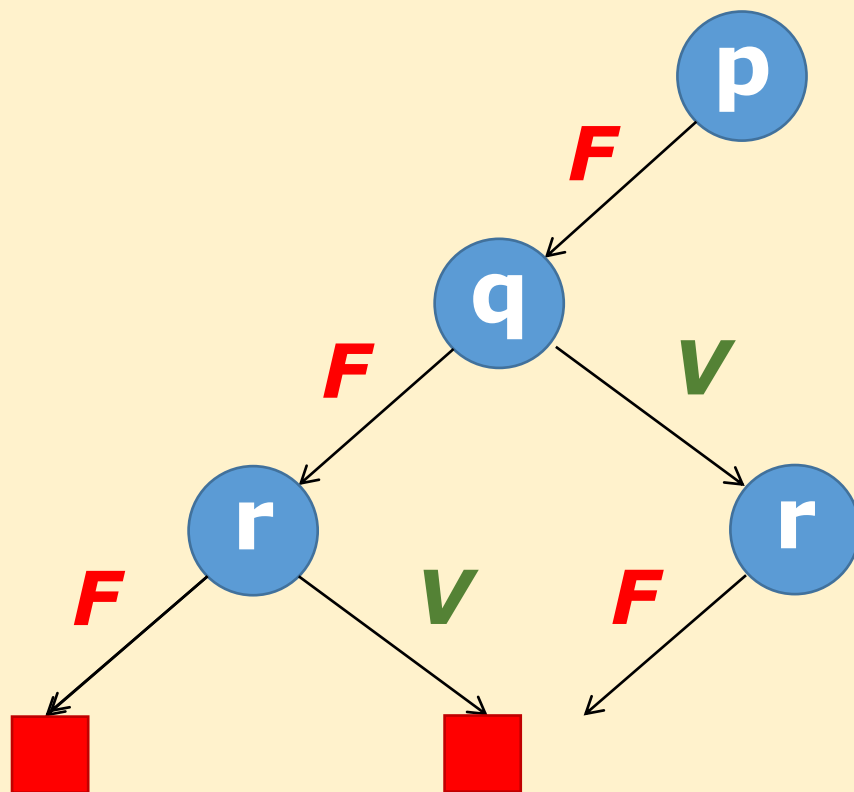
$\{ \quad rs, \ r\bar{s}, \ \bar{r}s, \ \bar{r}\bar{s}, \ \bar{r}s \quad \}$



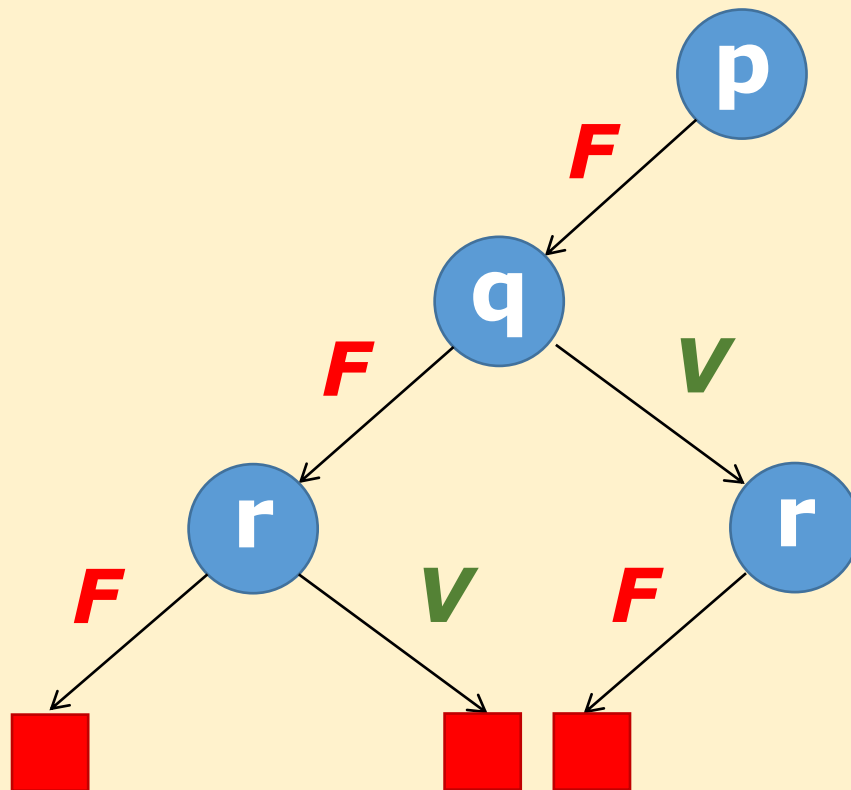
{ $\boxed{rs}$, $\boxed{r\bar{s}}$, $\bar{r}s$, $\overline{rs}$, $\bar{r}s$ }



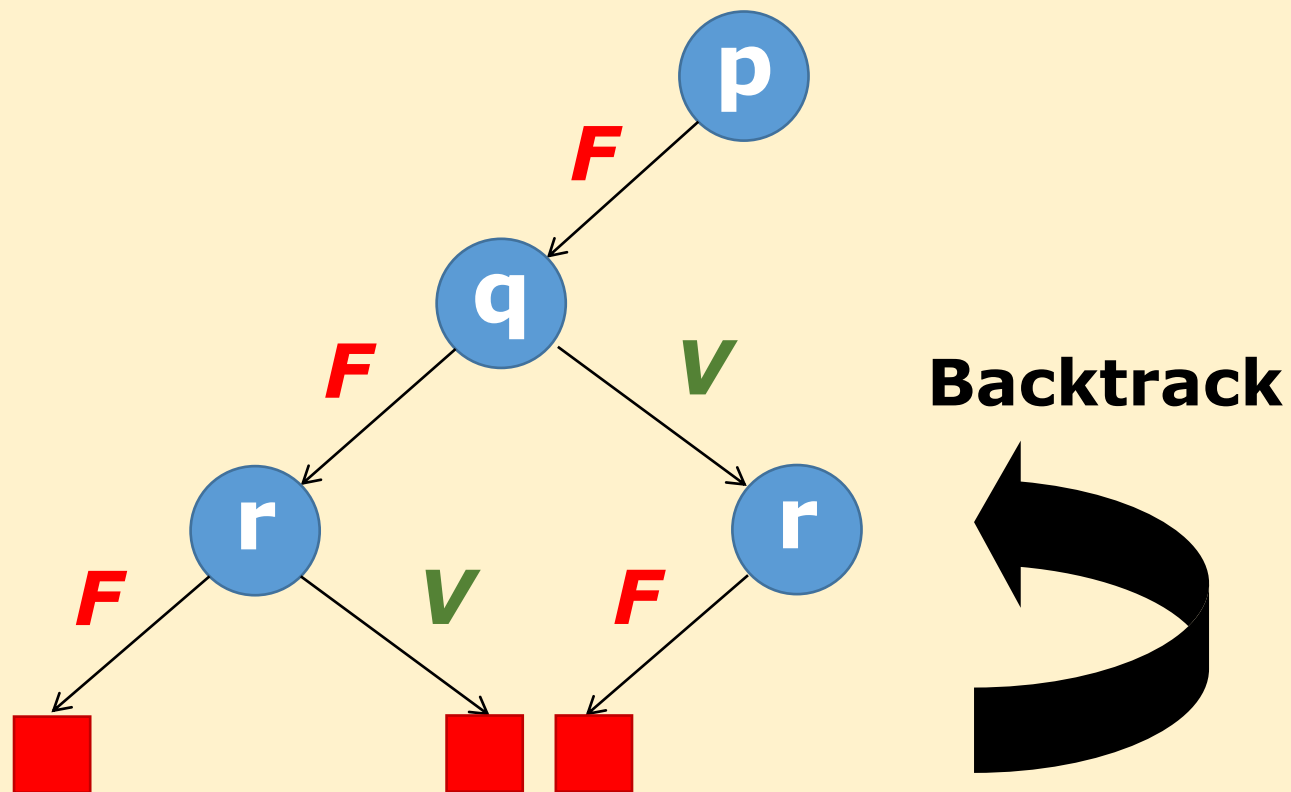
{ $s, \bar{s}, \bar{r}s, \overline{r\bar{s}}, \bar{r}s$ }



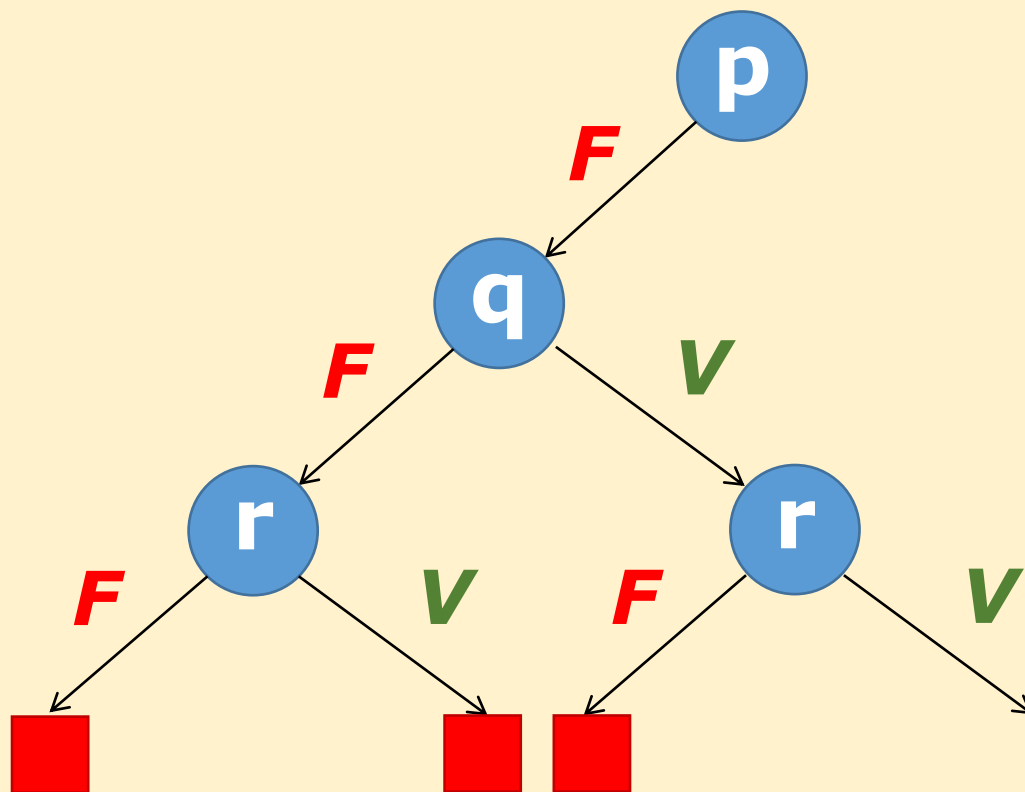
$\{ \quad rs, \ r\bar{s}, \ \bar{r}s, \ \bar{r}\bar{s}, \ \bar{r}s \quad \}$



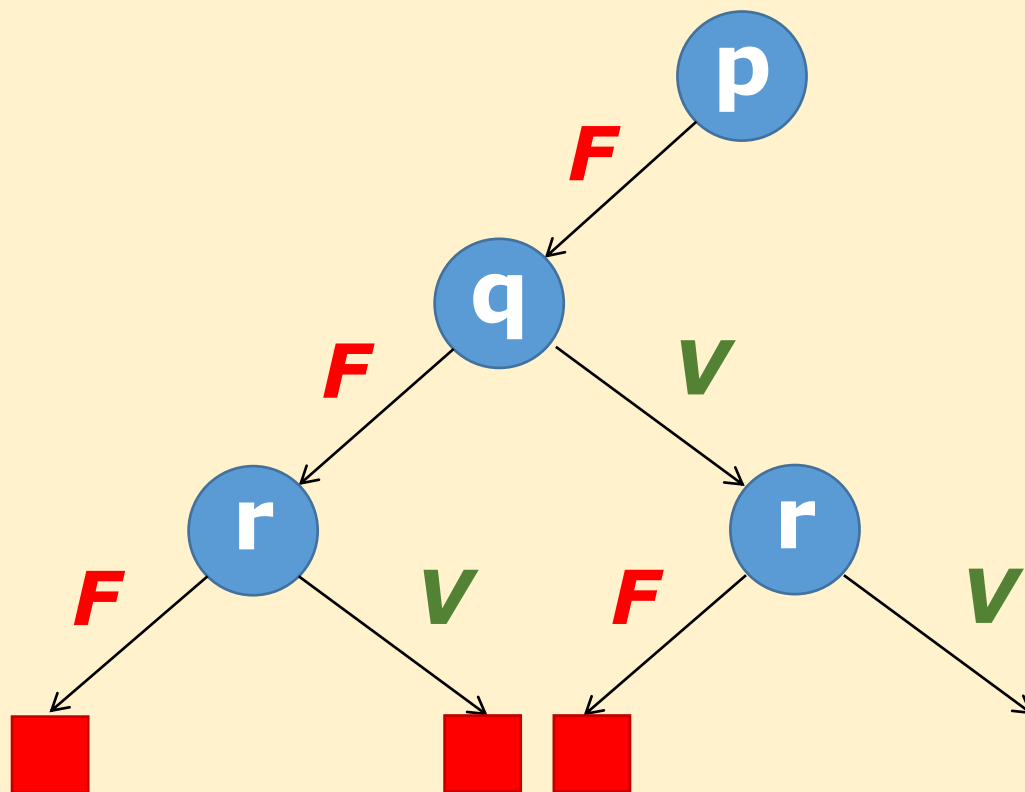
{ $rs, r\bar{s}, \bar{r}s, \bar{r}\bar{s}, \bar{r}s$ }



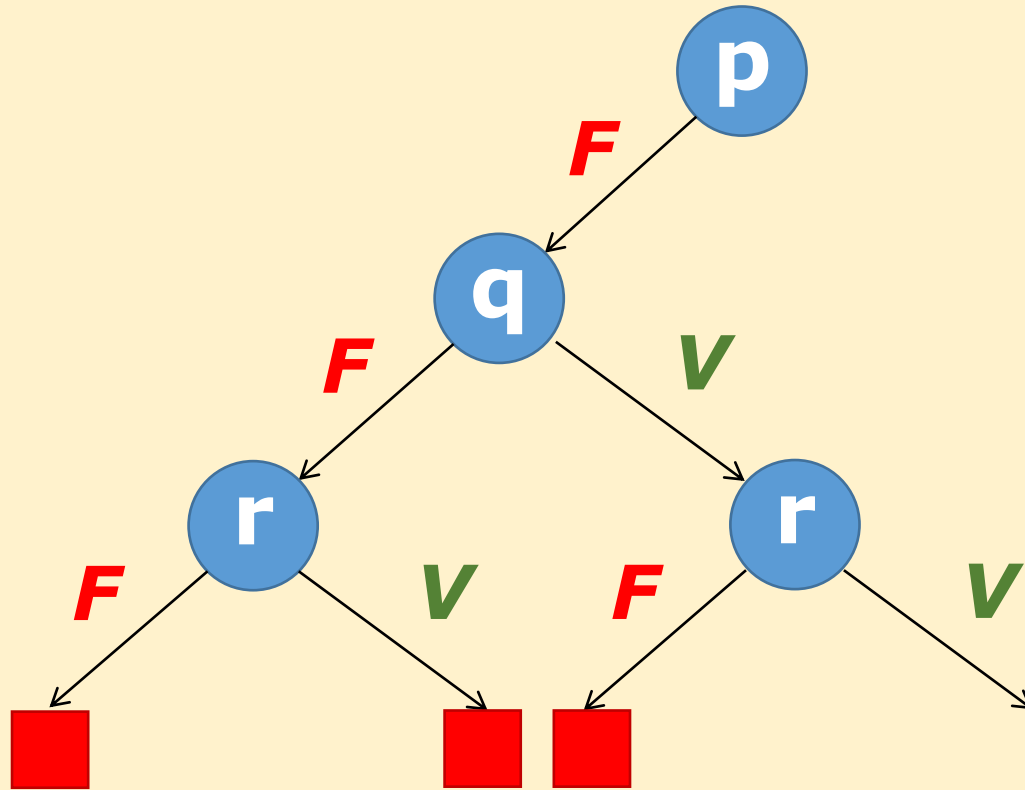
{ $\boxed{rs}$, $\boxed{r\bar{s}}$, $\bar{r}s$, $\overline{rs}$, $\bar{r}s$ }



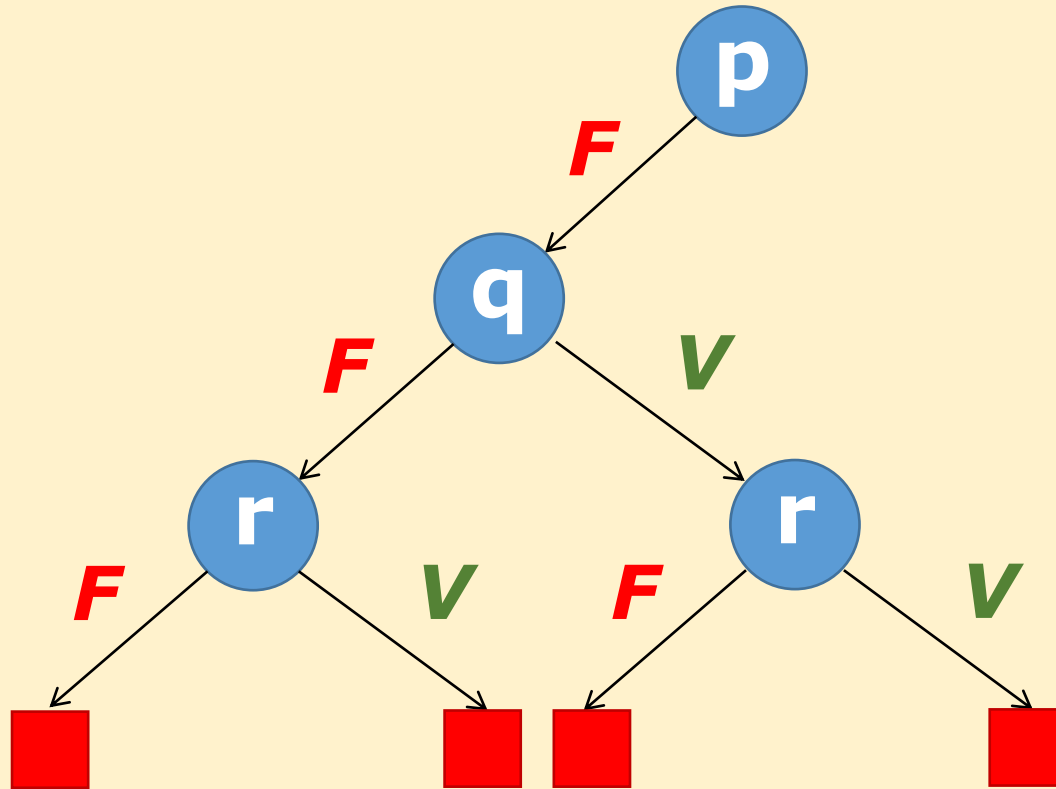
{ $\bar{r}s$, $\bar{r}s$, $\bar{r}s$ }



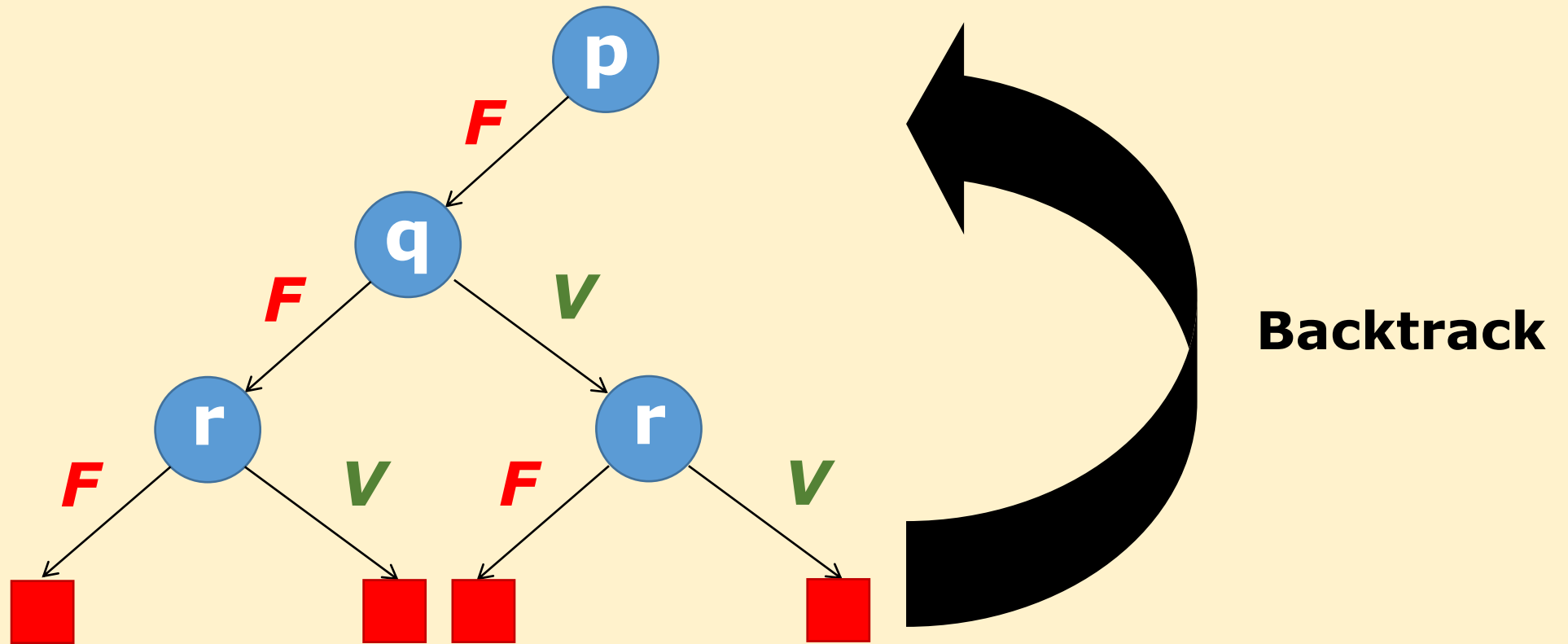
{ s, $\bar{s}$, s }



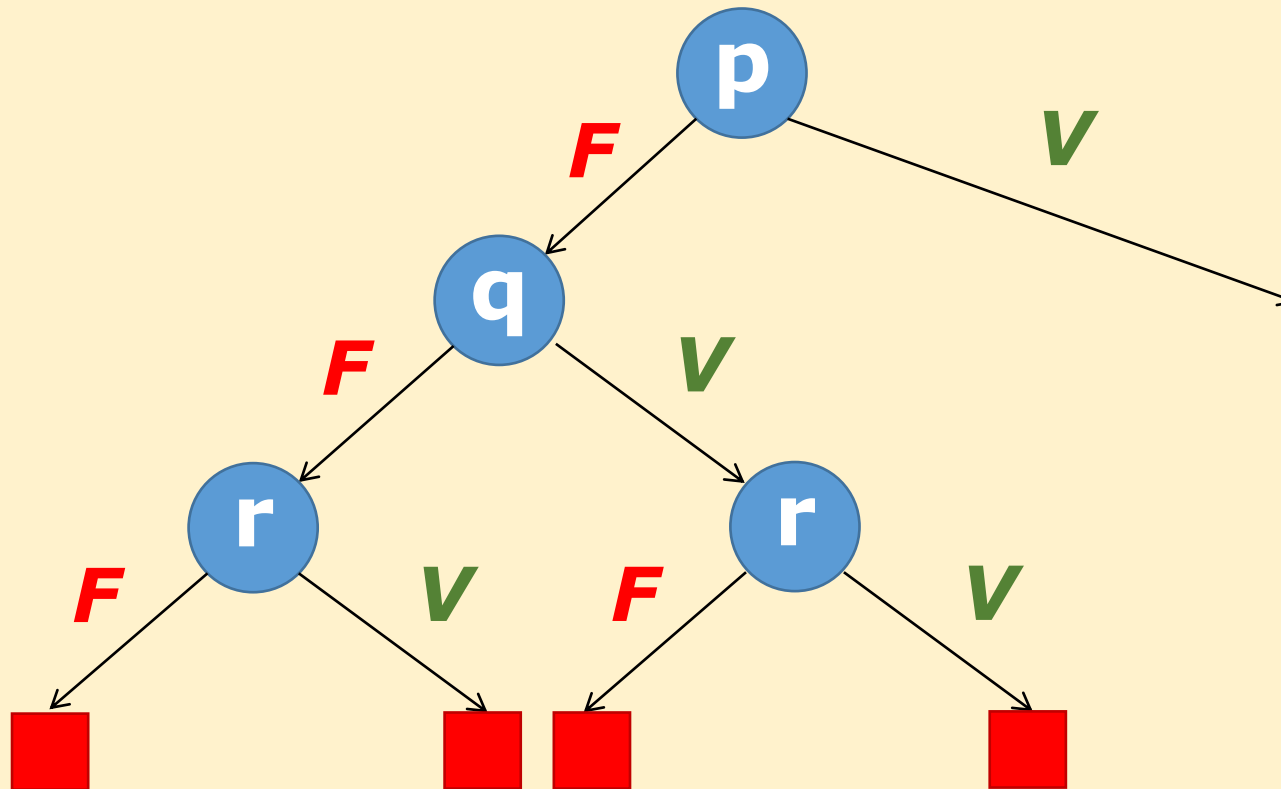
{ s, $\bar{s}$, s }



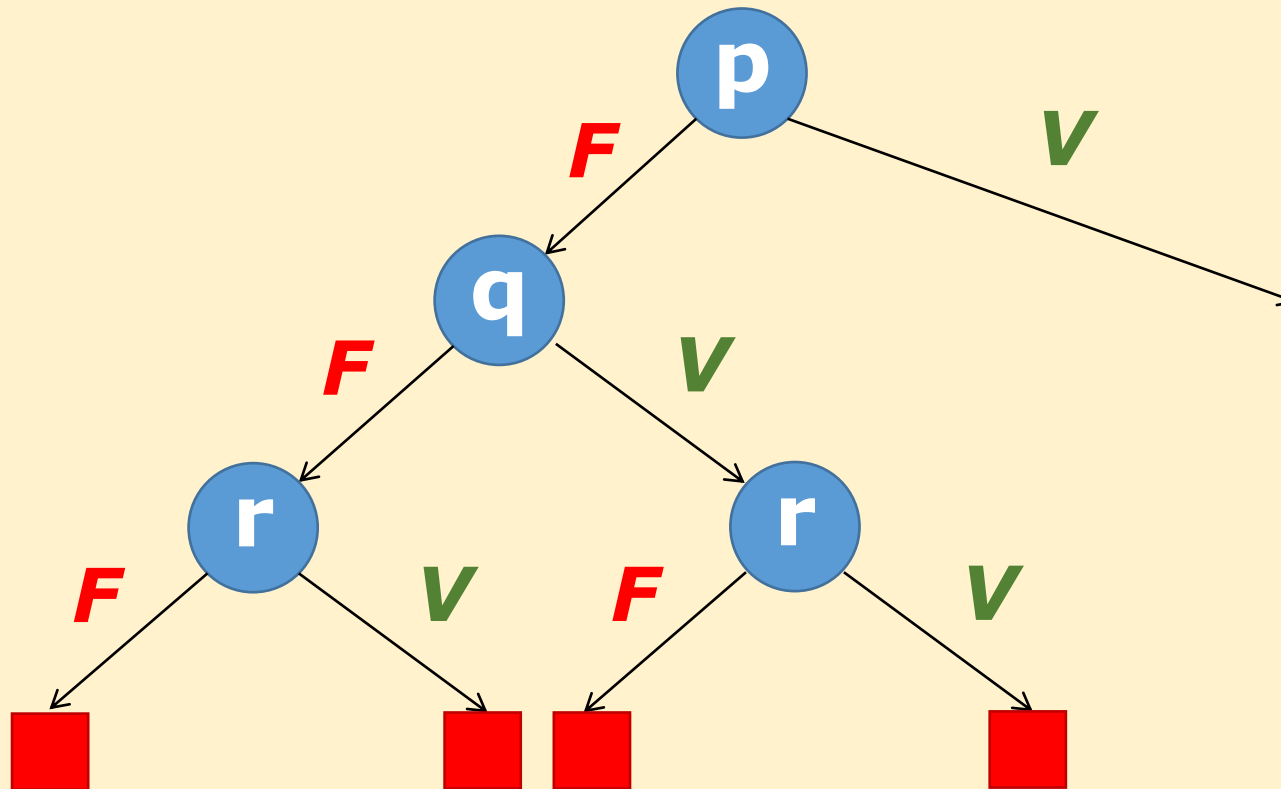
$\{\bar{p}qr, prs, pr\bar{s}, p\bar{r}s, p\bar{r}\bar{s}, \bar{q}rs, \bar{p}q\bar{r}, \bar{p}q\bar{r}\}$

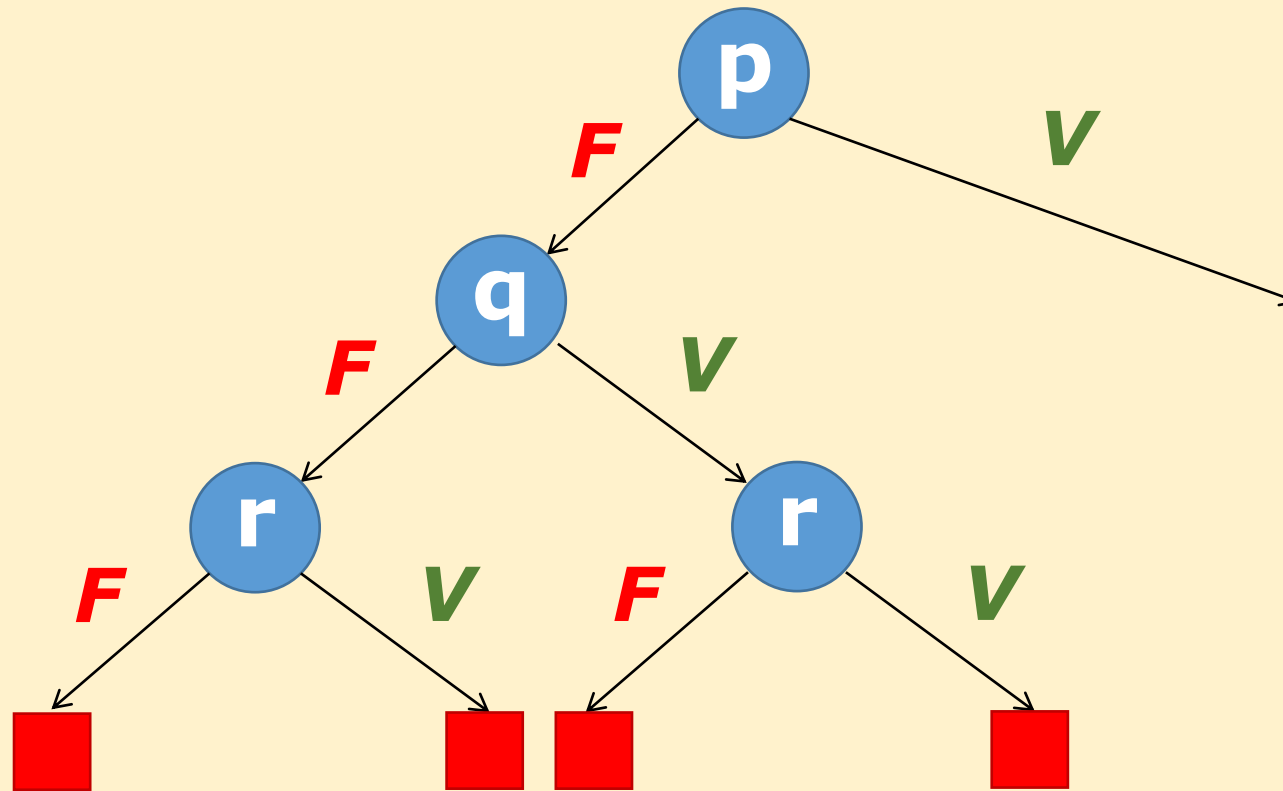


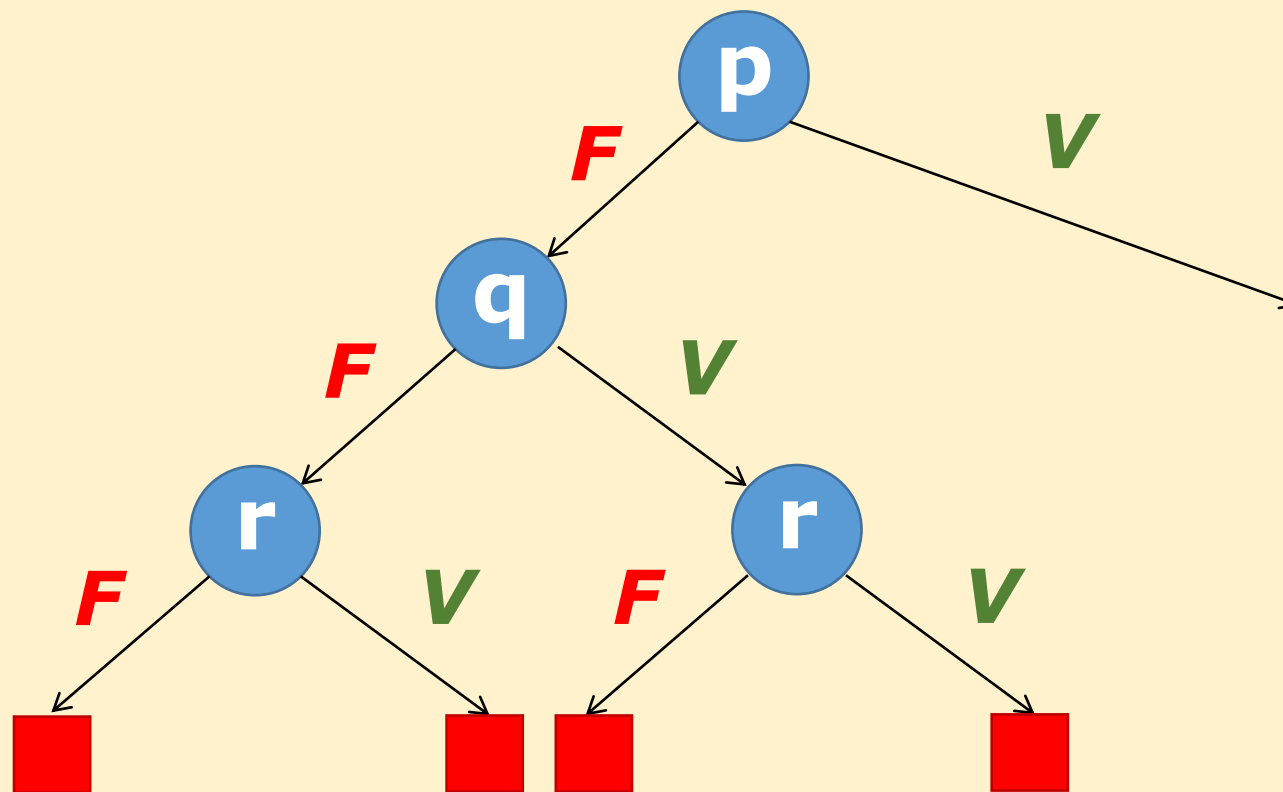
$\{\bar{p}qr, prs, pr\bar{s}, p\bar{r}s, p\bar{r}\bar{s}, \bar{q}rs, \bar{p}q\bar{r}, \bar{p}q\bar{r}\}$



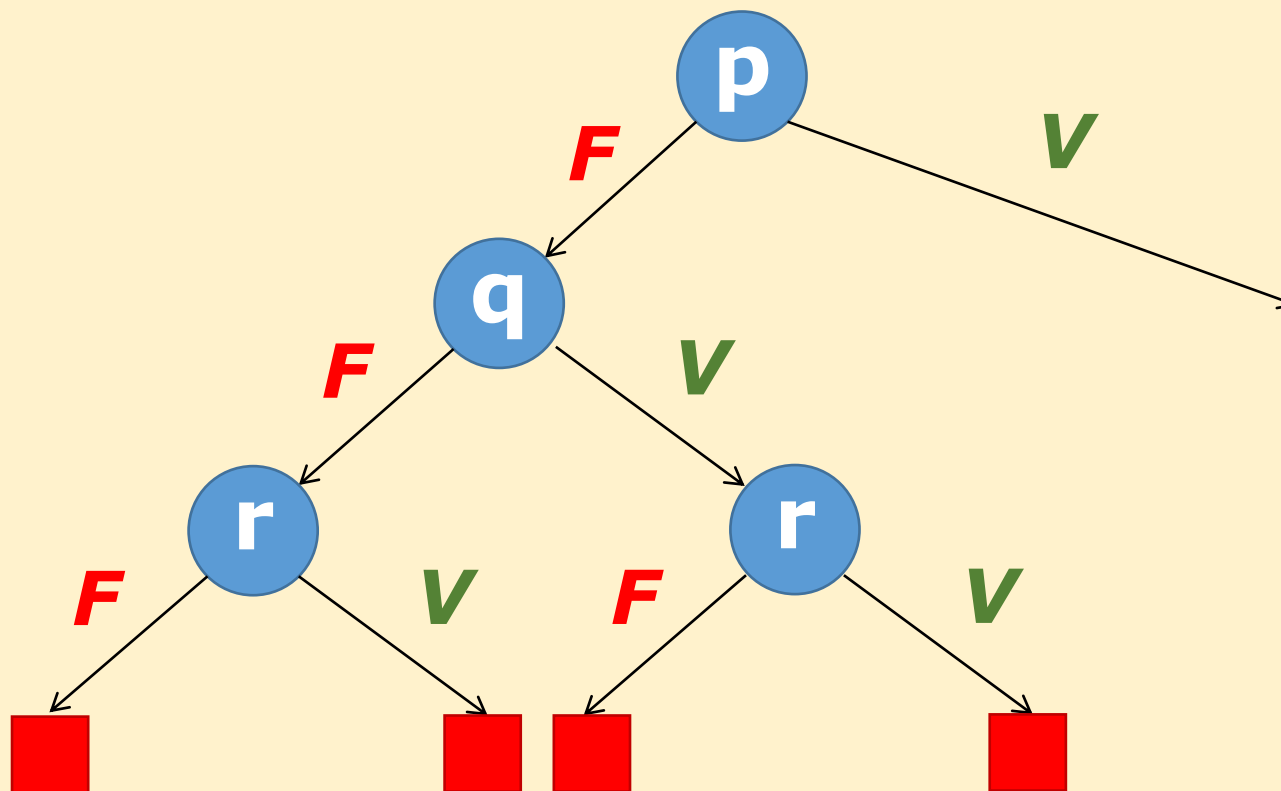
$\{\bar{p}qr, \boxed{prs}, \boxed{pr\bar{s}}, \boxed{p\bar{r}s}, \boxed{p\bar{r}\bar{s}}, \bar{q}rs, \bar{p}q\bar{r}, \bar{p}q\bar{r}\}$



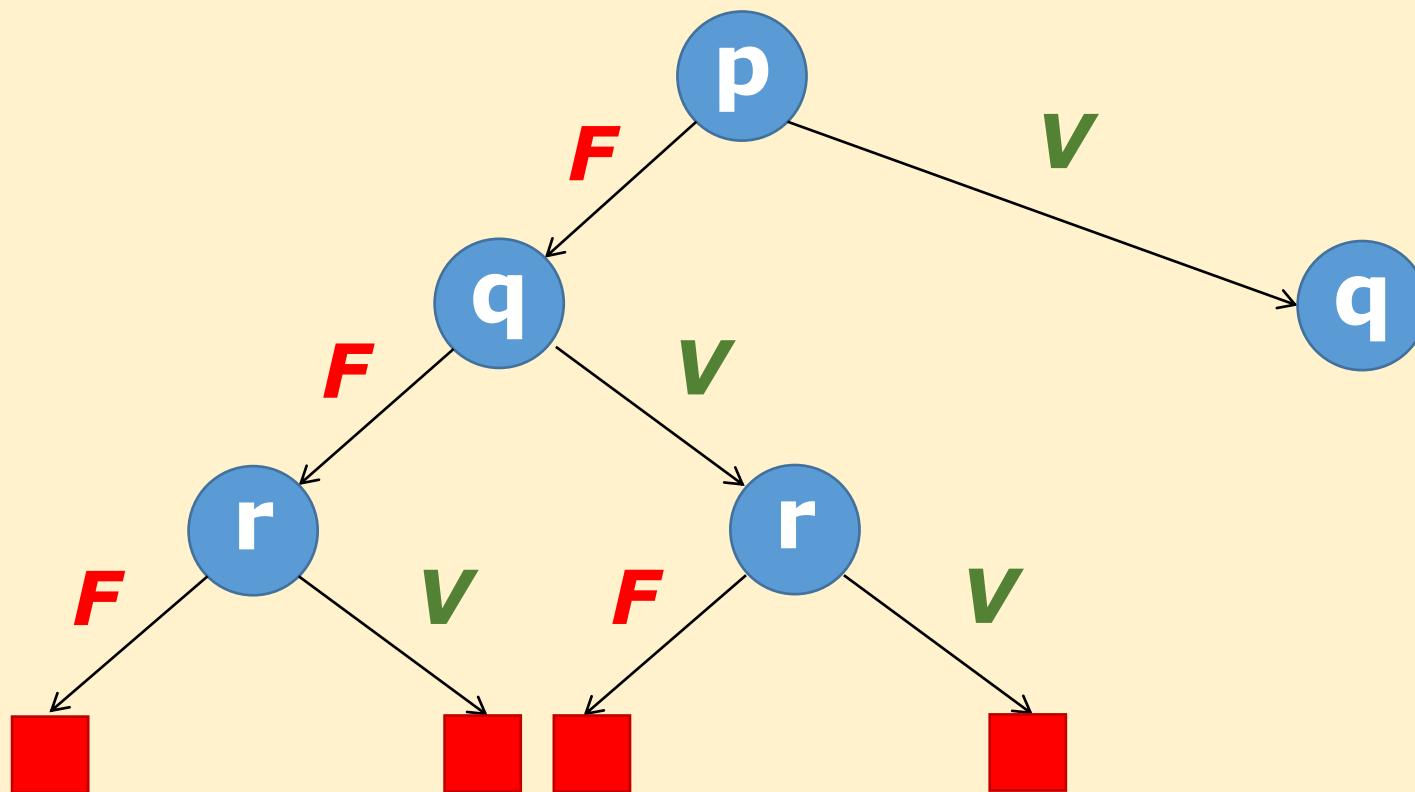
$\{\bar{p}qr$ $, \bar{q}rs, \bar{p}q\bar{r}, \bar{p}q\bar{r}\}$ 

$\{\bar{p}qr$ $, \bar{q}rs, \bar{p}q\bar{r}, \bar{p}\bar{q}r\}$ 

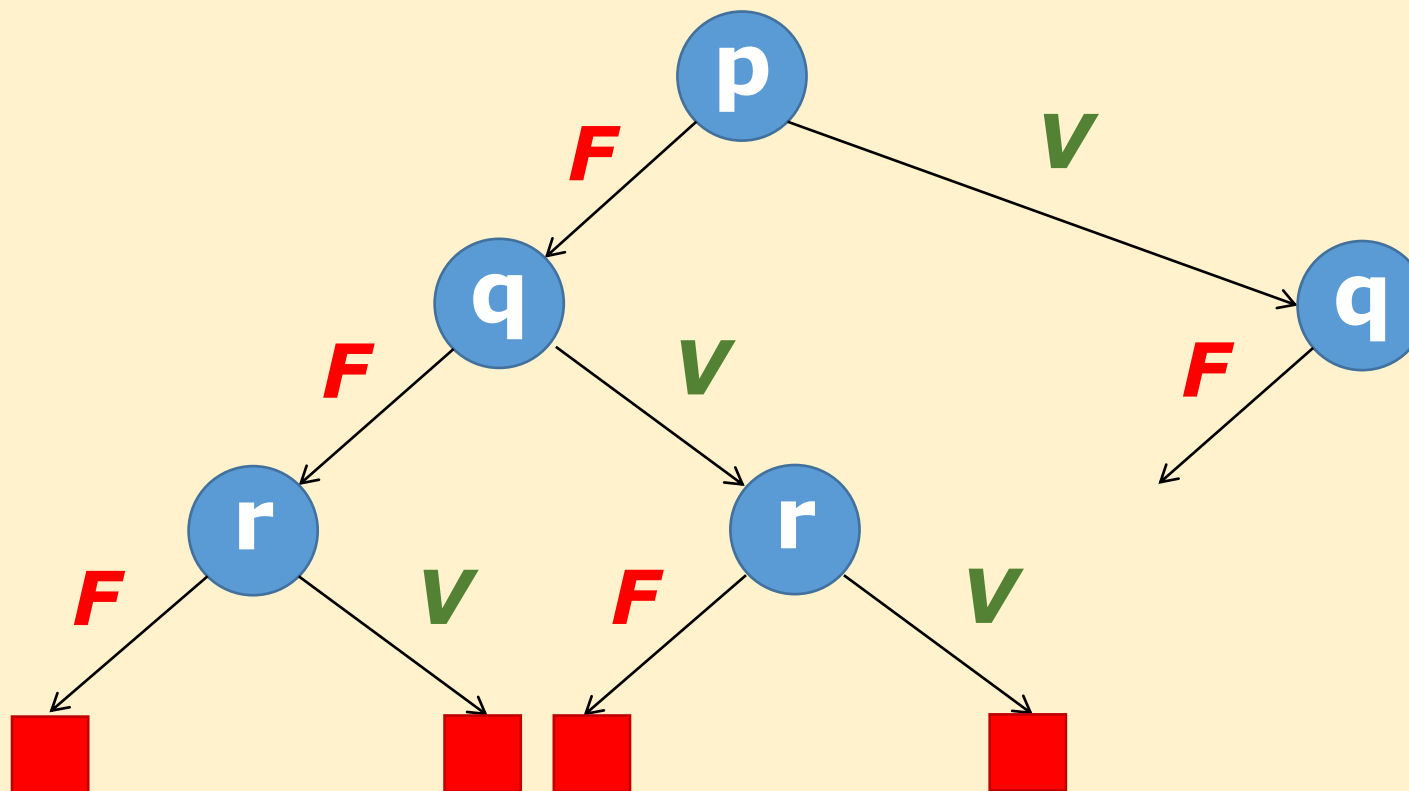
$\{ qr, \bar{q}rs, q\bar{r}, \bar{q}\bar{r} \}$



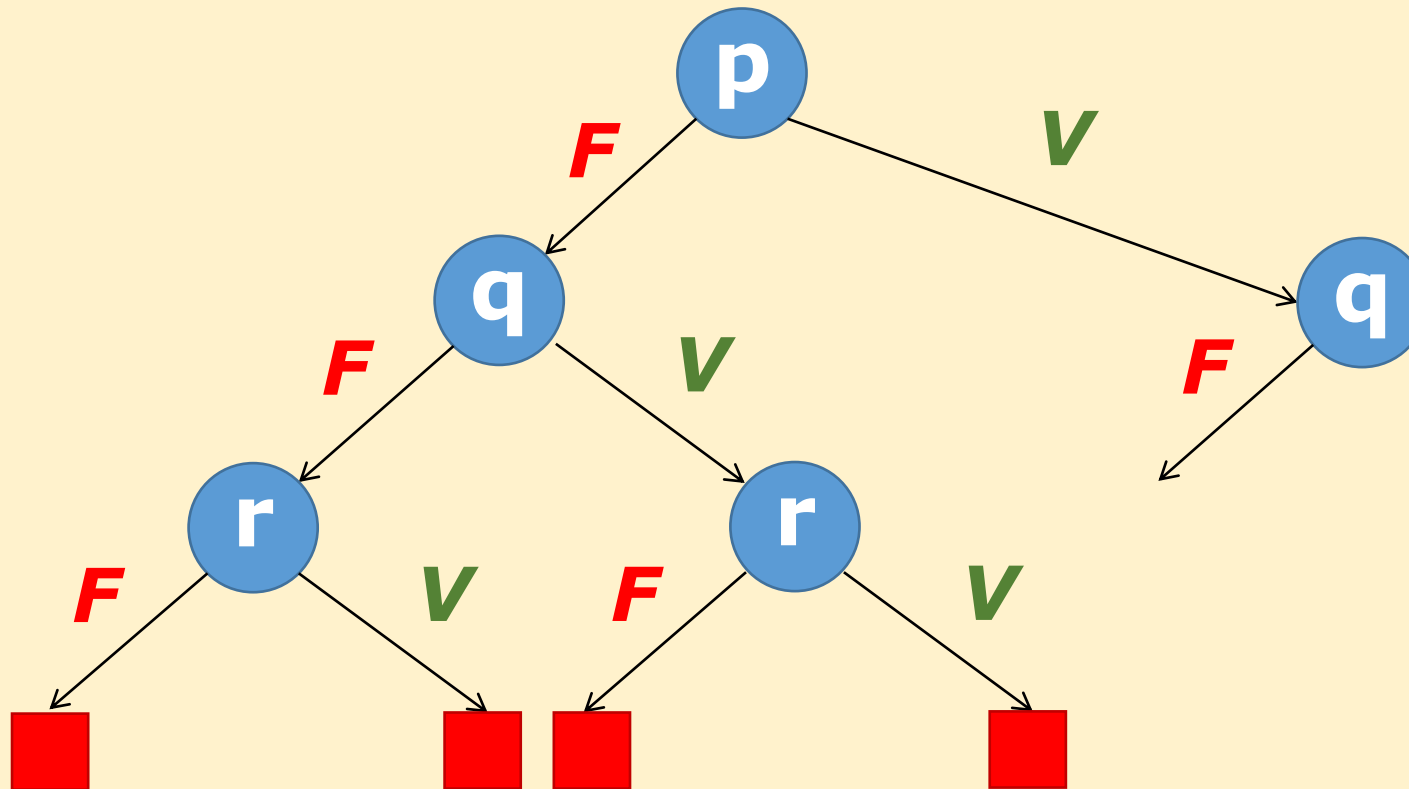
$\{ qr$, $\overline{qrs}$, $q\overline{r}$, $\overline{q}r\}$



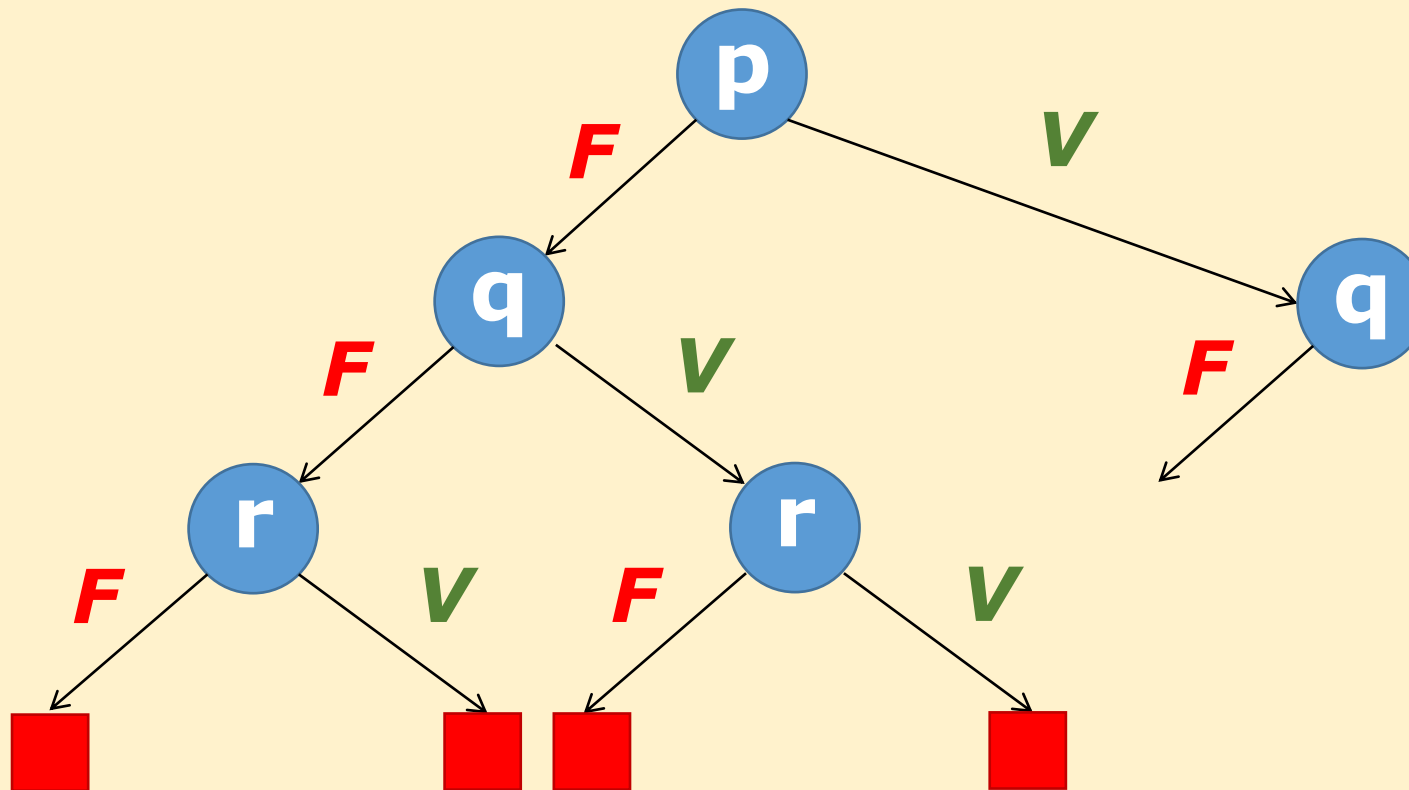
$\{ qr$, $\overline{qrs}$, $q\overline{r}$, $\overline{q}r\}$



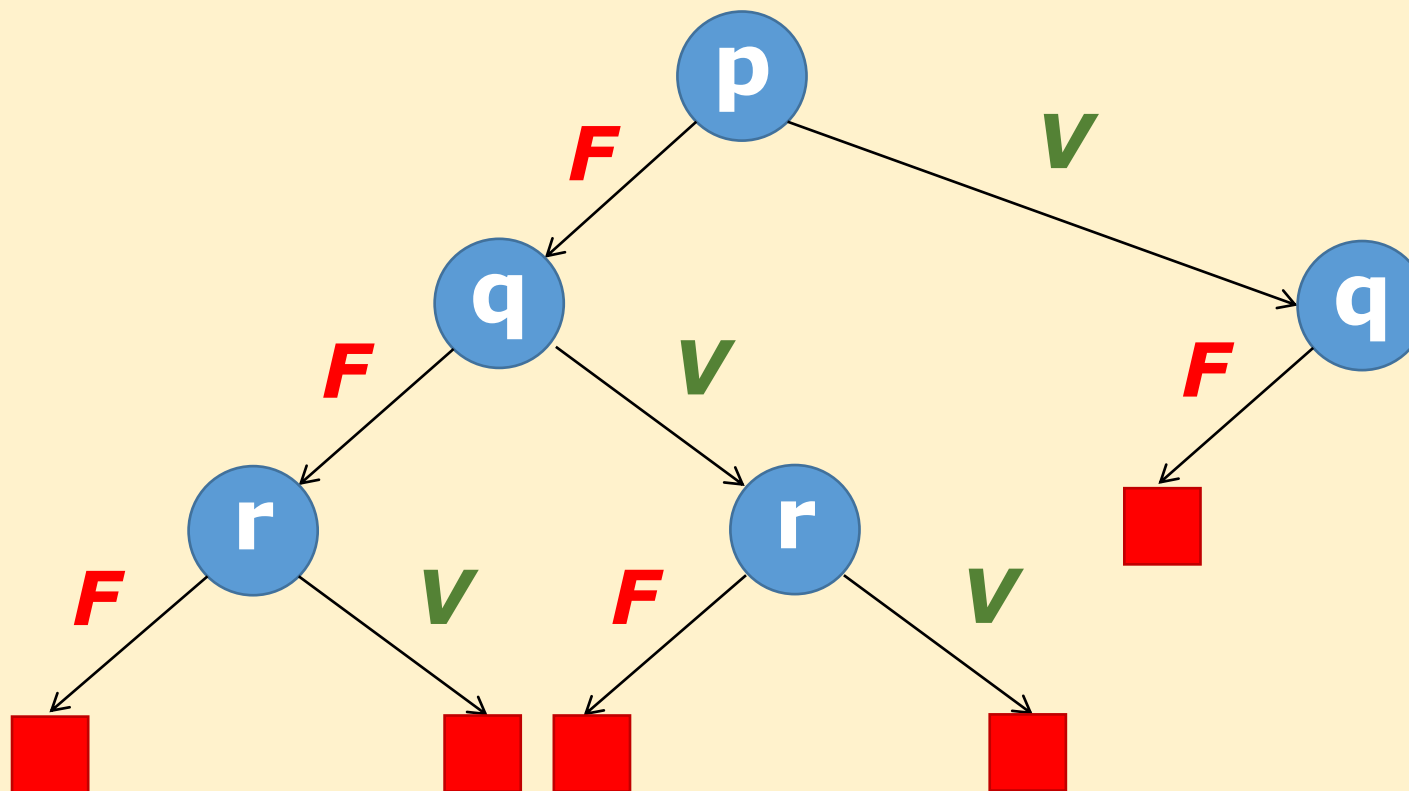
$\{ \boxed{qr}, \overline{qrs}, \boxed{q\bar{r}}, \overline{qr} \}$



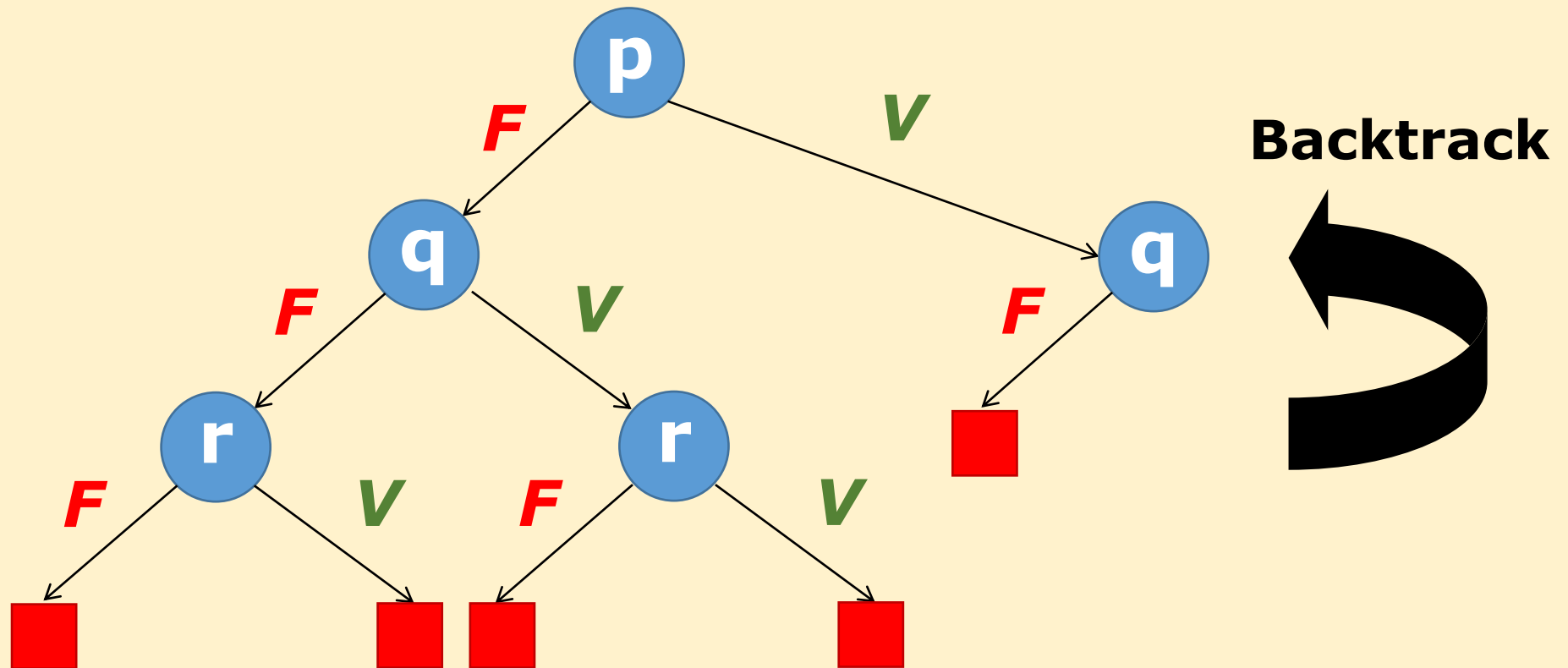
$\{ \quad r \quad , \overline{qrs}, \quad \bar{r}, \quad \overline{qr} \}$



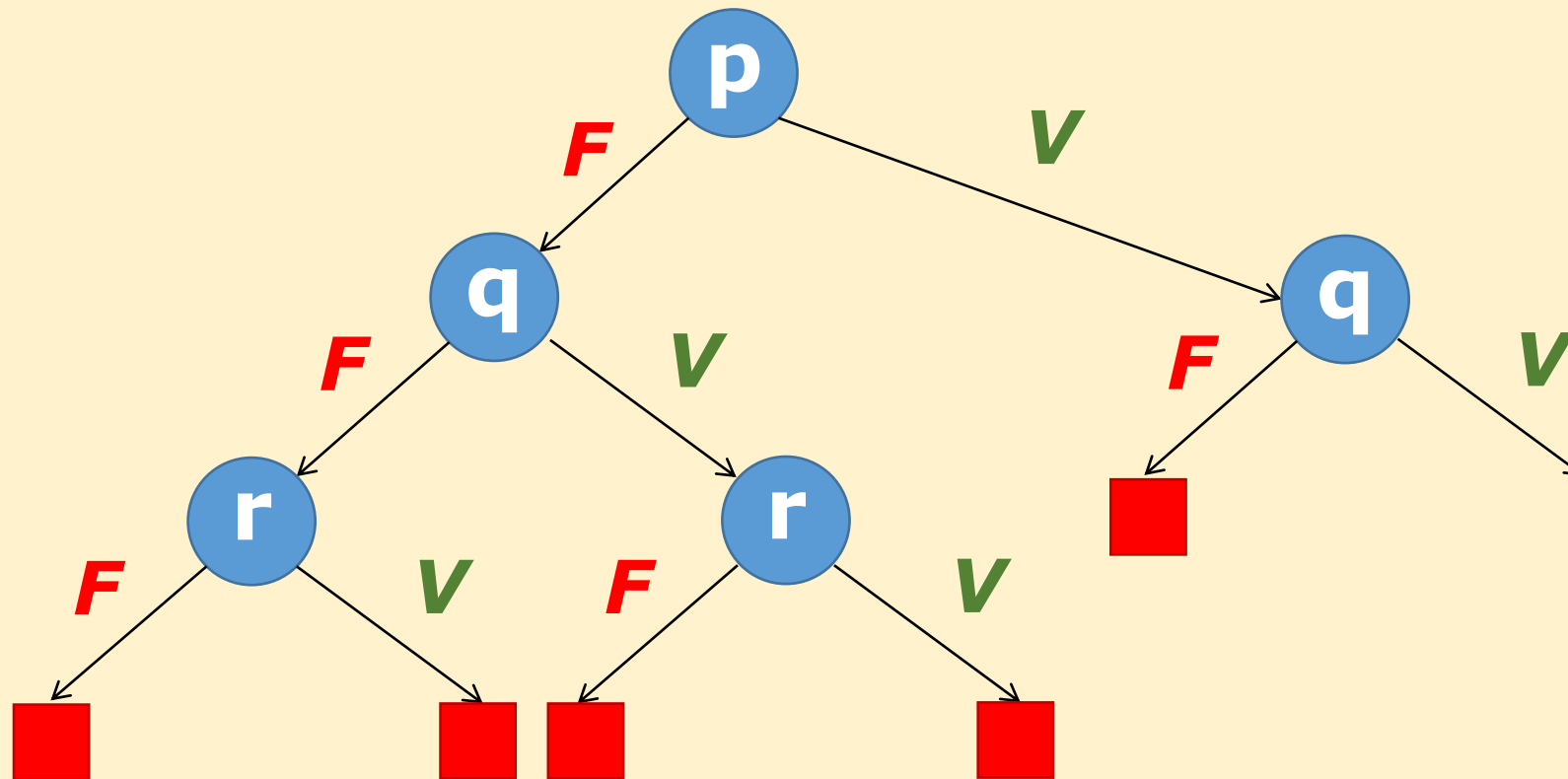
$\{ \quad r \quad , \overline{qrs}, \quad \bar{r}, \quad \overline{qr} \}$



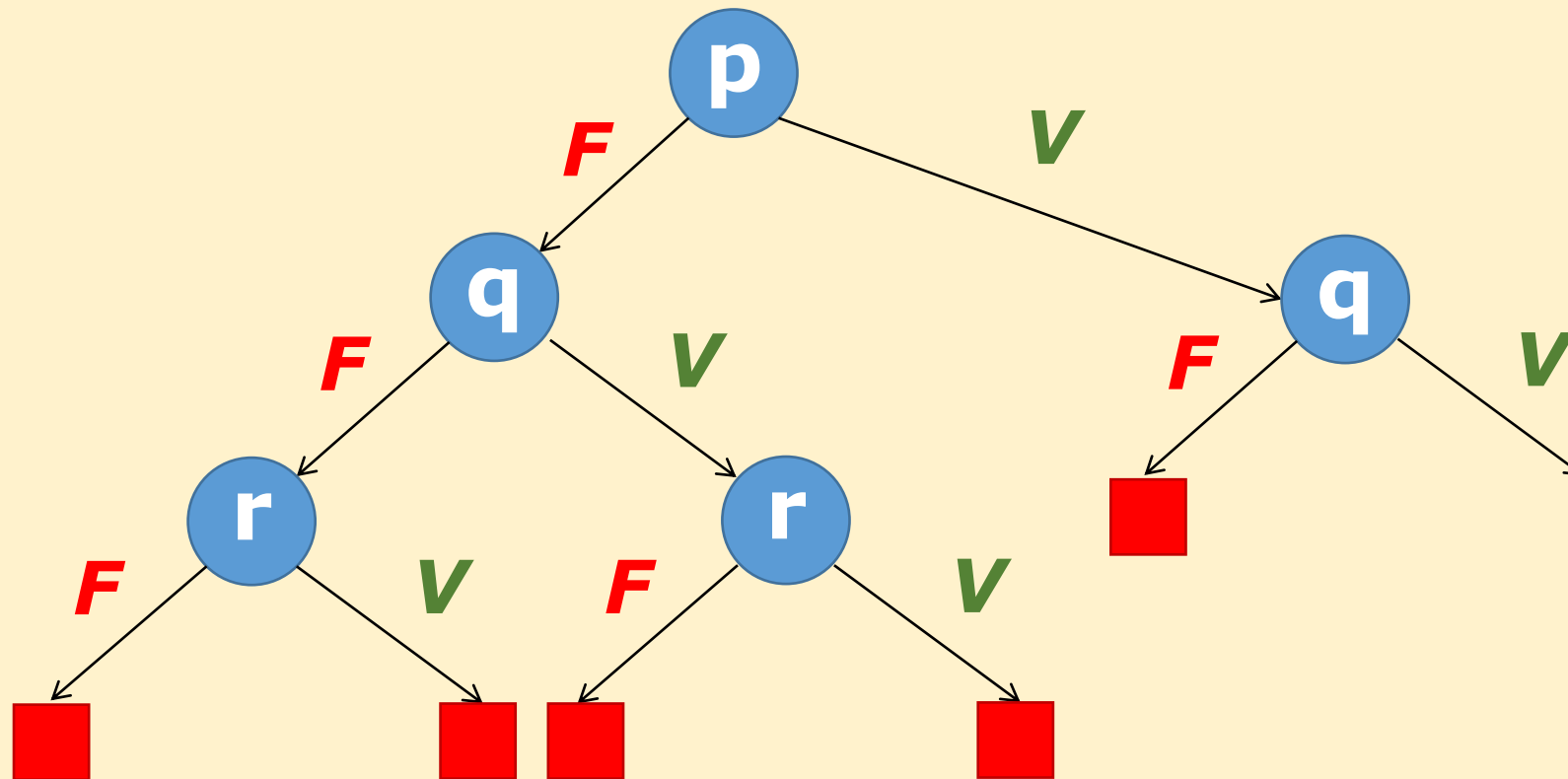
{ qr , $\overline{qrs}$, $q\overline{r}$, $\overline{q}r$ }



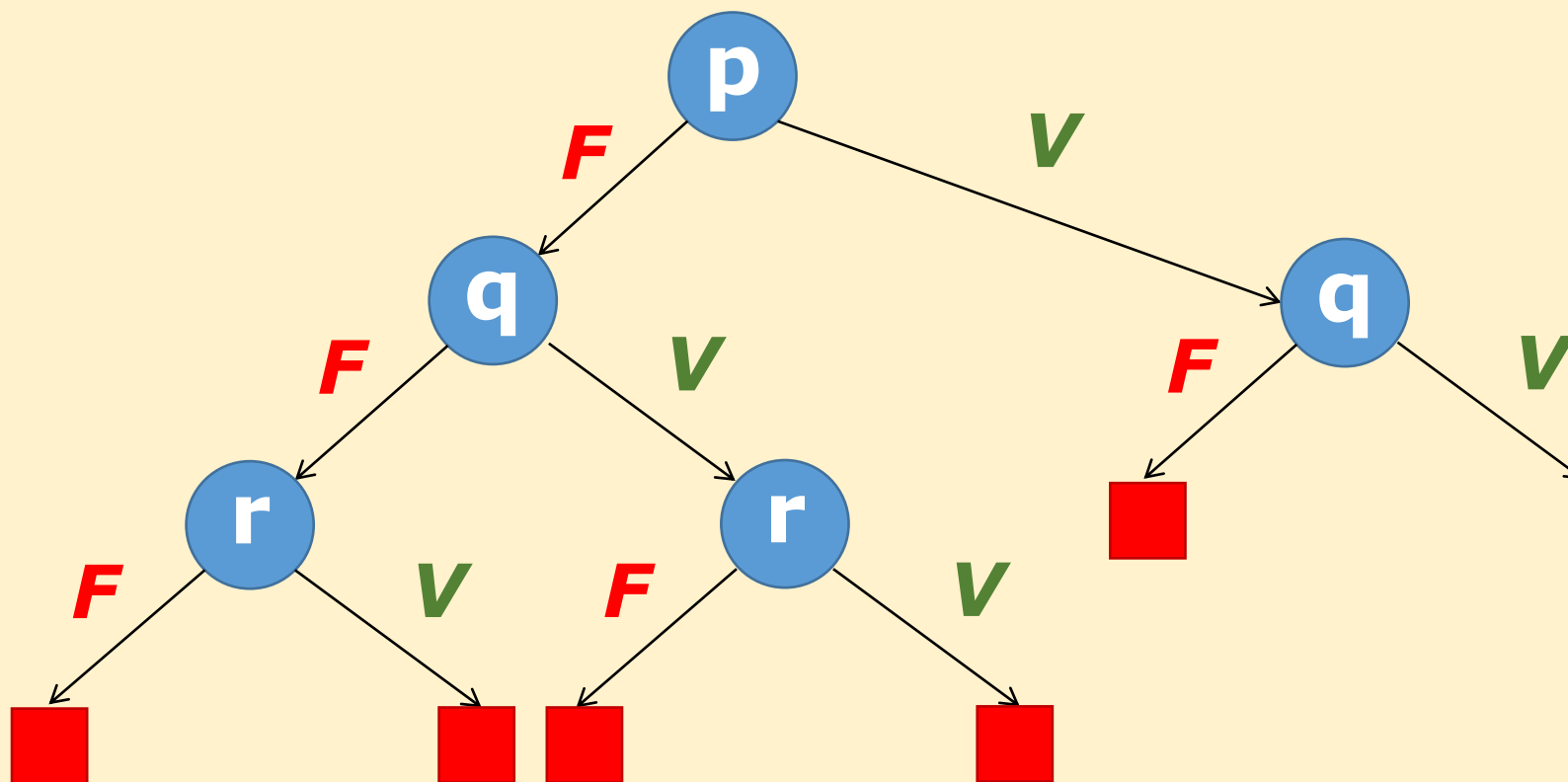
$\{ qr$, $\overline{qrs}$, $q\overline{r}$, $\overline{q}r\}$



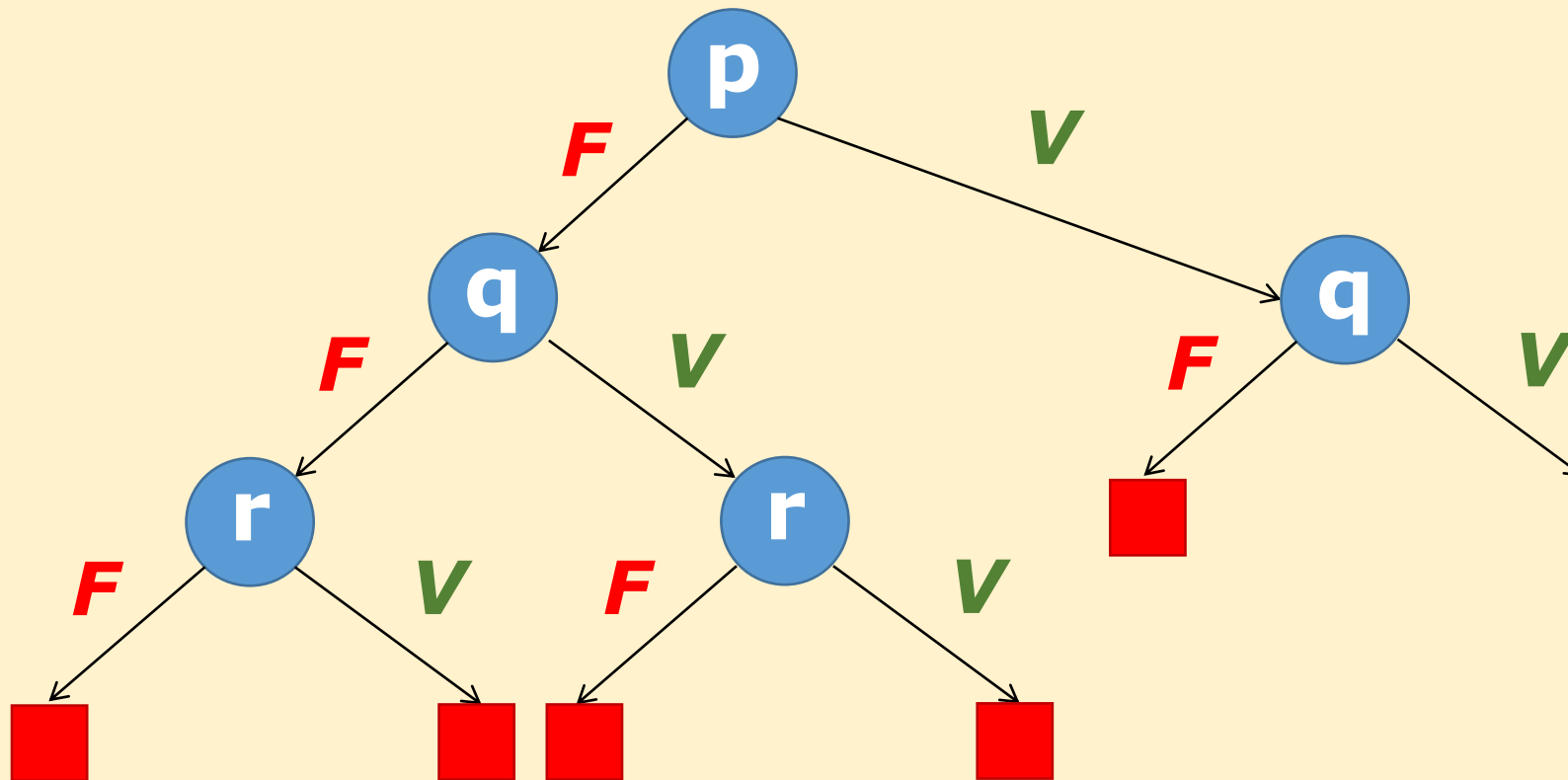
{ $\boxed{qr}$, $\overline{qrs}$, $\boxed{q\bar{r}}$, $\overline{q}r$ }



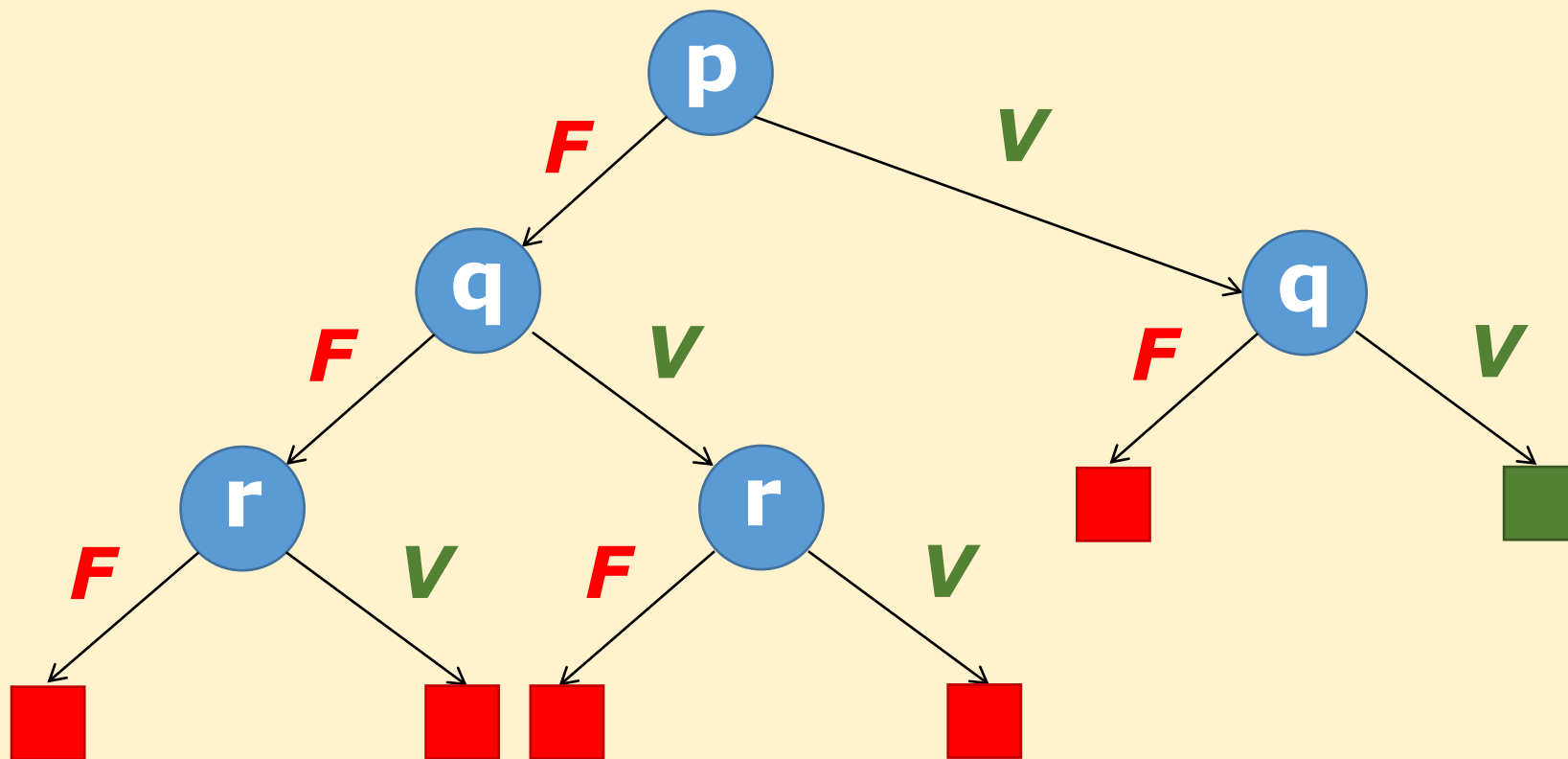
{ $\overline{qrs}, \quad \overline{qr}$ }



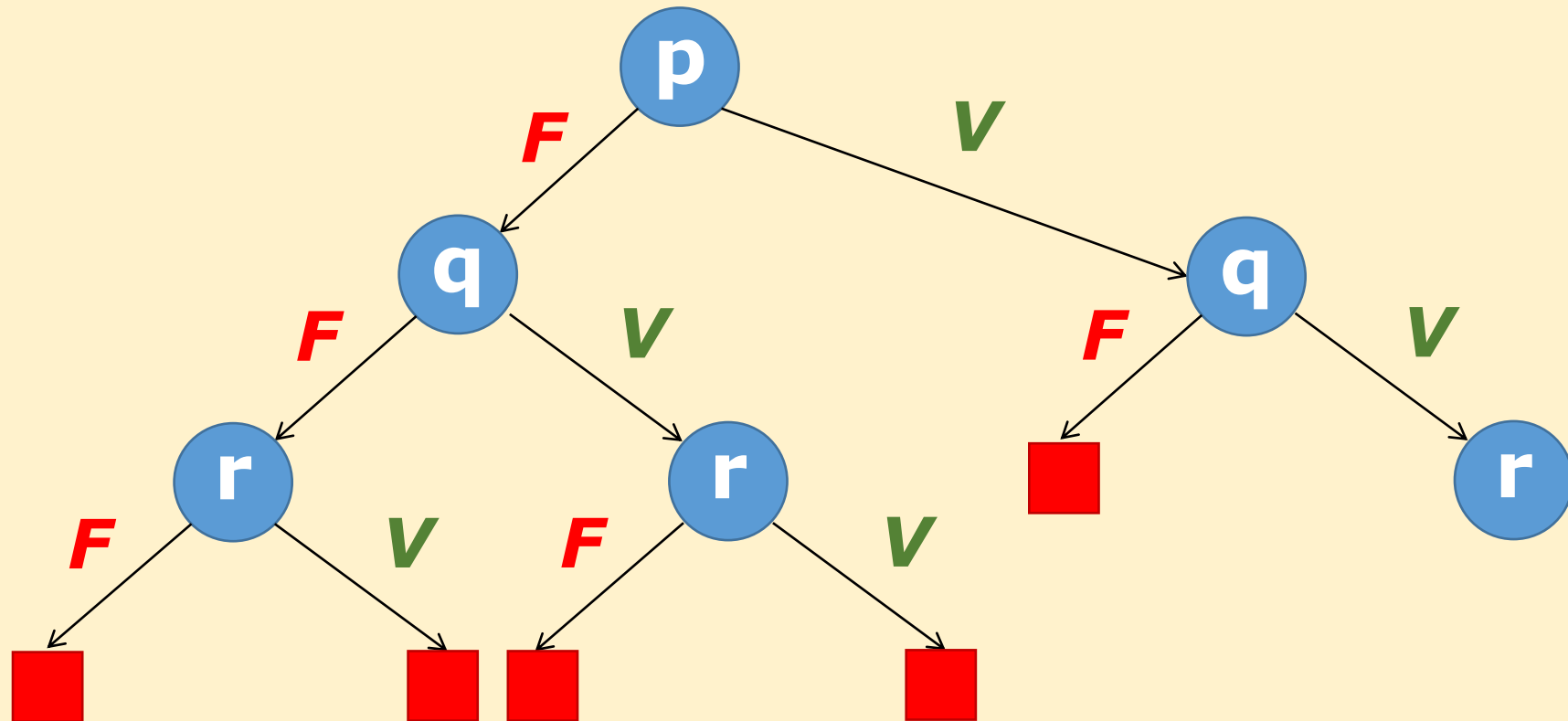
{ $\boxed{\bar{q}rs}, \quad \boxed{\bar{q}r}$



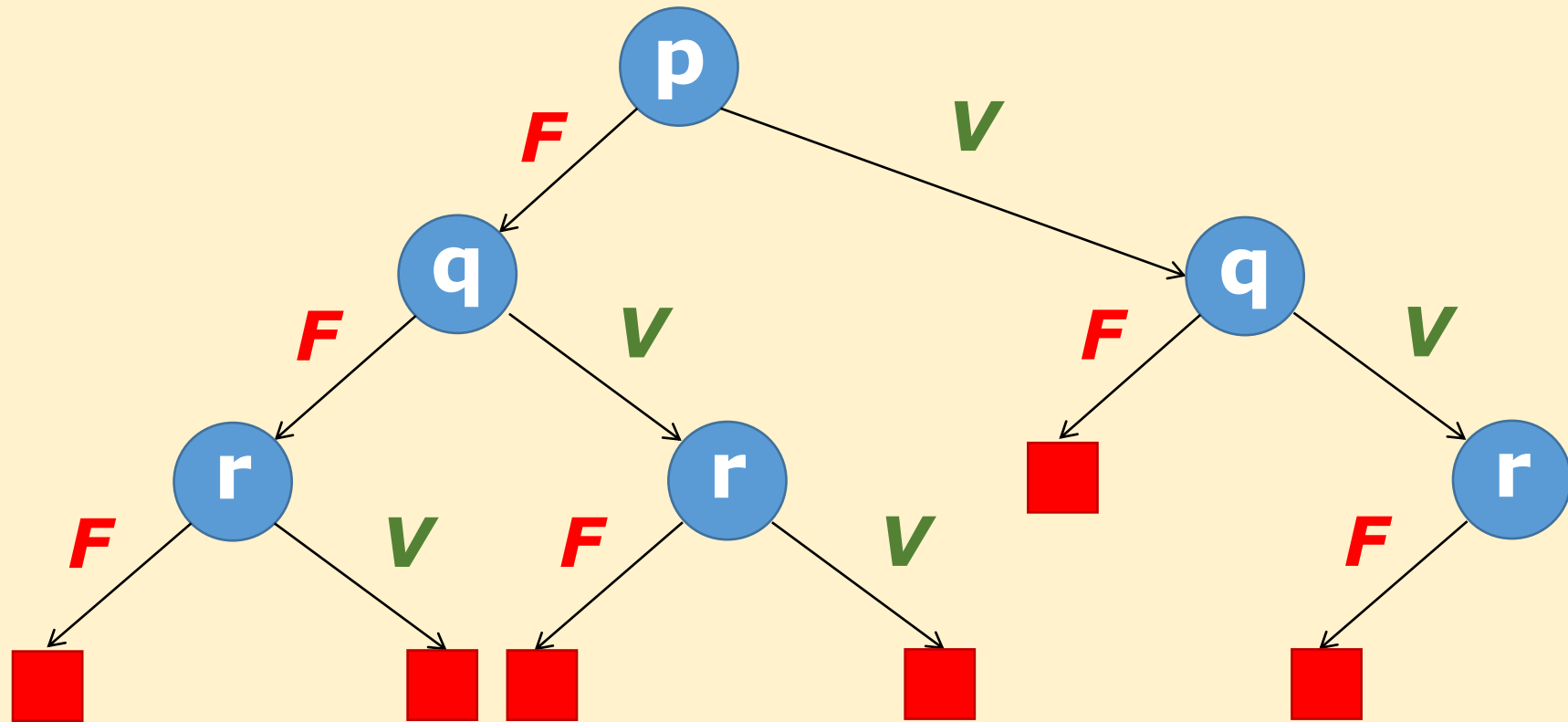
{ $\bar{r}s,$ r }



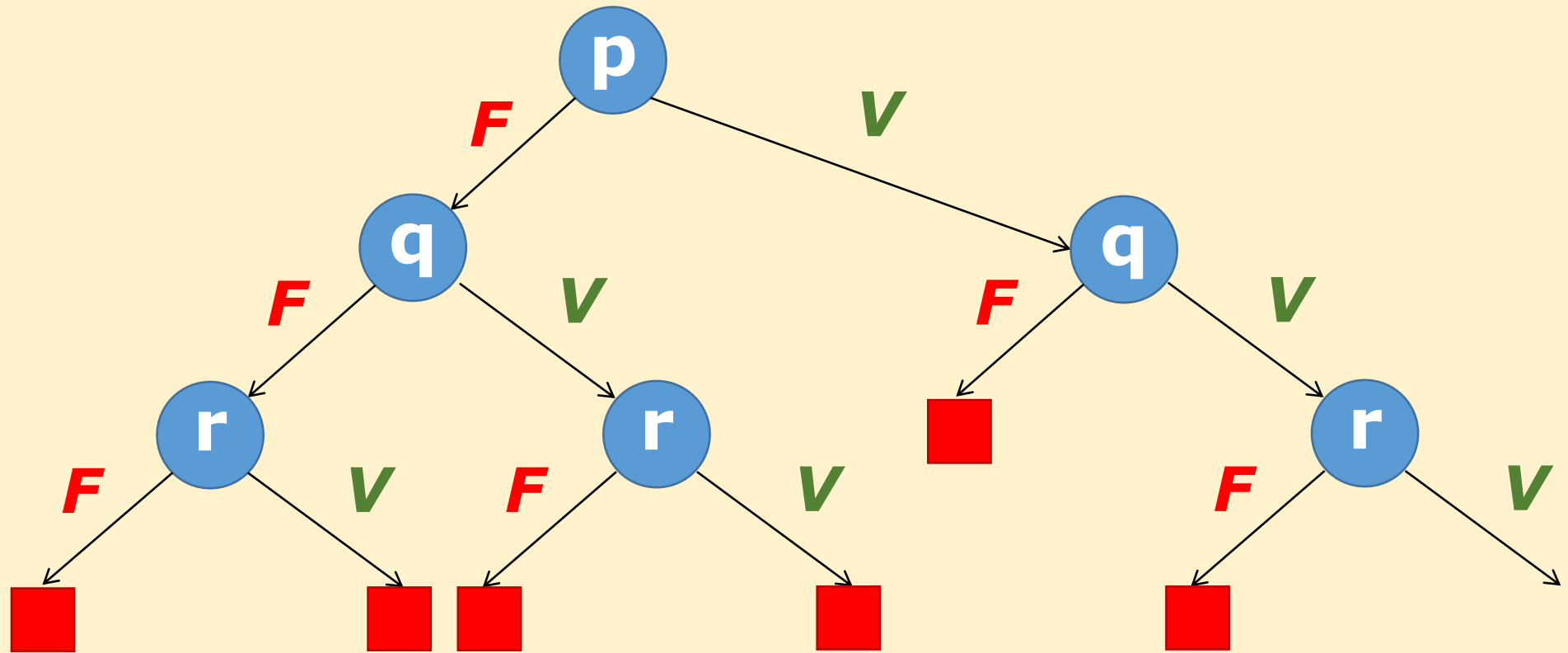
{ $\bar{r}s,$ r }

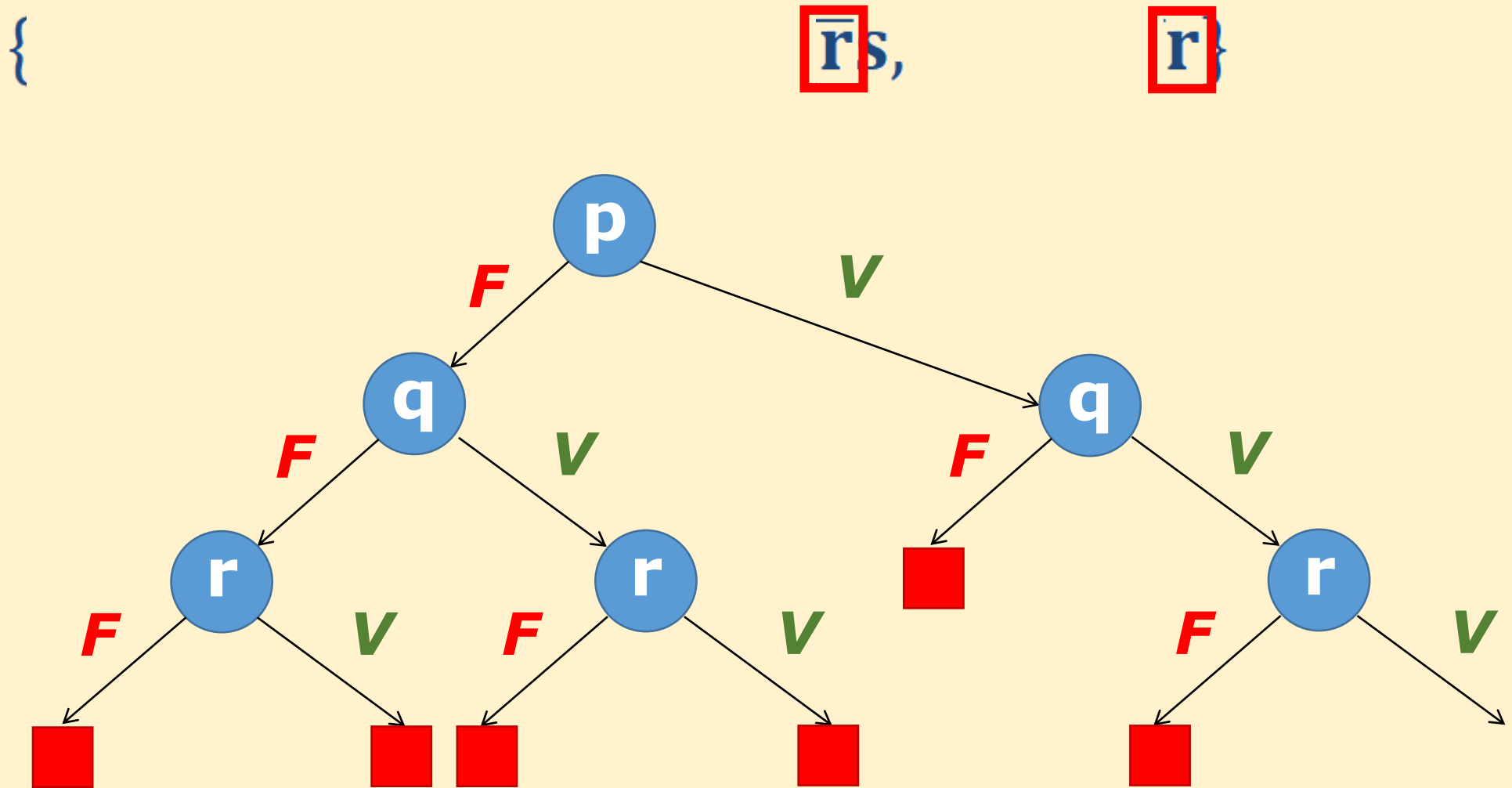


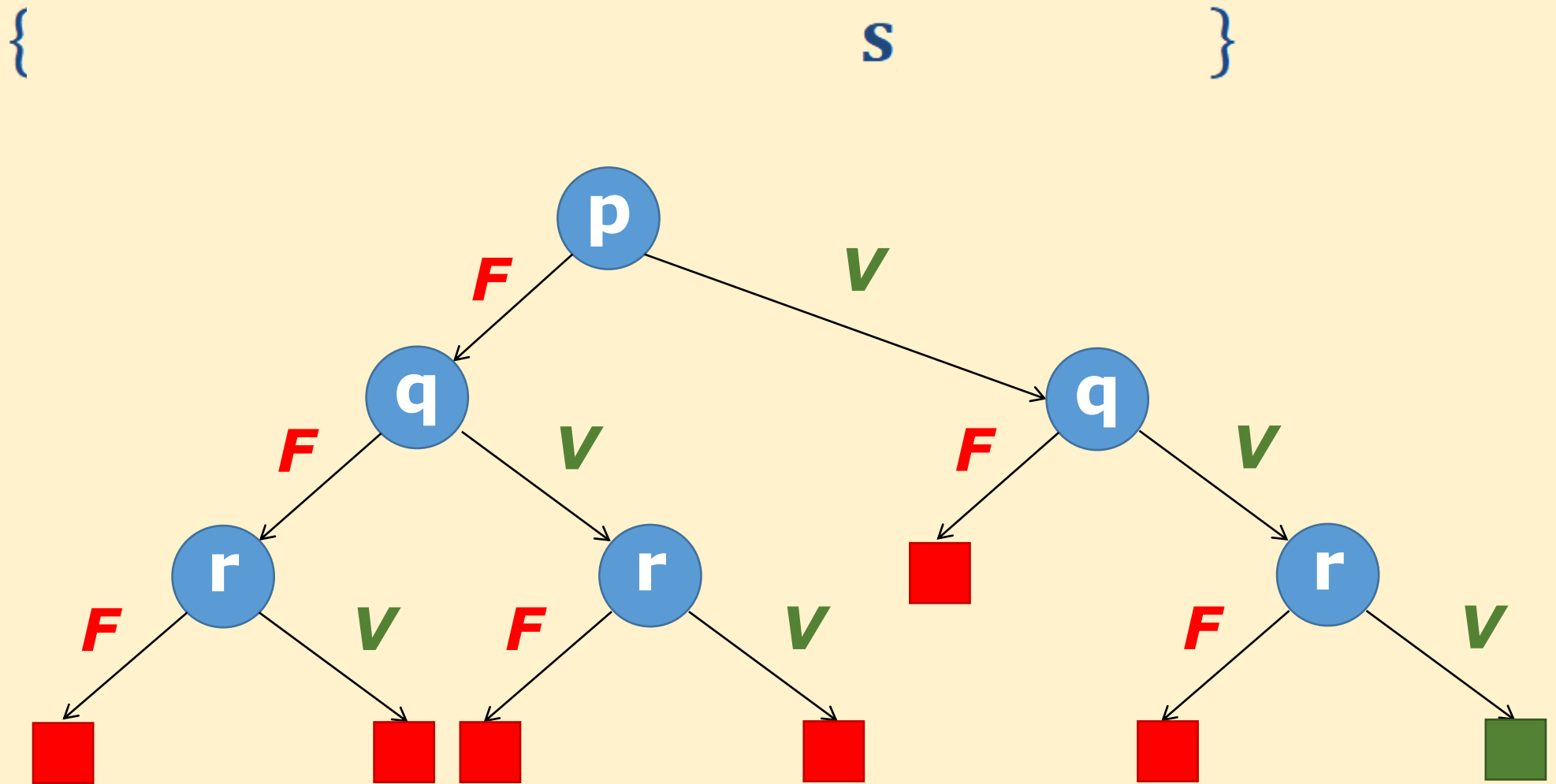
{ $\bar{r}s,$ r }

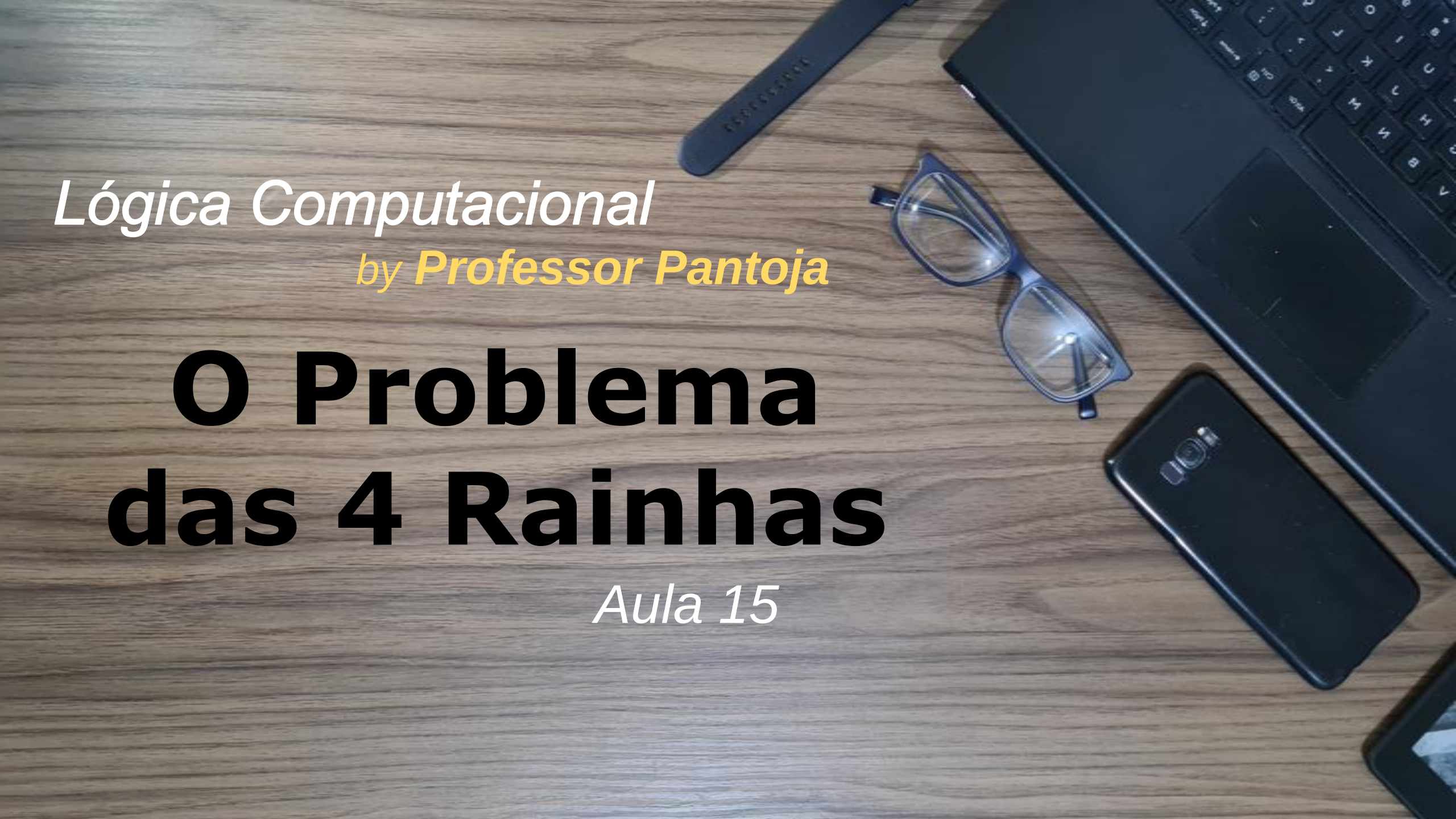


{ $\bar{r}s,$ r }







A top-down view of a wooden desk. On the right side, there is a black laptop with its keyboard visible. Below the laptop, a pair of blue-rimmed glasses lies on the desk. To the left of the glasses, a black pen is positioned diagonally. Further down and to the right, a black smartphone is lying flat. The background is the natural wood grain of the desk.

Lógica Computacional
by Professor Pantoja

O Problema das 4 Rainhas

Aula 15

- Codificação**

	1	2	3	4
1	11	12	13	14
2	21	22	23	24
3	31	32	33	34
4	41	42	43	44

- **No mínimo uma**

- No mínimo uma

$$p1 \vee p2 \vee p3$$

- No mínimo uma

$$p1 \vee p2 \vee p3 = \mathbf{V}$$

- No mínimo uma

$$p1 \vee p2 \vee p3 = V$$


F **F** **V**

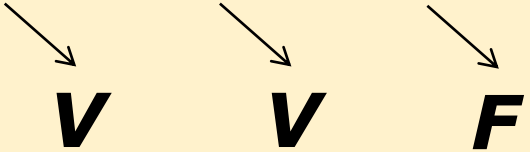
- No mínimo uma

$$\begin{array}{ccccc} p1 & \vee & p2 & \vee & p3 & = & V \\ \swarrow & & \swarrow & & \swarrow & & \\ F & & V & & F & & \end{array}$$

- No mínimo uma

$$\begin{array}{ccccc} p1 & \vee & p2 & \vee & p3 & = & V \\ \swarrow & & \swarrow & & \swarrow & & \\ V & & F & & F & & \end{array}$$

- No mínimo uma

$$p1 \vee p2 \vee p3 = V$$


V **V** **F**

- No mínimo uma

$$p1 \vee p2 \vee p3 = F$$


The diagram illustrates a logical clause $p1 \vee p2 \vee p3 = F$. Below each variable $p1$, $p2$, and $p3$, there is a bold F (False). A small arrow points from each variable to its corresponding F , indicating that each literal in the clause is false.

- No mínimo uma

$$p1 \vee p2 \vee p3$$

- Somente uma

$$(\overline{p1} \vee \overline{p2}) \wedge (\overline{p1} \vee \overline{p3}) \wedge (\overline{p2} \vee \overline{p3})$$

- No mínimo uma

$$p1 \vee p2 \vee p3$$

- Somente uma

$$(\overline{p1} \vee \overline{p2}) \wedge (\overline{p1} \vee \overline{p3}) \wedge (\overline{p2} \vee \overline{p3})$$


The diagram shows two arrows pointing from the first two literals of the first clause, $\overline{p1}$ and $\overline{p2}$, to the variable $p1$. This indicates that the clause is satisfied if $p1$ is false.

- No mínimo uma

$$p1 \vee p2 \vee p3$$

- Somente uma

$$(\overline{p1} \vee \overline{p2}) \wedge (\overline{p1} \vee \overline{p3}) \wedge (\overline{p2} \vee \overline{p3})$$

V V V F

- No mínimo uma

$$p1 \vee p2 \vee p3$$

- Somente uma

$$(\overline{p1} \vee \overline{p2}) \wedge (\overline{p1} \vee \overline{p3}) \wedge (\overline{p2} \vee \overline{p3}) = V$$



V V V F V F

- No mínimo uma

$$p1 \vee p2 \vee p3$$

- Somente uma

$$(\overline{p1} \vee \overline{p2}) \wedge (\overline{p1} \vee \overline{p3}) \wedge (\overline{p2} \vee \overline{p3}) = F$$

Diagram illustrating the constraints for the 4 Queens problem using propositional logic. The expression $(\overline{p1} \vee \overline{p2}) \wedge (\overline{p1} \vee \overline{p3}) \wedge (\overline{p2} \vee \overline{p3}) = F$ is shown. Below the expression, arrows point to the literals $\overline{p1}$, $\overline{p2}$, $\overline{p1}$, $\overline{p3}$, $\overline{p2}$, and $\overline{p3}$ with the following truth values: F , V , F , F , V , and F respectively.

- No mínimo uma

$$p1 \vee p2 \vee p3$$

- Somente uma

$$(\overline{p1} \vee \overline{p2}) \wedge (\overline{p1} \vee \overline{p3}) \wedge (\overline{p2} \vee \overline{p3}) = \mathbf{V}$$


	1	2	3	4
1	11	12	13	14
2	21	22	23	24
3	31	32	33	34
4	41	42	43	44

	1	2	3	4
1	11	12	13	14
2	21	22	23	24
3	31	32	33	34
4	41	42	43	44

- Uma Rainha por linha

$11 \vee 12 \vee 13 \vee 14,$

	1	2	3	4
1	11	12	13	14
2	21	22	23	24
3	31	32	33	34
4	41	42	43	44

- Uma Rainha por linha

11 ∨ 12 ∨ 13 ∨ 14,
21 ∨ 22 ∨ 23 ∨ 24,

	1	2	3	4
1	11	12	13	14
2	21	22	23	24
3	31	32	33	34
4	41	42	43	44

- Uma Rainha por linha

11 ∨ 12 ∨ 13 ∨ 14,
21 ∨ 22 ∨ 23 ∨ 24,
31 ∨ 32 ∨ 33 ∨ 34,

	1	2	3	4
1	11	12	13	14
2	21	22	23	24
3	31	32	33	34
4	41	42	43	44

- Uma Rainha por linha

11 ∨ 12 ∨ 13 ∨ 14,
21 ∨ 22 ∨ 23 ∨ 24,
31 ∨ 32 ∨ 33 ∨ 34,
41 ∨ 42 ∨ 43 ∨ 44,

	1	2	3	4
1	11	12	13	14
2	21	22	23	24
3	31	32	33	34
4	41	42	43	44

- **Somente uma Rainha por linha**

	1	2	3	4
1	11	12	13	14
2	21	22	23	24
3	31	32	33	34
4	41	42	43	44

$\overline{11} \vee \overline{12},$

	1	2	3	4
1	11	12	13	14
2	21	22	23	24
3	31	32	33	34
4	41	42	43	44

$\overline{11} \vee \overline{12}, \overline{11} \vee \overline{13},$

	1	2	3	4
1	11	12	13	14
2	21	22	23	24
3	31	32	33	34
4	41	42	43	44

$$\overline{11} \vee \overline{12}, \overline{11} \vee \overline{13}, \overline{11} \vee \overline{14}$$

	1	2	3	4
1	11	12	13	14
2	21	22	23	24
3	31	32	33	34
4	41	42	43	44

$\overline{11} \vee \overline{12}, \overline{11} \vee \overline{13}, \overline{11} \vee \overline{14}$
 $, \overline{12} \vee \overline{13},$

	1	2	3	4
1	11	12	13	14
2	21	22	23	24
3	31	32	33	34
4	41	42	43	44

$\overline{11} \vee \overline{12}, \overline{11} \vee \overline{13}, \overline{11} \vee \overline{14}$
 $, \overline{12} \vee \overline{13}, \overline{12} \vee \overline{14},$

	1	2	3	4
1	11	12	13	14
2	21	22	23	24
3	31	32	33	34
4	41	42	43	44

$\overline{11} \vee \overline{12}, \overline{11} \vee \overline{13}, \overline{11} \vee \overline{14}$
 $, \overline{12} \vee \overline{13}, \overline{12} \vee \overline{14}, \overline{13} \vee \overline{14}$

- Não mais que uma Rainha por linha

$\overline{11} \vee \overline{12}, \overline{11} \vee \overline{13}, \overline{11} \vee \overline{14}, \overline{12} \vee \overline{13}, \overline{12} \vee \overline{14}, \overline{13} \vee \overline{14}$

$\overline{21} \vee \overline{22}, \overline{21} \vee \overline{23}, \overline{21} \vee \overline{24}, \overline{22} \vee \overline{23}, \overline{22} \vee \overline{24}, \overline{23} \vee \overline{24}$

$\overline{31} \vee \overline{32}, \overline{31} \vee \overline{33}, \overline{31} \vee \overline{34}, \overline{32} \vee \overline{33}, \overline{32} \vee \overline{34}, \overline{33} \vee \overline{34}$

$\overline{41} \vee \overline{42}, \overline{41} \vee \overline{43}, \overline{41} \vee \overline{44}, \overline{42} \vee \overline{43}, \overline{42} \vee \overline{44}, \overline{43} \vee \overline{44}$

	1	2	3	4
1	11	12	13	14
2	21	22	23	24
3	31	32	33	34
4	41	42	43	44

- **Somente uma Rainha por coluna**

	1	2	3	4
1	11	12	13	14
2	21	22	23	24
3	31	32	33	34
4	41	42	43	44

$\overline{11} \vee \overline{21},$

	1	2	3	4
1	11	12	13	14
2	21	22	23	24
3	31	32	33	34
4	41	42	43	44

$\overline{11} \vee \overline{21}, \overline{11} \vee \overline{31},$

	1	2	3	4
1	11	12	13	14
2	21	22	23	24
3	31	32	33	34
4	41	42	43	44

$\overline{11} \vee \overline{21}, \overline{11} \vee \overline{31}, \overline{11} \vee \overline{41},$

	1	2	3	4
1	11	12	13	14
2	21	22	23	24
3	31	32	33	34
4	41	42	43	44

$\overline{11} \vee \overline{21}, \overline{11} \vee \overline{31}, \overline{11} \vee \overline{41},$
 $\overline{21} \vee \overline{31},$

	1	2	3	4
1	11	12	13	14
2	21	22	23	24
3	31	32	33	34
4	41	42	43	44

$\overline{11} \vee \overline{21}, \overline{11} \vee \overline{31}, \overline{11} \vee \overline{41},$
 $\overline{21} \vee \overline{31}, \overline{21} \vee \overline{41},$

	1	2	3	4
1	11	12	13	14
2	21	22	23	24
3	31	32	33	34
4	41	42	43	44

$\overline{11} \vee \overline{21}, \overline{11} \vee \overline{31}, \overline{11} \vee \overline{41},$
 $\overline{21} \vee \overline{31}, \overline{21} \vee \overline{41}, \overline{31} \vee \overline{41}$

- Não mais que uma Rainha por coluna

$\overline{11} \vee \overline{21}, \overline{11} \vee \overline{31}, \overline{11} \vee \overline{41}, \overline{21} \vee \overline{31}, \overline{21} \vee \overline{41}, \overline{31} \vee \overline{41}$

$\overline{12} \vee \overline{22}, \overline{12} \vee \overline{32}, \overline{12} \vee \overline{42}, \overline{22} \vee \overline{32}, \overline{22} \vee \overline{42}, \overline{32} \vee \overline{42}$

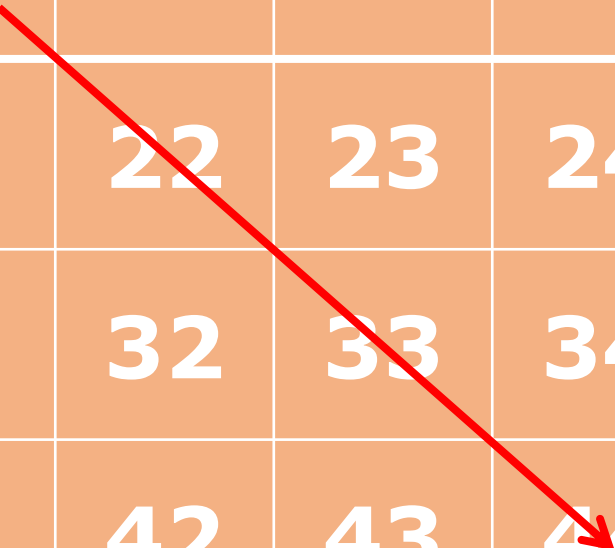
$\overline{13} \vee \overline{23}, \overline{13} \vee \overline{33}, \overline{13} \vee \overline{43}, \overline{23} \vee \overline{33}, \overline{23} \vee \overline{43}, \overline{33} \vee \overline{43}$

$\overline{14} \vee \overline{24}, \overline{14} \vee \overline{34}, \overline{14} \vee \overline{44}, \overline{24} \vee \overline{34}, \overline{24} \vee \overline{44}, \overline{34} \vee \overline{44}$

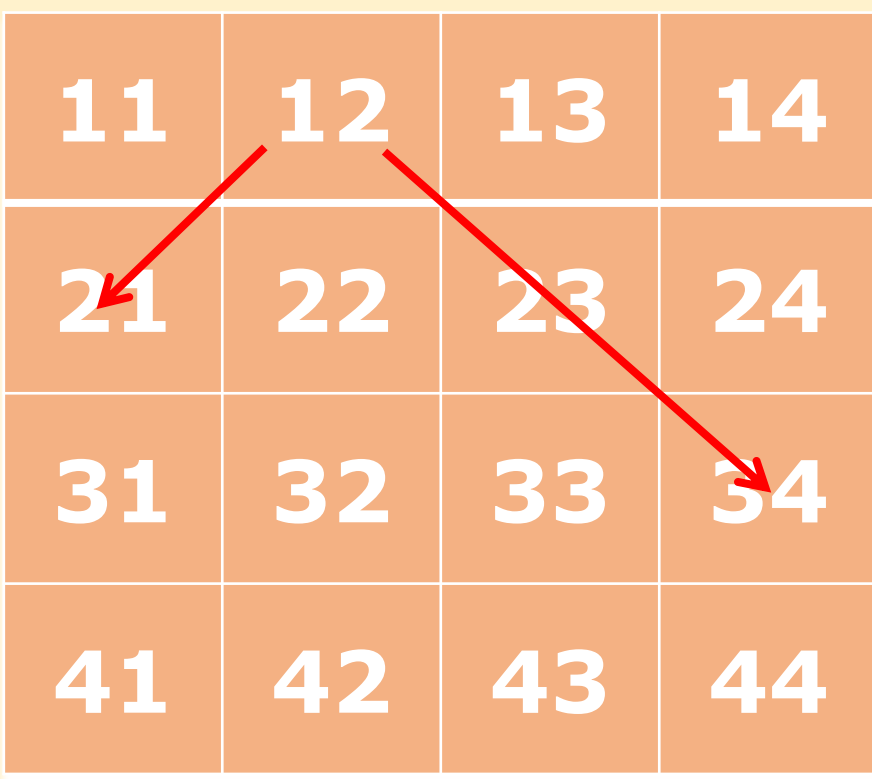
	1	2	3	4
1	11	12	13	14
2	21	22	23	24
3	31	32	33	34
4	41	42	43	44

- **Somente uma Rainha por diagonal**

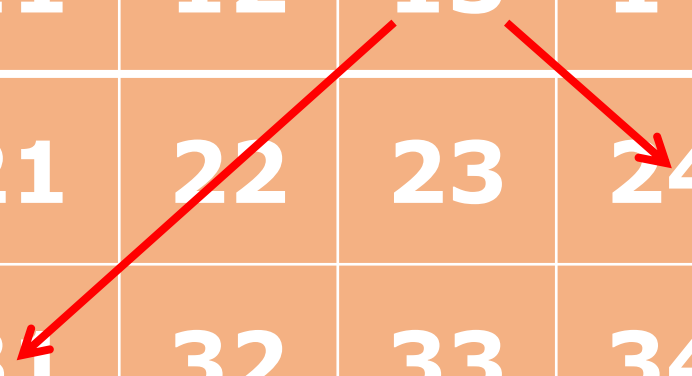
	1	2	3	4
1	11	12	13	14
2	21	22	23	24
3	31	32	33	34
4	41	42	43	44



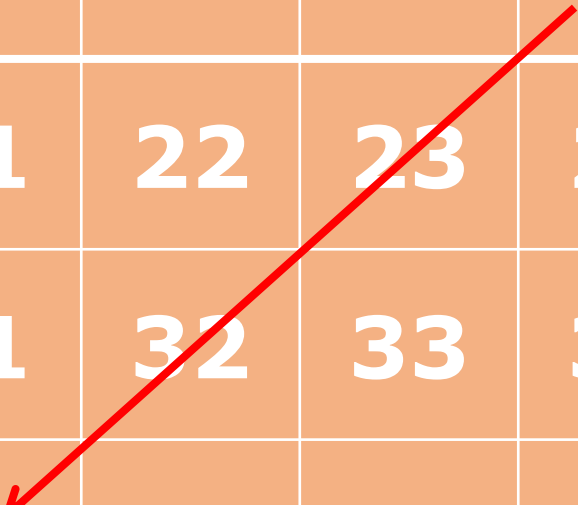
	1	2	3	4
1	11	12	13	14
2	21	22	23	24
3	31	32	33	34
4	41	42	43	44



	1	2	3	4
1	11	12	13	14
2	21	22	23	24
3	31	32	33	34
4	41	42	43	44



	1	2	3	4
1	11	12	13	14
2	21	22	23	24
3	31	32	33	34
4	41	42	43	44



	1	2	3	4
1	11	12	13	14
2	21	22	23	24
3	31	32	33	34
4	41	42	43	44

$\overline{11} \vee \overline{22},$

	1	2	3	4
1	11	12	13	14
2	21	22	23	24
3	31	32	33	34
4	41	42	43	44

$\overline{11} \vee \overline{22}, \overline{11} \vee \overline{33},$

	1	2	3	4
1	11	12	13	14
2	21	22	23	24
3	31	32	33	34
4	41	42	43	44

$\overline{11} \vee \overline{22}, \overline{11} \vee \overline{33}, \overline{11} \vee \overline{44}$

	1	2	3	4
1	11	12	13	14
2	21	22	23	24
3	31	32	33	34
4	41	42	43	44

$\overline{11} \vee \overline{22}, \overline{11} \vee \overline{33}, \overline{11} \vee \overline{44}$
 $\overline{12} \vee \overline{21},$

	1	2	3	4
1	11	12	13	14
2	21	22	23	24
3	31	32	33	34
4	41	42	43	44

$\overline{11} \vee \overline{22}, \overline{11} \vee \overline{33}, \overline{11} \vee \overline{44}$
 $\overline{12} \vee \overline{21}, \overline{12} \vee \overline{23},$

	1	2	3	4
1	11	12	13	14
2	21	22	23	24
3	31	32	33	34
4	41	42	43	44

$\overline{11} \vee \overline{22}, \overline{11} \vee \overline{33}, \overline{11} \vee \overline{44}$
 $\overline{12} \vee \overline{21}, \overline{12} \vee \overline{23}, \overline{12} \vee \overline{34}$

	1	2	3	4
1	11	12	13	14
2	21	22	23	24
3	31	32	33	34
4	41	42	43	44

$\overline{11} \vee \overline{22}, \overline{11} \vee \overline{33}, \overline{11} \vee \overline{44}$
 $\overline{12} \vee \overline{21}, \overline{12} \vee \overline{23}, \overline{12} \vee \overline{34}$
 $\overline{13} \vee \overline{22},$

	1	2	3	4
1	11	12	13	14
2	21	22	23	24
3	31	32	33	34
4	41	42	43	44

$\overline{11} \vee \overline{22}, \overline{11} \vee \overline{33}, \overline{11} \vee \overline{44}$
 $\overline{12} \vee \overline{21}, \overline{12} \vee \overline{23}, \overline{12} \vee \overline{34}$
 $\overline{13} \vee \overline{22}, \overline{13} \vee \overline{31},$

	1	2	3	4
1	11	12	13	14
2	21	22	23	24
3	31	32	33	34
4	41	42	43	44

$\overline{11} \vee \overline{22}, \overline{11} \vee \overline{33}, \overline{11} \vee \overline{44}$

$\overline{12} \vee \overline{21}, \overline{12} \vee \overline{23}, \overline{12} \vee \overline{34}$

$\overline{13} \vee \overline{22}, \overline{13} \vee \overline{31}, \overline{13} \vee \overline{24}$

	1	2	3	4
1	11	12	13	14
2	21	22	23	24
3	31	32	33	34
4	41	42	43	44

$\overline{11} \vee \overline{22}, \overline{11} \vee \overline{33}, \overline{11} \vee \overline{44}$
 $\overline{12} \vee \overline{21}, \overline{12} \vee \overline{23}, \overline{12} \vee \overline{34}$
 $\overline{13} \vee \overline{22}, \overline{13} \vee \overline{31}, \overline{13} \vee \overline{24}$
 $\overline{14} \vee \overline{23},$

	1	2	3	4
1	11	12	13	14
2	21	22	23	24
3	31	32	33	34
4	41	42	43	44

$\overline{11} \vee \overline{22}, \overline{11} \vee \overline{33}, \overline{11} \vee \overline{44}$
 $\overline{12} \vee \overline{21}, \overline{12} \vee \overline{23}, \overline{12} \vee \overline{34}$
 $\overline{13} \vee \overline{22}, \overline{13} \vee \overline{31}, \overline{13} \vee \overline{24}$
 $\overline{14} \vee \overline{23}, \overline{14} \vee \overline{32},$

	1	2	3	4
1	11	12	13	14
2	21	22	23	24
3	31	32	33	34
4	41	42	43	44

$\overline{11} \vee \overline{22}, \overline{11} \vee \overline{33}, \overline{11} \vee \overline{44}$
 $\overline{12} \vee \overline{21}, \overline{12} \vee \overline{23}, \overline{12} \vee \overline{34}$
 $\overline{13} \vee \overline{22}, \overline{13} \vee \overline{31}, \overline{13} \vee \overline{24}$
 $\overline{14} \vee \overline{23}, \overline{14} \vee \overline{32}, \overline{14} \vee \overline{41}$

- Não mais que uma Rainha por diagonal

$\overline{11} \vee \overline{22}, \overline{11} \vee \overline{33}, \overline{11} \vee \overline{44}$

$\overline{13} \vee \overline{22}, \overline{13} \vee \overline{31}, \overline{13} \vee \overline{24}$

$\overline{21} \vee \overline{32}, \overline{21} \vee \overline{43}$

$\overline{23} \vee \overline{32}, \overline{23} \vee \overline{41}, \overline{23} \vee \overline{34}$

$\overline{31} \vee \overline{42}$

$\overline{33} \vee \overline{42}, \overline{33} \vee \overline{44}$

$\overline{12} \vee \overline{21}, \overline{12} \vee \overline{23}, \overline{12} \vee \overline{34}$

$\overline{14} \vee \overline{23}, \overline{14} \vee \overline{32}, \overline{14} \vee \overline{41}$

$\overline{22} \vee \overline{31}, \overline{22} \vee \overline{33}, \overline{22} \vee \overline{44}$

$\overline{24} \vee \overline{33}, \overline{24} \vee \overline{42}$

$\overline{32} \vee \overline{41}, \overline{32} \vee \overline{43}$

$\overline{34} \vee \overline{43}$

$11 \vee 12 \vee 13 \vee 14,$
 $21 \vee 22 \vee 23 \vee 24,$
 $31 \vee 32 \vee 33 \vee 34,$
 $41 \vee 42 \vee 43 \vee 44,$
 $\overline{11} \vee \overline{12}, \overline{11} \vee \overline{13}, \overline{11} \vee \overline{14}, \overline{12} \vee \overline{13}, \overline{12} \vee \overline{14}, \overline{13} \vee \overline{14}$
 $\overline{21} \vee \overline{22}, \overline{21} \vee \overline{23}, \overline{21} \vee \overline{24}, \overline{22} \vee \overline{23}, \overline{22} \vee \overline{24}, \overline{23} \vee \overline{24}$
 $\overline{31} \vee \overline{32}, \overline{31} \vee \overline{33}, \overline{31} \vee \overline{34}, \overline{32} \vee \overline{33}, \overline{32} \vee \overline{34}, \overline{33} \vee \overline{34}$
 $\overline{41} \vee \overline{42}, \overline{41} \vee \overline{43}, \overline{41} \vee \overline{44}, \overline{42} \vee \overline{43}, \overline{42} \vee \overline{44}, \overline{43} \vee \overline{44}$
 $\overline{11} \vee \overline{21}, \overline{11} \vee \overline{31}, \overline{11} \vee \overline{41}, \overline{21} \vee \overline{31}, \overline{21} \vee \overline{41}, \overline{31} \vee \overline{41}$
 $\overline{12} \vee \overline{22}, \overline{12} \vee \overline{32}, \overline{12} \vee \overline{42}, \overline{22} \vee \overline{32}, \overline{22} \vee \overline{42}, \overline{32} \vee \overline{42}$
 $\overline{13} \vee \overline{23}, \overline{13} \vee \overline{33}, \overline{13} \vee \overline{43}, \overline{23} \vee \overline{33}, \overline{23} \vee \overline{43}, \overline{33} \vee \overline{43}$
 $\overline{14} \vee \overline{24}, \overline{14} \vee \overline{34}, \overline{14} \vee \overline{44}, \overline{24} \vee \overline{34}, \overline{24} \vee \overline{44}, \overline{34} \vee \overline{44}$
 $\overline{12} \vee \overline{21}, \overline{12} \vee \overline{23}, \overline{12} \vee \overline{34}$
 $\overline{14} \vee \overline{23}, \overline{14} \vee \overline{32}, \overline{14} \vee \overline{41}$
 $\overline{22} \vee \overline{31}, \overline{22} \vee \overline{33}, \overline{22} \vee \overline{44}$
 $\overline{24} \vee \overline{33}, \overline{24} \vee \overline{42}$
 $\overline{32} \vee \overline{41}, \overline{32} \vee \overline{43}$
 $\overline{34} \vee \overline{43}$
 $\overline{11} \vee \overline{22}, \overline{11} \vee \overline{33}, \overline{11} \vee \overline{44}$
 $\overline{13} \vee \overline{22}, \overline{13} \vee \overline{31}, \overline{13} \vee \overline{24}$
 $\overline{21} \vee \overline{32}, \overline{21} \vee \overline{43}$
 $\overline{23} \vee \overline{32}, \overline{23} \vee \overline{41}, \overline{23} \vee \overline{34}$
 $\overline{31} \vee \overline{42}$
 $\overline{33} \vee \overline{42}, \overline{33} \vee \overline{44}$

Escolhas**12** $11 \vee 12 \vee 13 \vee 14,$ $21 \vee 22 \vee 23 \vee 24,$ $31 \vee 32 \vee 33 \vee 34,$ $41 \vee 42 \vee 43 \vee 44,$ $\overline{11} \vee \overline{12}, \overline{11} \vee \overline{13}, \overline{11} \vee \overline{14}, \overline{12} \vee \overline{13}, \overline{12} \vee \overline{14}, \overline{13} \vee \overline{14}$ $\overline{21} \vee \overline{22}, \overline{21} \vee \overline{23}, \overline{21} \vee \overline{24}, \overline{22} \vee \overline{23}, \overline{22} \vee \overline{24}, \overline{23} \vee \overline{24}$ $\overline{31} \vee \overline{32}, \overline{31} \vee \overline{33}, \overline{31} \vee \overline{34}, \overline{32} \vee \overline{33}, \overline{32} \vee \overline{34}, \overline{33} \vee \overline{34}$ $\overline{41} \vee \overline{42}, \overline{41} \vee \overline{43}, \overline{41} \vee \overline{44}, \overline{42} \vee \overline{43}, \overline{42} \vee \overline{44}, \overline{43} \vee \overline{44}$ $\overline{11} \vee \overline{21}, \overline{11} \vee \overline{31}, \overline{11} \vee \overline{41}, \overline{21} \vee \overline{31}, \overline{21} \vee \overline{41}, \overline{31} \vee \overline{41}$ $\overline{12} \vee \overline{22}, \overline{12} \vee \overline{32}, \overline{12} \vee \overline{42}, \overline{22} \vee \overline{32}, \overline{22} \vee \overline{42}, \overline{32} \vee \overline{42}$ $\overline{13} \vee \overline{23}, \overline{13} \vee \overline{33}, \overline{13} \vee \overline{43}, \overline{23} \vee \overline{33}, \overline{23} \vee \overline{43}, \overline{33} \vee \overline{43}$ $\overline{14} \vee \overline{24}, \overline{14} \vee \overline{34}, \overline{14} \vee \overline{44}, \overline{24} \vee \overline{34}, \overline{24} \vee \overline{44}, \overline{34} \vee \overline{44}$ $\overline{12} \vee \overline{21}, \overline{12} \vee \overline{23}, \overline{12} \vee \overline{34}$ $\overline{14} \vee \overline{23}, \overline{14} \vee \overline{32}, \overline{14} \vee \overline{41}$ $\overline{22} \vee \overline{31}, \overline{22} \vee \overline{33}, \overline{22} \vee \overline{44}$ $\overline{24} \vee \overline{33}, \overline{24} \vee \overline{42}$ $\overline{32} \vee \overline{41}, \overline{32} \vee \overline{43}$ $\overline{34} \vee \overline{43}$ $\overline{11} \vee \overline{22}, \overline{11} \vee \overline{33}, \overline{11} \vee \overline{44}$ $\overline{13} \vee \overline{22}, \overline{13} \vee \overline{31}, \overline{13} \vee \overline{24}$ $\overline{21} \vee \overline{32}, \overline{21} \vee \overline{43}$ $\overline{23} \vee \overline{32}, \overline{23} \vee \overline{41}, \overline{23} \vee \overline{34}$ $\overline{31} \vee \overline{42}$ $\overline{33} \vee \overline{42}, \overline{33} \vee \overline{44}$

$11 \vee 12 \vee 13 \vee 14,$
 $21 \vee 22 \vee 23 \vee 24,$
 $31 \vee 32 \vee 33 \vee 34,$
 $41 \vee 42 \vee 43 \vee 44,$

$\overline{11} \vee \overline{12}, \overline{11} \vee \overline{13}, \overline{11} \vee \overline{14}, \overline{12} \vee \overline{13}, \overline{12} \vee \overline{14}, \overline{13} \vee \overline{14}$
 $\overline{21} \vee \overline{22}, \overline{21} \vee \overline{23}, \overline{21} \vee \overline{24}, \overline{22} \vee \overline{23}, \overline{22} \vee \overline{24}, \overline{23} \vee \overline{24}$
 $\overline{31} \vee \overline{32}, \overline{31} \vee \overline{33}, \overline{31} \vee \overline{34}, \overline{32} \vee \overline{33}, \overline{32} \vee \overline{34}, \overline{33} \vee \overline{34}$
 $\overline{41} \vee \overline{42}, \overline{41} \vee \overline{43}, \overline{41} \vee \overline{44}, \overline{42} \vee \overline{43}, \overline{42} \vee \overline{44}, \overline{43} \vee \overline{44}$

$\overline{11} \vee \overline{21}, \overline{11} \vee \overline{31}, \overline{11} \vee \overline{41}, \overline{21} \vee \overline{31}, \overline{21} \vee \overline{41}, \overline{31} \vee \overline{41}$
 $\overline{12} \vee \overline{22}, \overline{12} \vee \overline{32}, \overline{12} \vee \overline{42}, \overline{22} \vee \overline{32}, \overline{22} \vee \overline{42}, \overline{32} \vee \overline{42}$
 $\overline{13} \vee \overline{23}, \overline{13} \vee \overline{33}, \overline{13} \vee \overline{43}, \overline{23} \vee \overline{33}, \overline{23} \vee \overline{43}, \overline{33} \vee \overline{43}$
 $\overline{14} \vee \overline{24}, \overline{14} \vee \overline{34}, \overline{14} \vee \overline{44}, \overline{24} \vee \overline{34}, \overline{24} \vee \overline{44}, \overline{34} \vee \overline{44}$

$\overline{12} \vee \overline{21}, \overline{12} \vee \overline{23}, \overline{12} \vee \overline{34}$
 $\overline{14} \vee \overline{23}, \overline{14} \vee \overline{32}, \overline{14} \vee \overline{41}$
 $\overline{22} \vee \overline{31}, \overline{22} \vee \overline{33}, \overline{22} \vee \overline{44}$
 $\overline{24} \vee \overline{33}, \overline{24} \vee \overline{42}$
 $\overline{32} \vee \overline{41}, \overline{32} \vee \overline{43}$
 $\overline{34} \vee \overline{43}$
 $\overline{11} \vee \overline{22}, \overline{11} \vee \overline{33}, \overline{11} \vee \overline{44}$
 $\overline{13} \vee \overline{22}, \overline{13} \vee \overline{31}, \overline{13} \vee \overline{24}$
 $\overline{21} \vee \overline{32}, \overline{21} \vee \overline{43}$
 $\overline{23} \vee \overline{32}, \overline{23} \vee \overline{41}, \overline{23} \vee \overline{34}$
 $\overline{31} \vee \overline{42}$
 $\overline{33} \vee \overline{42}, \overline{33} \vee \overline{44}$

Escolhas

12

Escolhas

12

 $21 \vee 22 \vee 23 \vee 24,$ $31 \vee 32 \vee 33 \vee 34,$ $41 \vee 42 \vee 43 \vee 44,$ $\overline{11} \vee \overline{12} \vee \overline{11} \vee \overline{13}, \overline{11} \vee \overline{14}, \overline{12} \vee \overline{13}, \overline{12} \vee \overline{14}, \overline{13} \vee \overline{14}$ $\overline{21} \vee \overline{22}, \overline{21} \vee \overline{23}, \overline{21} \vee \overline{24}, \overline{22} \vee \overline{23}, \overline{22} \vee \overline{24}, \overline{23} \vee \overline{24}$ $\overline{31} \vee \overline{32}, \overline{31} \vee \overline{33}, \overline{31} \vee \overline{34}, \overline{32} \vee \overline{33}, \overline{32} \vee \overline{34}, \overline{33} \vee \overline{34}$ $\overline{41} \vee \overline{42}, \overline{41} \vee \overline{43}, \overline{41} \vee \overline{44}, \overline{42} \vee \overline{43}, \overline{42} \vee \overline{44}, \overline{43} \vee \overline{44}$ $\overline{11} \vee \overline{21}, \overline{11} \vee \overline{31}, \overline{11} \vee \overline{41}, \overline{21} \vee \overline{31}, \overline{21} \vee \overline{41}, \overline{31} \vee \overline{41}$ $\overline{12} \vee \overline{22}, \overline{12} \vee \overline{32}, \overline{12} \vee \overline{42}, \overline{22} \vee \overline{32}, \overline{22} \vee \overline{42}, \overline{32} \vee \overline{42}$ $\overline{13} \vee \overline{23}, \overline{13} \vee \overline{33}, \overline{13} \vee \overline{43}, \overline{23} \vee \overline{33}, \overline{23} \vee \overline{43}, \overline{33} \vee \overline{43}$ $\overline{14} \vee \overline{24}, \overline{14} \vee \overline{34}, \overline{14} \vee \overline{44}, \overline{24} \vee \overline{34}, \overline{24} \vee \overline{44}, \overline{34} \vee \overline{44}$ $\overline{12} \vee \overline{21}, \overline{12} \vee \overline{23}, \overline{12} \vee \overline{34}$ $\overline{14} \vee \overline{23}, \overline{14} \vee \overline{32}, \overline{14} \vee \overline{41}$ $\overline{22} \vee \overline{31}, \overline{22} \vee \overline{33}, \overline{22} \vee \overline{44}$ $\overline{24} \vee \overline{33}, \overline{24} \vee \overline{42}$ $\overline{32} \vee \overline{41}, \overline{32} \vee \overline{43}$ $\overline{34} \vee \overline{43}$ $\overline{11} \vee \overline{22}, \overline{11} \vee \overline{33}, \overline{11} \vee \overline{44}$ $\overline{13} \vee \overline{22}, \overline{13} \vee \overline{31}, \overline{13} \vee \overline{24}$ $\overline{21} \vee \overline{32}, \overline{21} \vee \overline{43}$ $\overline{23} \vee \overline{32}, \overline{23} \vee \overline{41}, \overline{23} \vee \overline{34}$ $\overline{31} \vee \overline{42}$ $\overline{33} \vee \overline{42}, \overline{33} \vee \overline{44}$

Escolhas

12

 $21 \vee 22 \vee 23 \vee 24,$ $31 \vee 32 \vee 33 \vee 34,$ $41 \vee 42 \vee 43 \vee 44,$ $\overline{11}, \overline{11} \vee \overline{13}, \overline{11} \vee \overline{14}, \overline{13}, \overline{14}, \overline{13} \vee \overline{14}$ $\overline{21} \vee \overline{22}, \overline{21} \vee \overline{23}, \overline{21} \vee \overline{24}, \overline{22} \vee \overline{23}, \overline{22} \vee \overline{24}, \overline{23} \vee \overline{24}$ $\overline{31} \vee \overline{32}, \overline{31} \vee \overline{33}, \overline{31} \vee \overline{34}, \overline{32} \vee \overline{33}, \overline{32} \vee \overline{34}, \overline{33} \vee \overline{34}$ $\overline{41} \vee \overline{42}, \overline{41} \vee \overline{43}, \overline{41} \vee \overline{44}, \overline{42} \vee \overline{43}, \overline{42} \vee \overline{44}, \overline{43} \vee \overline{44}$ $\overline{11} \vee \overline{21}, \overline{11} \vee \overline{31}, \overline{11} \vee \overline{41}, \overline{21} \vee \overline{31}, \overline{21} \vee \overline{41}, \overline{31} \vee \overline{41}$ $\overline{22}, \overline{32}, \overline{42}, \overline{22} \vee \overline{32}, \overline{22} \vee \overline{42}, \overline{32} \vee \overline{42}$ $\overline{13} \vee \overline{23}, \overline{13} \vee \overline{33}, \overline{13} \vee \overline{43}, \overline{23} \vee \overline{33}, \overline{23} \vee \overline{43}, \overline{33} \vee \overline{43}$ $\overline{14} \vee \overline{24}, \overline{14} \vee \overline{34}, \overline{14} \vee \overline{44}, \overline{24} \vee \overline{34}, \overline{24} \vee \overline{44}, \overline{34} \vee \overline{44}$ $\overline{21}, \overline{23}, \overline{34}$ $\overline{14} \vee \overline{23}, \overline{14} \vee \overline{32}, \overline{14} \vee \overline{41}$ $\overline{22} \vee \overline{31}, \overline{22} \vee \overline{33}, \overline{22} \vee \overline{44}$ $\overline{24} \vee \overline{33}, \overline{24} \vee \overline{42}$ $\overline{32} \vee \overline{41}, \overline{32} \vee \overline{43}$ $\overline{34} \vee \overline{43}$ $\overline{11} \vee \overline{22}, \overline{11} \vee \overline{33}, \overline{11} \vee \overline{44}$ $\overline{13} \vee \overline{22}, \overline{13} \vee \overline{31}, \overline{13} \vee \overline{24}$ $\overline{21} \vee \overline{32}, \overline{21} \vee \overline{43}$ $\overline{23} \vee \overline{32}, \overline{23} \vee \overline{41}, \overline{23} \vee \overline{34}$ $\overline{31} \vee \overline{42}$ $\overline{33} \vee \overline{42}, \overline{33} \vee \overline{44}$

Escolhas

12 $\overline{11}$ $21 \vee 22 \vee 23 \vee 24,$ $31 \vee 32 \vee 33 \vee 34,$ $41 \vee 42 \vee 43 \vee 44,$ $\overline{11} \quad , \overline{11} \vee \overline{13}, \overline{11} \vee \overline{14}, \quad \overline{13}, \quad \overline{14}, \overline{13} \vee \overline{14}$ $\overline{21} \vee \overline{22}, \overline{21} \vee \overline{23}, \overline{21} \vee \overline{24}, \overline{22} \vee \overline{23}, \overline{22} \vee \overline{24}, \overline{23} \vee \overline{24}$ $\overline{31} \vee \overline{32}, \overline{31} \vee \overline{33}, \overline{31} \vee \overline{34}, \overline{32} \vee \overline{33}, \overline{32} \vee \overline{34}, \overline{33} \vee \overline{34}$ $\overline{41} \vee \overline{42}, \overline{41} \vee \overline{43}, \overline{41} \vee \overline{44}, \overline{42} \vee \overline{43}, \overline{42} \vee \overline{44}, \overline{43} \vee \overline{44}$ $\overline{11} \vee \overline{21}, \overline{11} \vee \overline{31}, \overline{11} \vee \overline{41}, \overline{21} \vee \overline{31}, \overline{21} \vee \overline{41}, \overline{31} \vee \overline{41}$ $\overline{22}, \quad \overline{32}, \quad \overline{42}, \overline{22} \vee \overline{32}, \overline{22} \vee \overline{42}, \overline{32} \vee \overline{42}$ $\overline{13} \vee \overline{23}, \overline{13} \vee \overline{33}, \overline{13} \vee \overline{43}, \overline{23} \vee \overline{33}, \overline{23} \vee \overline{43}, \overline{33} \vee \overline{43}$ $\overline{14} \vee \overline{24}, \overline{14} \vee \overline{34}, \overline{14} \vee \overline{44}, \overline{24} \vee \overline{34}, \overline{24} \vee \overline{44}, \overline{34} \vee \overline{44}$ $\overline{21}, \quad \overline{23}, \quad \overline{34}$ $\overline{14} \vee \overline{23}, \overline{14} \vee \overline{32}, \overline{14} \vee \overline{41}$ $\overline{22} \vee \overline{31}, \overline{22} \vee \overline{33}, \overline{22} \vee \overline{44}$ $\overline{24} \vee \overline{33}, \overline{24} \vee \overline{42}$ $\overline{32} \vee \overline{41}, \overline{32} \vee \overline{43}$ $\overline{34} \vee \overline{43}$ $\overline{11} \vee \overline{22}, \overline{11} \vee \overline{33}, \overline{11} \vee \overline{44}$ $\overline{13} \vee \overline{22}, \overline{13} \vee \overline{31}, \overline{13} \vee \overline{24}$ $\overline{21} \vee \overline{32}, \overline{21} \vee \overline{43}$ $\overline{23} \vee \overline{32}, \overline{23} \vee \overline{41}, \overline{23} \vee \overline{34}$ $\overline{31} \vee \overline{42}$ $\overline{33} \vee \overline{42}, \overline{33} \vee \overline{44}$

Escolhas

12 $\overline{11}$ $21 \vee 22 \vee 23 \vee 24,$ $31 \vee 32 \vee 33 \vee 34,$ $41 \vee 42 \vee 43 \vee 44,$ $\boxed{11} \quad \boxed{11} \vee 13, \boxed{11} \vee 14, \quad 13, \quad 14, 13 \vee 14$ $\overline{21} \vee \overline{22}, \overline{21} \vee \overline{23}, \overline{21} \vee \overline{24}, \overline{22} \vee \overline{23}, \overline{22} \vee \overline{24}, \overline{23} \vee \overline{24}$ $\overline{31} \vee \overline{32}, \overline{31} \vee \overline{33}, \overline{31} \vee \overline{34}, \overline{32} \vee \overline{33}, \overline{32} \vee \overline{34}, \overline{33} \vee \overline{34}$ $\overline{41} \vee \overline{42}, \overline{41} \vee \overline{43}, \overline{41} \vee \overline{44}, \overline{42} \vee \overline{43}, \overline{42} \vee \overline{44}, \overline{43} \vee \overline{44}$ $\boxed{11} \vee 21, \boxed{11} \vee 31, \boxed{11} \vee 41, 21 \vee 31, 21 \vee 41, 31 \vee 41$ $\overline{22}, \quad \overline{32}, \quad \overline{42}, \overline{22} \vee \overline{32}, \overline{22} \vee \overline{42}, \overline{32} \vee \overline{42}$ $\overline{13} \vee \overline{23}, \overline{13} \vee \overline{33}, \overline{13} \vee \overline{43}, \overline{23} \vee \overline{33}, \overline{23} \vee \overline{43}, \overline{33} \vee \overline{43}$ $\overline{14} \vee \overline{24}, \overline{14} \vee \overline{34}, \overline{14} \vee \overline{44}, \overline{24} \vee \overline{34}, \overline{24} \vee \overline{44}, \overline{34} \vee \overline{44}$ $\overline{21}, \quad \overline{23}, \quad \overline{34}$ $\overline{14} \vee \overline{23}, \overline{14} \vee \overline{32}, \overline{14} \vee \overline{41}$ $\overline{22} \vee \overline{31}, \overline{22} \vee \overline{33}, \overline{22} \vee \overline{44}$ $\overline{24} \vee \overline{33}, \overline{24} \vee \overline{42}$ $\overline{32} \vee \overline{41}, \overline{32} \vee \overline{43}$ $\overline{34} \vee \overline{43}$ $\boxed{11} \vee 22, \boxed{11} \vee 33, \boxed{11} \vee 44$ $\overline{13} \vee \overline{22}, \overline{13} \vee \overline{31}, \overline{13} \vee \overline{24}$ $\overline{21} \vee \overline{32}, \overline{21} \vee \overline{43}$ $\overline{23} \vee \overline{32}, \overline{23} \vee \overline{41}, \overline{23} \vee \overline{34}$ $\overline{31} \vee \overline{42}$ $\overline{33} \vee \overline{42}, \overline{33} \vee \overline{44}$

Escolhas

12 11

21 ∨ 22 ∨ 23 ∨ 24,
31 ∨ 32 ∨ 33 ∨ 34,
41 ∨ 42 ∨ 43 ∨ 44,

13, 14, 13 ∨ 14

21 ∨ 22, 21 ∨ 23, 21 ∨ 24, 22 ∨ 23, 22 ∨ 24, 23 ∨ 24
31 ∨ 32, 31 ∨ 33, 31 ∨ 34, 32 ∨ 33, 32 ∨ 34, 33 ∨ 34
41 ∨ 42, 41 ∨ 43, 41 ∨ 44, 42 ∨ 43, 42 ∨ 44, 43 ∨ 44

21 ∨ 31, 21 ∨ 41, 31 ∨ 41

22, 32, 42, 22 ∨ 32, 22 ∨ 42, 32 ∨ 42

13 ∨ 23, 13 ∨ 33, 13 ∨ 43, 23 ∨ 33, 23 ∨ 43, 33 ∨ 43
14 ∨ 24, 14 ∨ 34, 14 ∨ 44, 24 ∨ 34, 24 ∨ 44, 34 ∨ 44

21, 23, 34
14 ∨ 23, 14 ∨ 32, 14 ∨ 41
22 ∨ 31, 22 ∨ 33, 22 ∨ 44
24 ∨ 33, 24 ∨ 42
32 ∨ 41, 32 ∨ 43
34 ∨ 43

13 ∨ 22, 13 ∨ 31, 13 ∨ 24
21 ∨ 32, 21 ∨ 43
23 ∨ 32, 23 ∨ 41, 23 ∨ 34
31 ∨ 42
33 ∨ 42, 33 ∨ 44

Escolhas

12 $\overline{11}$ $\overline{13}$

$21 \vee 22 \vee 23 \vee 24,$

$31 \vee 32 \vee 33 \vee 34,$

$41 \vee 42 \vee 43 \vee 44,$

$\overline{13}, \quad \overline{14}, \overline{13} \vee \overline{14}$

$\overline{21} \vee \overline{22}, \overline{21} \vee \overline{23}, \overline{21} \vee \overline{24}, \overline{22} \vee \overline{23}, \overline{22} \vee \overline{24}, \overline{23} \vee \overline{24}$

$\overline{31} \vee \overline{32}, \overline{31} \vee \overline{33}, \overline{31} \vee \overline{34}, \overline{32} \vee \overline{33}, \overline{32} \vee \overline{34}, \overline{33} \vee \overline{34}$

$\overline{41} \vee \overline{42}, \overline{41} \vee \overline{43}, \overline{41} \vee \overline{44}, \overline{42} \vee \overline{43}, \overline{42} \vee \overline{44}, \overline{43} \vee \overline{44}$

$\overline{21} \vee \overline{31}, \overline{21} \vee \overline{41}, \overline{31} \vee \overline{41}$

$\overline{22}, \quad \overline{32}, \quad \overline{42}, \overline{22} \vee \overline{32}, \overline{22} \vee \overline{42}, \overline{32} \vee \overline{42}$

$\overline{13} \vee \overline{23}, \overline{13} \vee \overline{33}, \overline{13} \vee \overline{43}, \overline{23} \vee \overline{33}, \overline{23} \vee \overline{43}, \overline{33} \vee \overline{43}$

$\overline{14} \vee \overline{24}, \overline{14} \vee \overline{34}, \overline{14} \vee \overline{44}, \overline{24} \vee \overline{34}, \overline{24} \vee \overline{44}, \overline{34} \vee \overline{44}$

$\overline{21}, \quad \overline{23}, \quad \overline{34}$

$\overline{14} \vee \overline{23}, \overline{14} \vee \overline{32}, \overline{14} \vee \overline{41}$

$\overline{22} \vee \overline{31}, \overline{22} \vee \overline{33}, \overline{22} \vee \overline{44}$

$\overline{24} \vee \overline{33}, \overline{24} \vee \overline{42}$

$\overline{32} \vee \overline{41}, \overline{32} \vee \overline{43}$

$\overline{34} \vee \overline{43}$

$\overline{13} \vee \overline{22}, \overline{13} \vee \overline{31}, \overline{13} \vee \overline{24}$

$\overline{21} \vee \overline{32}, \overline{21} \vee \overline{43}$

$\overline{23} \vee \overline{32}, \overline{23} \vee \overline{41}, \overline{23} \vee \overline{34}$

$\overline{31} \vee \overline{42}$

$\overline{33} \vee \overline{42}, \overline{33} \vee \overline{44}$

Escolhas

12 11 13

21 ∨ 22 ∨ 23 ∨ 24,
31 ∨ 32 ∨ 33 ∨ 34,
41 ∨ 42 ∨ 43 ∨ 44,

13 14, 13 ∨ 14

21 ∨ 22, 21 ∨ 23, 21 ∨ 24, 22 ∨ 23, 22 ∨ 24, 23 ∨ 24
31 ∨ 32, 31 ∨ 33, 31 ∨ 34, 32 ∨ 33, 32 ∨ 34, 33 ∨ 34
41 ∨ 42, 41 ∨ 43, 41 ∨ 44, 42 ∨ 43, 42 ∨ 44, 43 ∨ 44

21 ∨ 31, 21 ∨ 41, 31 ∨ 41

22, 32, 42, 22 ∨ 32, 22 ∨ 42, 32 ∨ 42
13 ∨ 23, 13 ∨ 33, 13 ∨ 43, 23 ∨ 33, 23 ∨ 43, 33 ∨ 43
14 ∨ 24, 14 ∨ 34, 14 ∨ 44, 24 ∨ 34, 24 ∨ 44, 34 ∨ 44

21, 23, 34
14 ∨ 23, 14 ∨ 32, 14 ∨ 41
22 ∨ 31, 22 ∨ 33, 22 ∨ 44
24 ∨ 33, 24 ∨ 42
32 ∨ 41, 32 ∨ 43
34 ∨ 43

13 ∨ 22, 13 ∨ 31, 13 ∨ 24
21 ∨ 32, 21 ∨ 43
23 ∨ 32, 23 ∨ 41, 23 ∨ 34
31 ∨ 42
33 ∨ 42, 33 ∨ 44

Escolhas $21 \vee 22 \vee 23 \vee 24,$ $31 \vee 32 \vee 33 \vee 34,$ $41 \vee 42 \vee 43 \vee 44,$ $\overline{14}$ $\overline{21} \vee \overline{22}, \overline{21} \vee \overline{23}, \overline{21} \vee \overline{24}, \overline{22} \vee \overline{23}, \overline{22} \vee \overline{24}, \overline{23} \vee \overline{24}$ $\overline{31} \vee \overline{32}, \overline{31} \vee \overline{33}, \overline{31} \vee \overline{34}, \overline{32} \vee \overline{33}, \overline{32} \vee \overline{34}, \overline{33} \vee \overline{34}$ $\overline{41} \vee \overline{42}, \overline{41} \vee \overline{43}, \overline{41} \vee \overline{44}, \overline{42} \vee \overline{43}, \overline{42} \vee \overline{44}, \overline{43} \vee \overline{44}$ $\overline{21} \vee \overline{31}, \overline{21} \vee \overline{41}, \overline{31} \vee \overline{41}$ $\overline{22}, \quad \overline{32}, \quad \overline{42}, \overline{22} \vee \overline{32}, \overline{22} \vee \overline{42}, \overline{32} \vee \overline{42}$ $\overline{23} \vee \overline{33}, \overline{23} \vee \overline{43}, \overline{33} \vee \overline{43}$ $\overline{14} \vee \overline{24}, \overline{14} \vee \overline{34}, \overline{14} \vee \overline{44}, \overline{24} \vee \overline{34}, \overline{24} \vee \overline{44}, \overline{34} \vee \overline{44}$ $\overline{21}, \quad \overline{23}, \quad \overline{34}$ $\overline{14} \vee \overline{23}, \overline{14} \vee \overline{32}, \overline{14} \vee \overline{41}$ $\overline{22} \vee \overline{31}, \overline{22} \vee \overline{33}, \overline{22} \vee \overline{44}$ $\overline{24} \vee \overline{33}, \overline{24} \vee \overline{42}$ $\overline{32} \vee \overline{41}, \overline{32} \vee \overline{43}$ $\overline{34} \vee \overline{43}$ $\overline{21} \vee \overline{32}, \overline{21} \vee \overline{43}$ $\overline{23} \vee \overline{32}, \overline{23} \vee \overline{41}, \overline{23} \vee \overline{34}$ $\overline{31} \vee \overline{42}$ $\overline{33} \vee \overline{42}, \overline{33} \vee \overline{44}$ **12 11 13**

Escolhas

$21 \vee 22 \vee 23 \vee 24,$
 $31 \vee 32 \vee 33 \vee 34,$
 $41 \vee 42 \vee 43 \vee 44,$

$\overline{14}$

$\overline{21} \vee \overline{22}, \overline{21} \vee \overline{23}, \overline{21} \vee \overline{24}, \overline{22} \vee \overline{23}, \overline{22} \vee \overline{24}, \overline{23} \vee \overline{24}$
 $\overline{31} \vee \overline{32}, \overline{31} \vee \overline{33}, \overline{31} \vee \overline{34}, \overline{32} \vee \overline{33}, \overline{32} \vee \overline{34}, \overline{33} \vee \overline{34}$
 $\overline{41} \vee \overline{42}, \overline{41} \vee \overline{43}, \overline{41} \vee \overline{44}, \overline{42} \vee \overline{43}, \overline{42} \vee \overline{44}, \overline{43} \vee \overline{44}$

$\overline{22}, \quad \overline{32}, \quad \overline{42}, \overline{22} \vee \overline{32}, \overline{22} \vee \overline{42}, \overline{32} \vee \overline{42}$
 $\overline{23} \vee \overline{33}, \overline{23} \vee \overline{43}, \overline{33} \vee \overline{43}$

$\overline{14} \vee \overline{24}, \overline{14} \vee \overline{34}, \overline{14} \vee \overline{44}, \overline{24} \vee \overline{34}, \overline{24} \vee \overline{44}, \overline{34} \vee \overline{44}$

$\overline{21}, \quad \overline{23}, \quad \overline{34}$
 $\overline{14} \vee \overline{23}, \overline{14} \vee \overline{32}, \overline{14} \vee \overline{41}$
 $\overline{22} \vee \overline{31}, \overline{22} \vee \overline{33}, \overline{22} \vee \overline{44}$
 $\overline{24} \vee \overline{33}, \overline{24} \vee \overline{42}$
 $\overline{32} \vee \overline{41}, \overline{32} \vee \overline{43}$
 $\overline{34} \vee \overline{43}$

$\overline{21} \vee \overline{32}, \overline{21} \vee \overline{43}$
 $\overline{23} \vee \overline{32}, \overline{23} \vee \overline{41}, \overline{23} \vee \overline{34}$
 $\overline{31} \vee \overline{42}$
 $\overline{33} \vee \overline{42}, \overline{33} \vee \overline{44}$

$\overline{12} \quad \overline{11} \quad \overline{13}$
 $\overline{14}$

$21 \vee 22 \vee 23 \vee 24,$
 $31 \vee 32 \vee 33 \vee 34,$
 $41 \vee 42 \vee 43 \vee 44,$

$\overline{21} \vee \overline{22}, \overline{21} \vee \overline{23}, \overline{21} \vee \overline{24}, \overline{22} \vee \overline{23}, \overline{22} \vee \overline{24}, \overline{23} \vee \overline{24}$
 $\overline{31} \vee \overline{32}, \overline{31} \vee \overline{33}, \overline{31} \vee \overline{34}, \overline{32} \vee \overline{33}, \overline{32} \vee \overline{34}, \overline{33} \vee \overline{34}$
 $\overline{41} \vee \overline{42}, \overline{41} \vee \overline{43}, \overline{41} \vee \overline{44}, \overline{42} \vee \overline{43}, \overline{42} \vee \overline{44}, \overline{43} \vee \overline{44}$

$\overline{21} \vee \overline{31}, \overline{21} \vee \overline{41}, \overline{31} \vee \overline{41}$
 $\overline{22}, \overline{32}, \overline{42}, \overline{22} \vee \overline{32}, \overline{22} \vee \overline{42}, \overline{32} \vee \overline{42}$
 $\overline{23} \vee \overline{33}, \overline{23} \vee \overline{43}, \overline{33} \vee \overline{43}$
 $\overline{14} \vee \overline{24}, \overline{14} \vee \overline{34}, \overline{14} \vee \overline{44}, \overline{24} \vee \overline{34}, \overline{24} \vee \overline{44}, \overline{34} \vee \overline{44}$

$\overline{21}, \overline{23}, \overline{34}$
 $\overline{14} \vee \overline{23}, \overline{14} \vee \overline{32}, \overline{14} \vee \overline{41}$
 $\overline{22} \vee \overline{31}, \overline{22} \vee \overline{33}, \overline{22} \vee \overline{44}$
 $\overline{24} \vee \overline{33}, \overline{24} \vee \overline{42}$
 $\overline{32} \vee \overline{41}, \overline{32} \vee \overline{43}$
 $\overline{34} \vee \overline{43}$

$\overline{21} \vee \overline{32}, \overline{21} \vee \overline{43}$
 $\overline{23} \vee \overline{32}, \overline{23} \vee \overline{41}, \overline{23} \vee \overline{34}$
 $\overline{31} \vee \overline{42}$
 $\overline{33} \vee \overline{42}, \overline{33} \vee \overline{44}$

Escolhas

$12 \quad \overline{11} \quad \overline{13}$
 $\overline{14}$

Escolhas $21 \vee 22 \vee 23 \vee 24,$ $31 \vee 32 \vee 33 \vee 34,$ $41 \vee 42 \vee 43 \vee 44,$ $\overline{21} \vee \overline{22}, \overline{21} \vee \overline{23}, \overline{21} \vee \overline{24}, \overline{22} \vee \overline{23}, \overline{22} \vee \overline{24}, \overline{23} \vee \overline{24}$ $\overline{31} \vee \overline{32}, \overline{31} \vee \overline{33}, \overline{31} \vee \overline{34}, \overline{32} \vee \overline{33}, \overline{32} \vee \overline{34}, \overline{33} \vee \overline{34}$ $\overline{41} \vee \overline{42}, \overline{41} \vee \overline{43}, \overline{41} \vee \overline{44}, \overline{42} \vee \overline{43}, \overline{42} \vee \overline{44}, \overline{43} \vee \overline{44}$ $\overline{22}, \quad \overline{32}, \quad \overline{42}, \overline{22} \vee \overline{32}, \overline{22} \vee \overline{42}, \overline{32} \vee \overline{42}$ $\overline{21} \vee \overline{31}, \overline{21} \vee \overline{41}, \overline{31} \vee \overline{41}$ $\overline{23} \vee \overline{33}, \overline{23} \vee \overline{43}, \overline{33} \vee \overline{43}$ $\overline{24} \vee \overline{34}, \overline{24} \vee \overline{44}, \overline{34} \vee \overline{44}$ $\overline{21}, \quad \overline{23}, \quad \overline{34}$ $\overline{22} \vee \overline{31}, \overline{22} \vee \overline{33}, \overline{22} \vee \overline{44}$ $\overline{24} \vee \overline{33}, \overline{24} \vee \overline{42}$ $\overline{32} \vee \overline{41}, \overline{32} \vee \overline{43}$ $\overline{34} \vee \overline{43}$ $\overline{21} \vee \overline{32}, \overline{21} \vee \overline{43}$ $\overline{23} \vee \overline{32}, \overline{23} \vee \overline{41}, \overline{23} \vee \overline{34}$ $\overline{31} \vee \overline{42}$ $\overline{33} \vee \overline{42}, \overline{33} \vee \overline{44}$ $\overline{12} \quad \overline{11} \quad \overline{13}$
 $\overline{14}$

Escolhas

$21 \vee 22 \vee 23 \vee 24,$
 $31 \vee 32 \vee 33 \vee 34,$
 $41 \vee 42 \vee 43 \vee 44,$

$\overline{21} \vee \overline{22}, \overline{21} \vee \overline{23}, \overline{21} \vee \overline{24}, \overline{22} \vee \overline{23}, \overline{22} \vee \overline{24}, \overline{23} \vee \overline{24}$
 $\overline{31} \vee \overline{32}, \overline{31} \vee \overline{33}, \overline{31} \vee \overline{34}, \overline{32} \vee \overline{33}, \overline{32} \vee \overline{34}, \overline{33} \vee \overline{34}$
 $\overline{41} \vee \overline{42}, \overline{41} \vee \overline{43}, \overline{41} \vee \overline{44}, \overline{42} \vee \overline{43}, \overline{42} \vee \overline{44}, \overline{43} \vee \overline{44}$

$\overline{22}, \quad \overline{32}, \quad \overline{42}, \overline{22} \vee \overline{32}, \overline{22} \vee \overline{42}, \overline{32} \vee \overline{42}$
 $\overline{23} \vee \overline{33}, \overline{23} \vee \overline{43}, \overline{33} \vee \overline{43}$
 $\overline{24} \vee \overline{34}, \overline{24} \vee \overline{44}, \overline{34} \vee \overline{44}$

$\overline{21}, \quad \overline{23}, \quad \overline{34}$

$\overline{22} \vee \overline{31}, \overline{22} \vee \overline{33}, \overline{22} \vee \overline{44}$
 $\overline{24} \vee \overline{33}, \overline{24} \vee \overline{42}$
 $\overline{32} \vee \overline{41}, \overline{32} \vee \overline{43}$
 $\overline{34} \vee \overline{43}$

$\overline{21} \vee \overline{32}, \overline{21} \vee \overline{43}$
 $\overline{23} \vee \overline{32}, \overline{23} \vee \overline{41}, \overline{23} \vee \overline{34}$
 $\overline{31} \vee \overline{42}$
 $\overline{33} \vee \overline{42}, \overline{33} \vee \overline{44}$

$\overline{12} \quad \overline{11} \quad \overline{13}$
 $\overline{14} \quad \overline{21}$

Escolhas

 $21 \vee 22 \vee 23 \vee 24,$ $31 \vee 32 \vee 33 \vee 34,$ $41 \vee 42 \vee 43 \vee 44,$ $\overline{21} \vee \overline{22}, \overline{21} \vee \overline{23}, \overline{21} \vee \overline{24}, \overline{22} \vee \overline{23}, \overline{22} \vee \overline{24}, \overline{23} \vee \overline{24}$ $\overline{31} \vee \overline{32}, \overline{31} \vee \overline{33}, \overline{31} \vee \overline{34}, \overline{32} \vee \overline{33}, \overline{32} \vee \overline{34}, \overline{33} \vee \overline{34}$ $\overline{41} \vee \overline{42}, \overline{41} \vee \overline{43}, \overline{41} \vee \overline{44}, \overline{42} \vee \overline{43}, \overline{42} \vee \overline{44}, \overline{43} \vee \overline{44}$ $\overline{22}, \overline{32}, \overline{42}, \overline{21} \vee \overline{31}, \overline{21} \vee \overline{41}, \overline{31} \vee \overline{41}$ $\overline{23} \vee \overline{33}, \overline{23} \vee \overline{43}, \overline{33} \vee \overline{43}$ $\overline{24} \vee \overline{34}, \overline{24} \vee \overline{44}, \overline{34} \vee \overline{44}$ $\overline{21}, \overline{23}, \overline{34}$ $\overline{22} \vee \overline{31}, \overline{22} \vee \overline{33}, \overline{22} \vee \overline{44}$ $\overline{24} \vee \overline{33}, \overline{24} \vee \overline{42}$ $\overline{32} \vee \overline{41}, \overline{32} \vee \overline{43}$ $\overline{34} \vee \overline{43}$ $\overline{21} \vee \overline{32}, \overline{21} \vee \overline{43}$ $\overline{23} \vee \overline{32}, \overline{23} \vee \overline{41}, \overline{23} \vee \overline{34}$ $\overline{31} \vee \overline{42}$ $\overline{33} \vee \overline{42}, \overline{33} \vee \overline{44}$

12	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	

21 $\vee 22 \vee 23 \vee 24,$
 $31 \vee 32 \vee 33 \vee 34,$
 $41 \vee 42 \vee 43 \vee 44,$

$\overline{22} \vee \overline{23}, \overline{22} \vee \overline{24}, \overline{23} \vee \overline{24}$

$\overline{31} \vee \overline{32}, \overline{31} \vee \overline{33}, \overline{31} \vee \overline{34}, \overline{32} \vee \overline{33}, \overline{32} \vee \overline{34}, \overline{33} \vee \overline{34}$

$\overline{41} \vee \overline{42}, \overline{41} \vee \overline{43}, \overline{41} \vee \overline{44}, \overline{42} \vee \overline{43}, \overline{42} \vee \overline{44}, \overline{43} \vee \overline{44}$

$\overline{31} \vee \overline{41}$

$\overline{22}, \quad \overline{32}, \quad \overline{42}, \overline{22} \vee \overline{32}, \overline{22} \vee \overline{42}, \overline{32} \vee \overline{42}$

$\overline{23} \vee \overline{33}, \overline{23} \vee \overline{43}, \overline{33} \vee \overline{43}$

$\overline{24} \vee \overline{34}, \overline{24} \vee \overline{44}, \overline{34} \vee \overline{44}$

$\overline{23}, \quad \overline{34}$

$\overline{22} \vee \overline{31}, \overline{22} \vee \overline{33}, \overline{22} \vee \overline{44}$

$\overline{24} \vee \overline{33}, \overline{24} \vee \overline{42}$

$\overline{32} \vee \overline{41}, \overline{32} \vee \overline{43}$

$\overline{34} \vee \overline{43}$

$\overline{23} \vee \overline{32}, \overline{23} \vee \overline{41}, \overline{23} \vee \overline{34}$

$\overline{31} \vee \overline{42}$

$\overline{33} \vee \overline{42}, \overline{33} \vee \overline{44}$

Escolhas

$\overline{12} \quad \overline{11} \quad \overline{13}$
 $\overline{14} \quad \overline{21}$

Escolhas

$22 \vee 23 \vee 24,$
 $31 \vee 32 \vee 33 \vee 34,$
 $41 \vee 42 \vee 43 \vee 44,$

$\overline{22} \vee \overline{23}, \overline{22} \vee \overline{24}, \overline{23} \vee \overline{24}$
 $\overline{31} \vee \overline{32}, \overline{31} \vee \overline{33}, \overline{31} \vee \overline{34}, \overline{32} \vee \overline{33}, \overline{32} \vee \overline{34}, \overline{33} \vee \overline{34}$
 $\overline{41} \vee \overline{42}, \overline{41} \vee \overline{43}, \overline{41} \vee \overline{44}, \overline{42} \vee \overline{43}, \overline{42} \vee \overline{44}, \overline{43} \vee \overline{44}$

$\overline{22}, \quad \overline{32}, \quad \overline{42}, \overline{22} \vee \overline{32}, \overline{22} \vee \overline{42}, \overline{32} \vee \overline{42}$
 $\overline{23} \vee \overline{33}, \overline{23} \vee \overline{43}, \overline{33} \vee \overline{43}$
 $\overline{24} \vee \overline{34}, \overline{24} \vee \overline{44}, \overline{34} \vee \overline{44}$

$\overline{23}, \quad \overline{34}$

$\overline{22} \vee \overline{31}, \overline{22} \vee \overline{33}, \overline{22} \vee \overline{44}$
 $\overline{24} \vee \overline{33}, \overline{24} \vee \overline{42}$
 $\overline{32} \vee \overline{41}, \overline{32} \vee \overline{43}$
 $\overline{34} \vee \overline{43}$

$\overline{23} \vee \overline{32}, \overline{23} \vee \overline{41}, \overline{23} \vee \overline{34}$
 $\overline{31} \vee \overline{42}$
 $\overline{33} \vee \overline{42}, \overline{33} \vee \overline{44}$

$\overline{12} \quad \overline{11} \quad \overline{13}$
 $\overline{14} \quad \overline{21}$

Escolhas

$22 \vee 23 \vee 24,$
 $31 \vee 32 \vee 33 \vee 34,$
 $41 \vee 42 \vee 43 \vee 44,$

$\overline{22} \vee \overline{23}, \overline{22} \vee \overline{24}, \overline{23} \vee \overline{24}$
 $\overline{31} \vee \overline{32}, \overline{31} \vee \overline{33}, \overline{31} \vee \overline{34}, \overline{32} \vee \overline{33}, \overline{32} \vee \overline{34}, \overline{33} \vee \overline{34}$
 $\overline{41} \vee \overline{42}, \overline{41} \vee \overline{43}, \overline{41} \vee \overline{44}, \overline{42} \vee \overline{43}, \overline{42} \vee \overline{44}, \overline{43} \vee \overline{44}$

$\overline{22}, \quad \overline{32}, \quad \overline{42}, \overline{22} \vee \overline{32}, \overline{22} \vee \overline{42}, \overline{32} \vee \overline{42}$
 $\overline{23} \vee \overline{33}, \overline{23} \vee \overline{43}, \overline{33} \vee \overline{43}$
 $\overline{24} \vee \overline{34}, \overline{24} \vee \overline{44}, \overline{34} \vee \overline{44}$

$\overline{23}, \quad \overline{34}$

$\overline{22} \vee \overline{31}, \overline{22} \vee \overline{33}, \overline{22} \vee \overline{44}$
 $\overline{24} \vee \overline{33}, \overline{24} \vee \overline{42}$
 $\overline{32} \vee \overline{41}, \overline{32} \vee \overline{43}$
 $\overline{34} \vee \overline{43}$

$\overline{23} \vee \overline{32}, \overline{23} \vee \overline{41}, \overline{23} \vee \overline{34}$
 $\overline{31} \vee \overline{42}$
 $\overline{33} \vee \overline{42}, \overline{33} \vee \overline{44}$

$\overline{12} \quad \overline{11} \quad \overline{13}$
 $\overline{14} \quad \overline{21} \quad \overline{22}$

Escolhas

$22 \vee 23 \vee 24,$
 $31 \vee 32 \vee 33 \vee 34,$
 $41 \vee 42 \vee 43 \vee 44,$

$\overline{22} \vee \overline{23}, \overline{22} \vee \overline{24}, \overline{23} \vee \overline{24}$
 $\overline{31} \vee \overline{32}, \overline{31} \vee \overline{33}, \overline{31} \vee \overline{34}, \overline{32} \vee \overline{33}, \overline{32} \vee \overline{34}, \overline{33} \vee \overline{34}$
 $\overline{41} \vee \overline{42}, \overline{41} \vee \overline{43}, \overline{41} \vee \overline{44}, \overline{42} \vee \overline{43}, \overline{42} \vee \overline{44}, \overline{43} \vee \overline{44}$

$\overline{22}$ $\overline{32},$ $\overline{42}, \overline{22} \vee \overline{32}, \overline{22} \vee \overline{42}, \overline{32} \vee \overline{42}$
 $\overline{23} \vee \overline{33}, \overline{23} \vee \overline{43}, \overline{33} \vee \overline{43}$
 $\overline{24} \vee \overline{34}, \overline{24} \vee \overline{44}, \overline{34} \vee \overline{44}$

$\overline{23},$ $\overline{34}$

$\overline{22} \vee \overline{31}, \overline{22} \vee \overline{33}, \overline{22} \vee \overline{44}$
 $\overline{24} \vee \overline{33}, \overline{24} \vee \overline{42}$
 $\overline{32} \vee \overline{41}, \overline{32} \vee \overline{43}$
 $\overline{34} \vee \overline{43}$

$\overline{23} \vee \overline{32}, \overline{23} \vee \overline{41}, \overline{23} \vee \overline{34}$
 $\overline{31} \vee \overline{42}$
 $\overline{33} \vee \overline{42}, \overline{33} \vee \overline{44}$

12	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$

Escolhas

22 ∨ 23 ∨ 24,
31 ∨ 32 ∨ 33 ∨ 34,
41 ∨ 42 ∨ 43 ∨ 44,

$\overline{23}, \quad \overline{34}$

12

11

13

14

21

22

$\overline{24} \vee \overline{33}, \overline{24} \vee \overline{42}$
 $\overline{32} \vee \overline{41}, \overline{32} \vee \overline{43}$
 $\overline{34} \vee \overline{43}$

$\overline{23} \vee \overline{24}$
 $\overline{31} \vee \overline{32}, \overline{31} \vee \overline{33}, \overline{31} \vee \overline{34}, \overline{32} \vee \overline{33}, \overline{32} \vee \overline{34}, \overline{33} \vee \overline{34}$
 $\overline{41} \vee \overline{42}, \overline{41} \vee \overline{43}, \overline{41} \vee \overline{44}, \overline{42} \vee \overline{43}, \overline{42} \vee \overline{44}, \overline{43} \vee \overline{44}$

$\overline{32}, \quad \overline{42},$

$\overline{31} \vee \overline{41}$

$\overline{32} \vee \overline{42}$

$\overline{23} \vee \overline{33}, \overline{23} \vee \overline{43}, \overline{33} \vee \overline{43}$

$\overline{24} \vee \overline{34}, \overline{24} \vee \overline{44}, \overline{34} \vee \overline{44}$

$\overline{23} \vee \overline{32}, \overline{23} \vee \overline{41}, \overline{23} \vee \overline{34}$
 $\overline{31} \vee \overline{42}$
 $\overline{33} \vee \overline{42}, \overline{33} \vee \overline{44}$

Escolhas

23 ∨ 24,
31 ∨ 32 ∨ 33 ∨ 34,
41 ∨ 42 ∨ 43 ∨ 44,

$\overline{23}, \quad \overline{34}$

$\overline{12} \quad \overline{11} \quad \overline{13}$
 $\overline{14} \quad \overline{21} \quad \overline{22}$

$\overline{24} \vee \overline{33}, \overline{24} \vee \overline{42}$
 $\overline{32} \vee \overline{41}, \overline{32} \vee \overline{43}$
 $\overline{34} \vee \overline{43}$

$\overline{23} \vee \overline{24}$
 $\overline{31} \vee \overline{32}, \overline{31} \vee \overline{33}, \overline{31} \vee \overline{34}, \overline{32} \vee \overline{33}, \overline{32} \vee \overline{34}, \overline{33} \vee \overline{34}$
 $\overline{41} \vee \overline{42}, \overline{41} \vee \overline{43}, \overline{41} \vee \overline{44}, \overline{42} \vee \overline{43}, \overline{42} \vee \overline{44}, \overline{43} \vee \overline{44}$

$\overline{31} \vee \overline{41}$
 $\overline{32} \vee \overline{42}$
 $\overline{23} \vee \overline{32}, \overline{23} \vee \overline{41}, \overline{23} \vee \overline{34}$
 $\overline{31} \vee \overline{42}$
 $\overline{33} \vee \overline{42}, \overline{33} \vee \overline{44}$
 $\overline{32}, \quad \overline{42},$
 $\overline{23} \vee \overline{33}, \overline{23} \vee \overline{43}, \overline{33} \vee \overline{43}$
 $\overline{24} \vee \overline{34}, \overline{24} \vee \overline{44}, \overline{34} \vee \overline{44}$

Escolhas

23 ∨ 24,
31 ∨ 32 ∨ 33 ∨ 34,
41 ∨ 42 ∨ 43 ∨ 44,

$\overline{23}, \quad \overline{34}$

12 $\overline{11}$ $\overline{13}$
 $\overline{14}$ $\overline{21}$ $\overline{22}$
 $\overline{23}$

$\overline{24} \vee \overline{33}, \overline{24} \vee \overline{42}$
 $\overline{32} \vee \overline{41}, \overline{32} \vee \overline{43}$
 $\overline{34} \vee \overline{43}$

$\overline{23} \vee \overline{24}$
 $\overline{31} \vee \overline{32}, \overline{31} \vee \overline{33}, \overline{31} \vee \overline{34}, \overline{32} \vee \overline{33}, \overline{32} \vee \overline{34}, \overline{33} \vee \overline{34}$
 $\overline{41} \vee \overline{42}, \overline{41} \vee \overline{43}, \overline{41} \vee \overline{44}, \overline{42} \vee \overline{43}, \overline{42} \vee \overline{44}, \overline{43} \vee \overline{44}$

$\overline{32}, \quad \overline{42}, \quad \overline{31} \vee \overline{41} \quad \overline{23} \vee \overline{32}, \overline{23} \vee \overline{41}, \overline{23} \vee \overline{34}$
 $\overline{32} \vee \overline{42} \quad \overline{31} \vee \overline{42}$
 $\overline{23} \vee \overline{33}, \overline{23} \vee \overline{43}, \overline{33} \vee \overline{43} \quad \overline{33} \vee \overline{42}, \overline{33} \vee \overline{44}$
 $\overline{24} \vee \overline{34}, \overline{24} \vee \overline{44}, \overline{34} \vee \overline{44}$

Escolhas

 $\overline{23}$ $\overline{34}$

12	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$		

23 $\vee$ 24,
 31 $\vee$ 32 $\vee$ 33 $\vee$ 34,
 41 $\vee$ 42 $\vee$ 43 $\vee$ 44,

$\overline{24} \vee \overline{33}, \overline{24} \vee \overline{42}$
 $\overline{32} \vee \overline{41}, \overline{32} \vee \overline{43}$
 $\overline{34} \vee \overline{43}$

 $\overline{23} \vee \overline{24}$

$\overline{31} \vee \overline{32}, \overline{31} \vee \overline{33}, \overline{31} \vee \overline{34}, \overline{32} \vee \overline{33}, \overline{32} \vee \overline{34}, \overline{33} \vee \overline{34}$
 $\overline{41} \vee \overline{42}, \overline{41} \vee \overline{43}, \overline{41} \vee \overline{44}, \overline{42} \vee \overline{43}, \overline{42} \vee \overline{44}, \overline{43} \vee \overline{44}$

 $\overline{31} \vee \overline{41}$ $\overline{23} \vee \overline{32}, \overline{23} \vee \overline{41}, \overline{23} \vee \overline{34}$ $\overline{32}, \overline{42},$ $\overline{32} \vee \overline{42}$ $\overline{31} \vee \overline{42}$ $\overline{23} \vee \overline{33}, \overline{23} \vee \overline{43}, \overline{33} \vee \overline{43}$ $\overline{33} \vee \overline{42}, \overline{33} \vee \overline{44}$ $\overline{24} \vee \overline{34}, \overline{24} \vee \overline{44}, \overline{34} \vee \overline{44}$

$\boxed{23} \vee 24,$
 $31 \vee 32 \vee 33 \vee 34,$
 $41 \vee 42 \vee 43 \vee 44,$
 $\overline{31} \vee \overline{32}, \overline{31} \vee \overline{33}, \overline{31} \vee \overline{34}, \overline{32} \vee \overline{33}, \overline{32} \vee \overline{34}, \overline{33} \vee \overline{34}$
 $\overline{41} \vee \overline{42}, \overline{41} \vee \overline{43}, \overline{41} \vee \overline{44}, \overline{42} \vee \overline{43}, \overline{42} \vee \overline{44}, \overline{43} \vee \overline{44}$
 $\overline{32}, \quad \overline{42},$
 $\overline{31} \vee \overline{41}$
 $\overline{32} \vee \overline{42}$
 $\overline{33} \vee \overline{43}$
 $\overline{24} \vee \overline{34}, \overline{24} \vee \overline{44}, \overline{34} \vee \overline{44}$
 $\overline{24} \vee \overline{33}, \overline{24} \vee \overline{42}$
 $\overline{32} \vee \overline{41}, \overline{32} \vee \overline{43}$
 $\overline{34} \vee \overline{43}$
 $\overline{31} \vee \overline{42}$
 $\overline{33} \vee \overline{42}, \overline{33} \vee \overline{44}$

Escolhas

 $\overline{34}$

12	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$		

Escolhas $\overline{34}$

12	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$		

24,

 $31 \vee 32 \vee 33 \vee 34,$ $41 \vee 42 \vee 43 \vee 44,$ $\overline{24} \vee \overline{33}, \overline{24} \vee \overline{42}$ $\overline{32} \vee \overline{41}, \overline{32} \vee \overline{43}$ $\overline{34} \vee \overline{43}$ $\overline{31} \vee \overline{32}, \overline{31} \vee \overline{33}, \overline{31} \vee \overline{34}, \overline{32} \vee \overline{33}, \overline{32} \vee \overline{34}, \overline{33} \vee \overline{34}$ $\overline{41} \vee \overline{42}, \overline{41} \vee \overline{43}, \overline{41} \vee \overline{44}, \overline{42} \vee \overline{43}, \overline{42} \vee \overline{44}, \overline{43} \vee \overline{44}$ $\overline{31} \vee \overline{41}$ $\overline{32} \vee \overline{42}$ $\overline{33} \vee \overline{43}$ $\overline{24} \vee \overline{34}, \overline{24} \vee \overline{44}, \overline{34} \vee \overline{44}$ $\overline{32}, \quad \overline{42},$ $\overline{31} \vee \overline{42}$ $\overline{33} \vee \overline{42}, \overline{33} \vee \overline{44}$

Escolhas

 $\overline{34}$

12	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$		

 $24,$ $31 \vee 32 \vee 33 \vee 34,$ $41 \vee 42 \vee 43 \vee 44,$ $\overline{24} \vee \overline{33}, \overline{24} \vee \overline{42}$ $\overline{32} \vee \overline{41}, \overline{32} \vee \overline{43}$ $\overline{34} \vee \overline{43}$ $\overline{31} \vee \overline{32}, \overline{31} \vee \overline{33}, \overline{31} \vee \overline{34}, \overline{32} \vee \overline{33}, \overline{32} \vee \overline{34}, \overline{33} \vee \overline{34}$ $\overline{41} \vee \overline{42}, \overline{41} \vee \overline{43}, \overline{41} \vee \overline{44}, \overline{42} \vee \overline{43}, \overline{42} \vee \overline{44}, \overline{43} \vee \overline{44}$ $\overline{31} \vee \overline{41}$ $\overline{32} \vee \overline{42}$ $\overline{33} \vee \overline{43}$ $\overline{24} \vee \overline{34}, \overline{24} \vee \overline{44}, \overline{34} \vee \overline{44}$ $\overline{32}, \quad \overline{42},$ $\overline{31} \vee \overline{42}$ $\overline{33} \vee \overline{42}, \overline{33} \vee \overline{44}$

Escolhas

$\overline{34}$

$12 \quad \overline{11} \quad \overline{13}$
 $\overline{14} \quad \overline{21} \quad \overline{22}$
 $\overline{23} \quad \overline{32}$

$24,$
 $31 \vee 32 \vee 33 \vee 34,$
 $41 \vee 42 \vee 43 \vee 44,$

$\overline{24} \vee \overline{33}, \overline{24} \vee \overline{42}$
 $\overline{32} \vee \overline{41}, \overline{32} \vee \overline{43}$
 $\overline{34} \vee \overline{43}$

$\overline{31} \vee \overline{32}, \overline{31} \vee \overline{33}, \overline{31} \vee \overline{34}, \overline{32} \vee \overline{33}, \overline{32} \vee \overline{34}, \overline{33} \vee \overline{34}$
 $\overline{41} \vee \overline{42}, \overline{41} \vee \overline{43}, \overline{41} \vee \overline{44}, \overline{42} \vee \overline{43}, \overline{42} \vee \overline{44}, \overline{43} \vee \overline{44}$

$\overline{32}, \quad \overline{42}, \quad \overline{31} \vee \overline{41}$
 $\overline{32} \vee \overline{42} \quad \overline{31} \vee \overline{42}$
 $\overline{33} \vee \overline{43} \quad \overline{33} \vee \overline{42}, \overline{33} \vee \overline{44}$
 $\overline{24} \vee \overline{34}, \overline{24} \vee \overline{44}, \overline{34} \vee \overline{44}$

Escolhas

 $\overline{34}$

12	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$	$\overline{32}$	

24,
 $31 \vee 32 \vee 33 \vee 34$,
 $41 \vee 42 \vee 43 \vee 44$,

$\overline{24} \vee \overline{33}, \overline{24} \vee \overline{42}$
 $\overline{32} \vee \overline{41}, \overline{32} \vee \overline{43}$
 $\overline{34} \vee \overline{43}$

$\overline{31} \vee \overline{32}, \overline{31} \vee \overline{33}, \overline{31} \vee \overline{34}, \overline{32} \vee \overline{33}, \overline{32} \vee \overline{34}, \overline{33} \vee \overline{34}$
 $\overline{41} \vee \overline{42}, \overline{41} \vee \overline{43}, \overline{41} \vee \overline{44}, \overline{42} \vee \overline{43}, \overline{42} \vee \overline{44}, \overline{43} \vee \overline{44}$

$\overline{32}$ 42,

$\overline{31} \vee \overline{41}$
 $\overline{32} \vee \overline{42}$
 $\overline{33} \vee \overline{43}$

$\overline{31} \vee \overline{42}$
 $\overline{33} \vee \overline{42}, \overline{33} \vee \overline{44}$

$\overline{24} \vee \overline{34}, \overline{24} \vee \overline{44}, \overline{34} \vee \overline{44}$

24,
31 ∨ 32 ∨ 33 ∨ 34,
41 ∨ 42 ∨ 43 ∨ 44,

31 ∨ 33, 31 ∨ 34, 33 ∨ 34
41 ∨ 42, 41 ∨ 43, 41 ∨ 44, 42 ∨ 43, 42 ∨ 44, 43 ∨ 44
31 ∨ 41
42,
33 ∨ 43
24 ∨ 34, 24 ∨ 44, 34 ∨ 44

34
24 ∨ 33, 24 ∨ 42
34 ∨ 43

31 ∨ 42
33 ∨ 42, 33 ∨ 44

Escolhas

12 11 13

14 21 22

23 32

24,
31 ∨ 33 ∨ 34,
41 ∨ 42 ∨ 43 ∨ 44,

31 ∨ 33, 31 ∨ 34, 33 ∨ 34
41 ∨ 42, 41 ∨ 43, 41 ∨ 44, 42 ∨ 43, 42 ∨ 44, 43 ∨ 44
31 ∨ 41
42,
33 ∨ 43
24 ∨ 34, 24 ∨ 44, 34 ∨ 44

34
24 ∨ 33, 24 ∨ 42
34 ∨ 43

31 ∨ 42
33 ∨ 42, 33 ∨ 44

Escolhas

12 11 13
14 21 22
23 32

Escolhas

$\overline{34}$

12	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$	$\overline{32}$	$\overline{34}$

24,
31 $\vee$ 33 $\vee$ 34,
41 $\vee$ 42 $\vee$ 43 $\vee$ 44,

$\overline{24} \vee \overline{33}, \overline{24} \vee \overline{42}$

$\overline{34} \vee \overline{43}$

$\overline{31} \vee \overline{33}, \overline{31} \vee \overline{34}, \overline{33} \vee \overline{34}$
 $\overline{41} \vee \overline{42}, \overline{41} \vee \overline{43}, \overline{41} \vee \overline{44}, \overline{42} \vee \overline{43}, \overline{42} \vee \overline{44}, \overline{43} \vee \overline{44}$

$\overline{31} \vee \overline{41}$

$\overline{42},$

$\overline{31} \vee \overline{42}$

$\overline{33} \vee \overline{43}$

$\overline{33} \vee \overline{42}, \overline{33} \vee \overline{44}$

$\overline{24} \vee \overline{34}, \overline{24} \vee \overline{44}, \overline{34} \vee \overline{44}$

Escolhas

 $\overline{34}$

12	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$	$\overline{32}$	$\overline{34}$

24,
 $31 \vee 33 \vee 34$,
 $41 \vee 42 \vee 43 \vee 44$,

 $\overline{24} \vee \overline{33}, \overline{24} \vee \overline{42}$ $\overline{34} \vee \overline{43}$

$\overline{31} \vee \overline{33}, \overline{31} \vee \overline{34}, \overline{33} \vee \overline{34}$
 $\overline{41} \vee \overline{42}, \overline{41} \vee \overline{43}, \overline{41} \vee \overline{44}, \overline{42} \vee \overline{43}, \overline{42} \vee \overline{44}, \overline{43} \vee \overline{44}$

 $\overline{31} \vee \overline{41}$ $\overline{42}$, $\overline{31} \vee \overline{42}$ $\overline{33} \vee \overline{43}$ $\overline{33} \vee \overline{42}, \overline{33} \vee \overline{44}$ $\overline{24} \vee \overline{34}, \overline{24} \vee \overline{44}, \overline{34} \vee \overline{44}$

Escolhas

12	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$	$\overline{32}$	$\overline{34}$

$31 \vee$ $33 \vee$ 34 ,
 $41 \vee 42 \vee 43 \vee 44$,

$\overline{24} \vee \overline{33}, \overline{24} \vee \overline{42}$

$\overline{31} \vee \overline{33},$
 $\overline{41} \vee \overline{42}, \overline{41} \vee \overline{43}, \overline{41} \vee \overline{44}, \overline{42} \vee \overline{43}, \overline{42} \vee \overline{44}, \overline{43} \vee \overline{44}$

$\overline{31} \vee \overline{41}$

$\overline{42},$

$\overline{33} \vee \overline{43}$

$\overline{31} \vee \overline{42}$

$\overline{33} \vee \overline{42}, \overline{33} \vee \overline{44}$

$\overline{24} \vee \overline{44}$

Escolhas

12	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$	$\overline{32}$	$\overline{34}$

24,
31 $\vee$ 33
41 $\vee$ 42 $\vee$ 43 $\vee$ 44,

$\overline{24} \vee \overline{33}, \overline{24} \vee \overline{42}$

$\overline{31} \vee \overline{33},$
 $\overline{41} \vee \overline{42}, \overline{41} \vee \overline{43}, \overline{41} \vee \overline{44}, \overline{42} \vee \overline{43}, \overline{42} \vee \overline{44}, \overline{43} \vee \overline{44}$

$\overline{42},$
 $\overline{31} \vee \overline{41}$
 $\overline{33} \vee \overline{43}$
 $\overline{24} \vee \overline{44}$
 $\overline{31} \vee \overline{42}$
 $\overline{33} \vee \overline{42}, \overline{33} \vee \overline{44}$

Escolhas

12	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$	$\overline{32}$	$\overline{34}$
$\overline{42}$		

24,
 $31 \vee 33$
 $41 \vee 42 \vee 43 \vee 44,$

$\overline{24} \vee \overline{33}, \overline{24} \vee \overline{42}$

$\overline{31} \vee \overline{33},$
 $\overline{41} \vee \overline{42}, \overline{41} \vee \overline{43}, \overline{41} \vee \overline{44}, \overline{42} \vee \overline{43}, \overline{42} \vee \overline{44}, \overline{43} \vee \overline{44}$

$\overline{31} \vee \overline{41}$

$\overline{42},$

$\overline{31} \vee \overline{42}$

$\overline{33} \vee \overline{43}$

$\overline{33} \vee \overline{42}, \overline{33} \vee \overline{44}$

$\overline{24} \vee \overline{44}$

Escolhas

12	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$	$\overline{32}$	$\overline{34}$
$\overline{42}$		

24,
 $31 \vee 33$
 $41 \vee 42 \vee 43 \vee 44,$

$\overline{24} \vee \overline{33}, \overline{24} \vee \boxed{\overline{42}}$

$\overline{31} \vee \overline{33},$
 $\overline{41} \vee \boxed{\overline{42}} \vee \overline{41} \vee \overline{43}, \overline{41} \vee \overline{44}, \boxed{\overline{42}} \vee \overline{43}, \boxed{\overline{42}} \vee \overline{44}, \overline{43} \vee \overline{44}$

$\overline{31} \vee \overline{41}$

$\boxed{\overline{42}}$

$\overline{33} \vee \overline{43}$

$\overline{31} \vee \boxed{\overline{42}}$
 $\overline{33} \vee \boxed{\overline{42}} \vee \overline{33} \vee \overline{44}$

$\overline{24} \vee \overline{44}$

Escolhas

12	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$	$\overline{32}$	$\overline{34}$
$\overline{42}$		

24,
 31 $\vee$ 33
 41 $\vee$ **42** $\vee$ 43 $\vee$ 44,

$\overline{24} \vee \overline{33},$

$\overline{31} \vee \overline{33},$
 $\overline{41} \vee \overline{43}, \overline{41} \vee \overline{44},$

$\overline{43} \vee \overline{44}$

$\overline{31} \vee \overline{41}$

$\overline{33} \vee \overline{43}$

$\overline{33} \vee \overline{44}$

$\overline{24} \vee \overline{44},$

Escolhas

12	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$	$\overline{32}$	$\overline{34}$
$\overline{42}$		

$24,$
 $31 \vee 33$
 $41 \vee 43 \vee 44,$

$\overline{24} \vee \overline{33},$

$\overline{31} \vee \overline{33},$
 $\overline{41} \vee \overline{43}, \overline{41} \vee \overline{44},$

$\overline{43} \vee \overline{44}$

$\overline{31} \vee \overline{41}$

$\overline{33} \vee \overline{43}$

$\overline{33} \vee \overline{44}$

$\overline{24} \vee \overline{44}$

Escolhas

12 11 13
14 21 22
23 32 34
42 24

24,
31 v 33
41 v 43 v 44,

24 v 33,

31 v 33,
41 v 43, 41 v 44,

43 v 44

31 v 41

33 v 43

33 v 44

24 v 44

Escolhas

12	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$	$\overline{32}$	$\overline{34}$
$\overline{42}$	24	

24

31 $\vee$

33

41 $\vee$ 43 $\vee$ 44, $\overline{24} \vee \overline{33},$ $\overline{31} \vee \overline{33},$ $\overline{41} \vee \overline{43}, \overline{41} \vee \overline{44},$ $\overline{43} \vee \overline{44}$ $\overline{31} \vee \overline{41}$ $\overline{33} \vee \overline{43}$ $\overline{33} \vee \overline{44}$ $\overline{24} \vee \overline{44},$

Escolhas

12	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$	$\overline{32}$	$\overline{34}$
$\overline{42}$	24	

31 $\vee$ 33
 41 $\vee$ 43 $\vee$ 44,

$\overline{24}$ $\vee$ $\overline{33}$,

$\overline{31} \vee \overline{33}$,
 $\overline{41} \vee \overline{43}, \overline{41} \vee \overline{44}$,

$\overline{43} \vee \overline{44}$

$\overline{31} \vee \overline{41}$

$\overline{33} \vee \overline{43}$

$\overline{33} \vee \overline{44}$

$\overline{24}$ $\vee$ $\overline{44}$,

Escolhas

12	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$	$\overline{32}$	$\overline{34}$
$\overline{42}$	24	

31 $\vee$ 33
 41 $\vee$ 43 $\vee$ 44,

$\overline{33}$,

$\overline{31} \vee \overline{33}$,
 $\overline{41} \vee \overline{43}, \overline{41} \vee \overline{44}$,

$\overline{43} \vee \overline{44}$

$\overline{31} \vee \overline{41}$

$\overline{33} \vee \overline{43}$

$\overline{33} \vee \overline{44}$

$\overline{44}$

Escolhas

12	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$	$\overline{32}$	$\overline{34}$
$\overline{42}$	$\overline{24}$	$\overline{33}$

31 $\vee$ 33
 41 $\vee$ 43 $\vee$ 44,

$\overline{33}$,

$\overline{31} \vee \overline{33}$,
 $\overline{41} \vee \overline{43}, \overline{41} \vee \overline{44}$,

$\overline{43} \vee \overline{44}$

$\overline{31} \vee \overline{41}$

$\overline{33} \vee \overline{43}$

$\overline{33} \vee \overline{44}$

$\overline{44}$

Escolhas

12	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$	$\overline{32}$	$\overline{34}$
$\overline{42}$	24	$\overline{33}$

31 $\vee$ 33
 41 $\vee$ 43 $\vee$ 44,

$\overline{33}$

$\overline{31} \vee \overline{33}$
 $\overline{41} \vee \overline{43}, \overline{41} \vee \overline{44},$

$\overline{43} \vee \overline{44}$

$\overline{31} \vee \overline{41}$

$\overline{33}$ $\vee$ 43

$\overline{33}$ $\vee$ 44

$\overline{44}$

$31 \vee$ **33** $41 \vee$ $43 \vee 44,$ $\overline{41} \vee \overline{43}, \overline{41} \vee \overline{44},$ $\overline{43} \vee \overline{44}$ $\overline{31} \vee \overline{41}$ $\overline{44}$ **Escolhas**

$\overline{12}$	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$	$\overline{32}$	$\overline{34}$
$\overline{42}$	$\overline{24}$	$\overline{33}$

Escolhas

12	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$	$\overline{32}$	$\overline{34}$
$\overline{42}$	24	$\overline{33}$

31**41 $\vee$ 43 $\vee$ 44,** **$\overline{41} \vee \overline{43}, \overline{41} \vee \overline{44},$** **$\overline{43} \vee \overline{44}$** **$\overline{31} \vee \overline{41}$** **$\overline{44}$**

Escolhas

12	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$	$\overline{32}$	$\overline{34}$
$\overline{42}$	24	$\overline{33}$
$\overline{44}$		

31**41 $\vee$ 43 $\vee$ 44,** **$\overline{41} \vee \overline{43}, \overline{41} \vee \overline{44},$** **$\overline{43} \vee \overline{44}$** **$\overline{31} \vee \overline{41}$** **$\overline{44}$**

Escolhas

12	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$	$\overline{32}$	$\overline{34}$
$\overline{42}$	24	$\overline{33}$
$\overline{44}$		

31

41 $\vee$ 43 $\vee$ 44, $\overline{41} \vee \overline{43}, \overline{41} \vee \overline{44},$ $\overline{43} \vee \overline{44}$ $\overline{31} \vee \overline{41}$ $\overline{44}$

31 $41 \vee$ $43 \vee 44,$ $\overline{41} \vee \overline{43},$ $\overline{31} \vee \overline{41}$ **Escolhas**

12	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$	$\overline{32}$	$\overline{34}$
$\overline{42}$	24	$\overline{33}$
$\overline{44}$		

31 $41 \vee$ 43 $\overline{41} \vee \overline{43},$ $\overline{31} \vee \overline{41}$ **Escolhas**

12	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$	$\overline{32}$	$\overline{34}$
$\overline{42}$	24	$\overline{33}$
$\overline{44}$		

31 $41 \vee$ 43 $\overline{41} \vee \overline{43},$ $\overline{31} \vee \overline{41}$ **Escolhas**

12	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$	$\overline{32}$	$\overline{34}$
$\overline{42}$	24	$\overline{33}$
$\overline{44}$	31	

31

 $41 \vee$

43

 $\overline{41} \vee \overline{43},$ $\overline{31} \vee \overline{41}$

Escolhas

12	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$	$\overline{32}$	$\overline{34}$
$\overline{42}$	24	$\overline{33}$
$\overline{44}$	31	

Escolhas

12	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$	$\overline{32}$	$\overline{34}$
$\overline{42}$	24	$\overline{33}$
$\overline{44}$	31	

 $41 \vee 43$
 $\overline{41} \vee \overline{43},$
 $\overline{31} \vee \overline{41}$

$41 \vee 43$

$\overline{41} \vee \overline{43},$

$\overline{41}$

Escolhas

12	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$	$\overline{32}$	$\overline{34}$
$\overline{42}$	24	$\overline{33}$
$\overline{44}$	31	

$41 \vee 43$

$\overline{41} \vee \overline{43},$

$\overline{41}$

Escolhas

12	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$	$\overline{32}$	$\overline{34}$
$\overline{42}$	24	$\overline{33}$
$\overline{44}$	31	$\overline{41}$

$41 \vee 43$

$\boxed{\overline{41}} \vee \overline{43},$

$\boxed{\overline{41}}$

Escolhas

$\overline{12}$	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$	$\overline{32}$	$\overline{34}$
$\overline{42}$	$\overline{24}$	$\overline{33}$
$\overline{44}$	$\overline{31}$	$\overline{41}$

41

v

43

Escolhas

12	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$	$\overline{32}$	$\overline{34}$
$\overline{42}$	$\overline{24}$	$\overline{33}$
$\overline{44}$	$\overline{31}$	$\overline{41}$

43

Escolhas

$\overline{12}$	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$	$\overline{32}$	$\overline{34}$
$\overline{42}$	$\overline{24}$	$\overline{33}$
$\overline{44}$	$\overline{31}$	$\overline{41}$

11	12	13	14
21	22	23	24
31	32	33	34
41	42	43	44

Escolhas

12	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$	$\overline{32}$	$\overline{34}$
$\overline{42}$	24	$\overline{33}$
$\overline{44}$	31	$\overline{41}$
	43	

11	12	13	14
21	22	23	24
31	32	33	34
41	42	43	44

Escolhas

12	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$	$\overline{32}$	$\overline{34}$
$\overline{42}$	24	$\overline{33}$
$\overline{44}$	31	$\overline{41}$
	43	

11	12	13	14
21	22	23	24
31	32	33	34
41	42	43	44

Escolhas

12	11	13
14	21	22
23	32	34
42	24	33
44	31	41
	43	

11	12	13	14
21	22	23	24
31	32	33	34
41	42	43	44

Escolhas

12	<u>11</u>	<u>13</u>
<u>14</u>	<u>21</u>	<u>22</u>
<u>23</u>	<u>32</u>	<u>34</u>
<u>42</u>	24	<u>33</u>
<u>44</u>	31	<u>41</u>
	43	

11	12	13	14
21	22	23	24
31	32	33	34
41	42	43	44

Escolhas

12	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$	$\overline{32}$	$\overline{34}$
$\overline{42}$	24	$\overline{33}$
$\overline{44}$	31	$\overline{41}$
	43	

11	12	13	14
21	22	23	24
31	32	33	34
41	42	43	44

Escolhas

12	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$	32	34
$\overline{42}$	24	$\overline{33}$
$\overline{44}$	31	$\overline{41}$
	43	

11	12	13	14
21	22	23	24
31	32	33	34
41	42	43	44

Escolhas

12	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$	$\overline{32}$	$\overline{34}$
$\overline{42}$	24	$\overline{33}$
$\overline{44}$	31	$\overline{41}$
	43	

11	12	13	14
21	22	23	24
31	32	33	34
41	42	43	44

Escolhas

12	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$	$\overline{32}$	$\overline{34}$
$\overline{42}$	24	$\overline{33}$
$\overline{44}$	31	$\overline{41}$
	43	

11	12	13	14
21	22	23	24
31	32	33	34
41	42	43	44

Escolhas

12	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$	$\overline{32}$	$\overline{34}$
$\overline{42}$	24	$\overline{33}$
$\overline{44}$	31	$\overline{41}$
	43	

11	12	13	14
21	22	23	24
31	32	33	34
41	42	43	44

Escolhas

12	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$	$\overline{32}$	$\overline{34}$
$\overline{42}$	24	$\overline{33}$
$\overline{44}$	31	$\overline{41}$
	43	

11	12	13	14
21	22	23	24
31	32	33	34
41	42	43	44

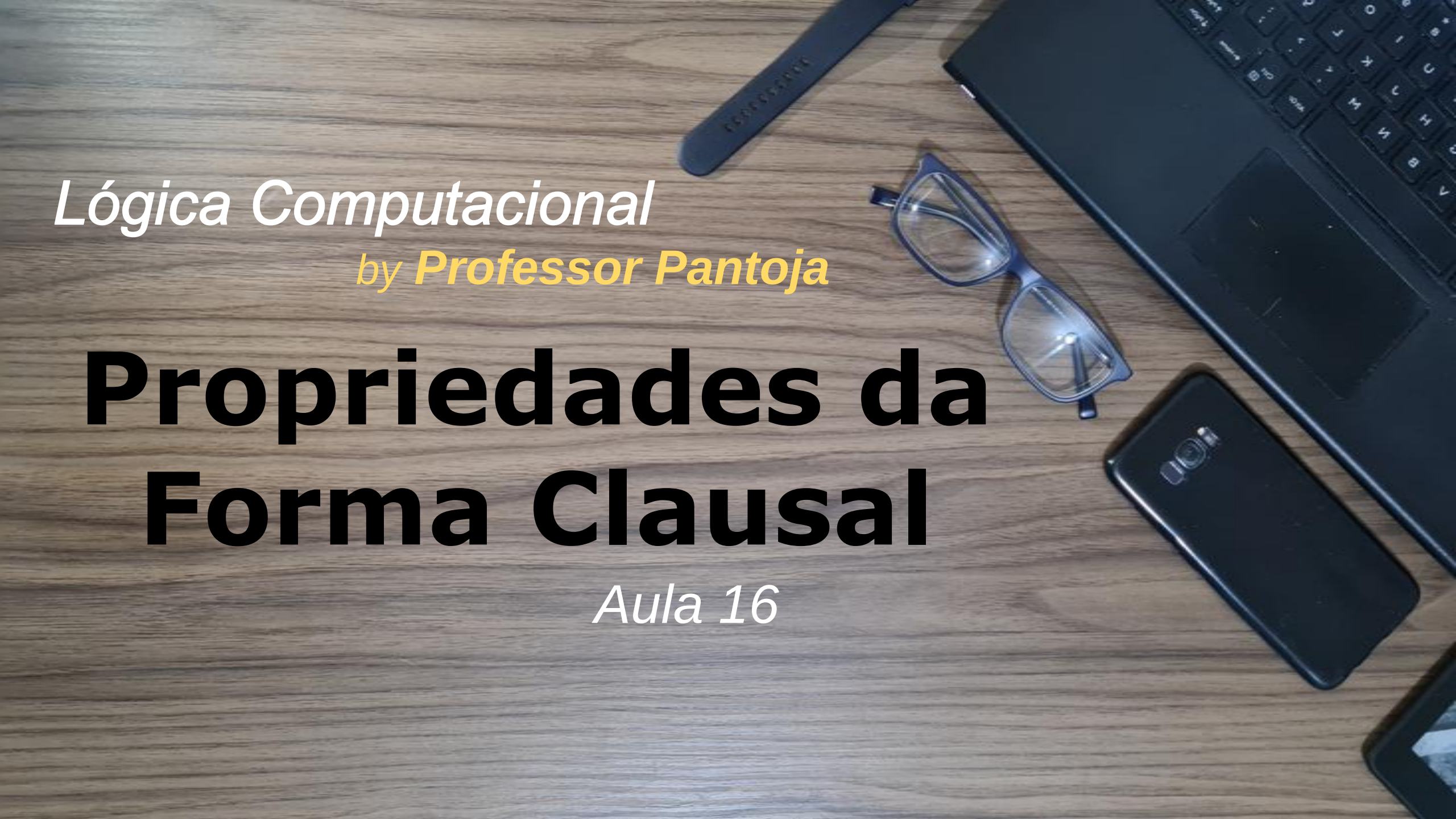
Escolhas

12	<u>11</u>	<u>13</u>
<u>14</u>	<u>21</u>	<u>22</u>
<u>23</u>	<u>32</u>	<u>34</u>
<u>42</u>	24	<u>33</u>
<u>44</u>	31	<u>41</u>
	43	

11	12	13	14
21	22	23	24
31	32	33	34
41	42	43	44

Escolhas

12	$\overline{11}$	$\overline{13}$
$\overline{14}$	$\overline{21}$	$\overline{22}$
$\overline{23}$	$\overline{32}$	$\overline{34}$
$\overline{42}$	24	$\overline{33}$
$\overline{44}$	31	$\overline{41}$
	43	

A top-down view of a wooden desk. On the right side, there is a black laptop with its keyboard visible. Below the laptop, a pair of blue-rimmed glasses lies on the desk. To the left of the glasses is a black pen. Further down and to the right is a black smartphone. The text is overlaid on the left side of the image.

Lógica Computacional
by Professor Pantoja

Propriedades da Forma Clausal

Aula 16

- ***Literais Puros***

- ***Literais Puros***
- ***Cláusulas Unitárias***

- ***Literais Puros***
- ***Cláusulas Unitárias***
- ***Se S e S' são conjuntos de cláusulas:***

- **Literais Puros**
- **Cláusulas Unitárias**
- **Se S e S' são conjuntos de cláusulas:**
 - **$S \approx S'$: S só é *Satisfazível* se e somente se S' for *Satisfazível*;**

- **Literais Puros**
- **Cláusulas Unitárias**
- **Se S e S' são conjuntos de cláusulas:**
 - **$S \approx S'$: S só é *Satisfazível* se e somente se S' for *Satisfazível*;**
 - **$S \equiv S'$: não é possível dizer que há equivalência, se $S \approx S'$.**

$$S = \{pqr, p\bar{q}, \bar{p}q\}$$

$$S' = \{p\bar{q}, \bar{p}q\}$$

$$p = F$$

$$q = F$$

$$r = F$$

$$S = \{pqr, p\bar{q}, \bar{p}q\}$$

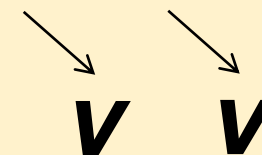
$$S' = \{p\bar{q}, \bar{p}q\}$$

↓
V

$$\begin{aligned} p &= F \\ q &= F \\ r &= F \end{aligned}$$

$$S = \{pqr, p\bar{q}, \bar{p}q\}$$

$$S' = \{p\bar{q}, \bar{p}q\}$$


V **V**

$$p = F$$

$$q = F$$

$$r = F$$

$$S = \{pq\bar{r}, p\bar{q}, \bar{p}q\}$$

↓
V

$$S' = \{p\bar{q}, \bar{p}q\}$$

↓ ↓
V **V**

$$p = F$$

$$q = F$$

$$r = F$$

$$S = \{pq\bar{r}, p\bar{q}, \bar{p}q\}$$

$\swarrow$ $\searrow$
V **V**

$$S' = \{p\bar{q}, \bar{p}q\}$$

$\swarrow$ $\searrow$
V **V**

$$p = F$$

$$q = F$$

$$r = F$$

$$S = \{pq\bar{r}, p\bar{q}, \bar{p}q\}$$

$\swarrow$ $\swarrow$ $\swarrow$
V **V** **V**

$$S' = \{p\bar{q}, \bar{p}q\}$$

$\swarrow$ $\swarrow$
V **V**

$$p = F$$

$$q = F$$

$$r = F$$

$$S = \{pq\bar{r}, p\bar{q}, \bar{p}q\}$$

$\swarrow \quad \swarrow \quad \swarrow$
V V V

$$S' = \{p\bar{q}, \bar{p}q\}$$

$\swarrow \quad \swarrow$
V V

$$\begin{aligned} p &= F \\ q &= F \\ r &= F \end{aligned}$$

$$S \approx S'$$

$$S = \{pq\bar{r}, p\bar{q}, \bar{p}q\}$$

$\swarrow \quad \swarrow \quad \swarrow$
V **V** **V**

$$S' = \{p\bar{q}, \bar{p}q\}$$

$\swarrow \quad \searrow$
V **F**

$$\begin{aligned} p &= F \\ q &= F \\ r &= F \end{aligned}$$

$$S \approx S'$$

$$S = \{pq\bar{r}, p\bar{q}, \bar{p}q\}$$

$\swarrow$ $\swarrow$ $\searrow$
V **V** **F**

$$S' = \{p\bar{q}, \bar{p}q\}$$

$\swarrow$ $\searrow$
V **F**

$$\begin{aligned} p &= F \\ q &= F \\ r &= F \end{aligned}$$

$$S \approx S'$$

$$S = \{pq\bar{r}, p\bar{q}, \bar{p}q\}$$

$$S' = \{p\bar{q}, \bar{p}q\}$$

$$\begin{aligned} p &= F \\ q &= F \\ r &= F \end{aligned}$$

$$\begin{aligned} p &= F \\ q &= F \\ r &= V \end{aligned}$$

$$S \not\equiv S'$$

$$S = \{pq\bar{r}, p\bar{q}, \bar{p}q\}$$

$$S' = \{p\bar{q}, \bar{p}q\}$$

↓
V

$$\begin{aligned} p &= F \\ q &= F \\ r &= F \end{aligned}$$

$$\begin{aligned} p &= F \\ q &= F \\ r &= V \end{aligned}$$

$$S \not\equiv S'$$

$$S = \{pq\bar{r}, p\bar{q}, \bar{p}q\}$$

$$S' = \{p\bar{q}, \bar{p}q\}$$

$\swarrow$ $\searrow$
V **V**

$$\begin{aligned} p &= F \\ q &= F \\ r &= F \end{aligned}$$

$$\begin{aligned} p &= F \\ q &= F \\ r &= V \end{aligned}$$

$$S \not\equiv S'$$

$$S = \{pq\bar{r}, p\bar{q}, \bar{p}q\}$$

↙
F

$$S' = \{p\bar{q}, \bar{p}q\}$$

↙ ↘
V **V**

$$\begin{aligned} p &= F \\ q &= F \\ r &= F \end{aligned}$$

$$\begin{aligned} p &= F \\ q &= F \\ r &= V \end{aligned}$$

$$S \not\equiv S'$$

$$S = \{pq\bar{r}, p\bar{q}, \bar{p}q\}$$

$\swarrow$ $\searrow$
F **V**

$$S' = \{p\bar{q}, \bar{p}q\}$$

$\swarrow$ $\searrow$
V **V**

$$\begin{aligned} p &= F \\ q &= F \\ r &= F \end{aligned}$$

$$\begin{aligned} p &= F \\ q &= F \\ r &= V \end{aligned}$$

$$S \neq S'$$

$$S = \{pq\bar{r}, p\bar{q}, \bar{p}q\}$$

$\swarrow$ $\swarrow$ $\swarrow$
F **V** **V**

$$S' = \{p\bar{q}, \bar{p}q\}$$

$\swarrow$ $\swarrow$
V **V**

$$\begin{aligned} p &= F \\ q &= F \\ r &= F \end{aligned}$$

$$\begin{aligned} p &= F \\ q &= F \\ r &= V \end{aligned}$$

$$S \not\equiv S'$$

- **Subsunção (subsumption)**

$$C1 \subseteq C2$$

- **Subsunção (subsumption)**

$$C1 \subseteq C2$$

$$S' - \{C2\}$$

- **Subsunção (subsumption)**

$$C1 \subseteq C2$$

$$S' - \{C2\}$$

$$S \approx S'$$

- **Subsunção (subsumption)**

$$C1 \subseteq C2$$

$$S' - \{C2\}$$

$$S \approx S'$$

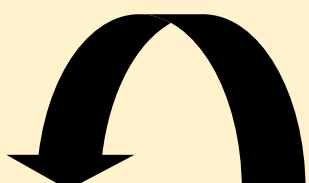
$$S = \{pr, \bar{p}q\bar{r}, q\bar{r}\}$$

- **Subsunção (subsumption)**

$$C1 \subseteq C2$$

$$S' - \{C2\}$$

$$S \approx S'$$

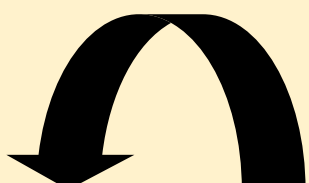

$$S = \{pr, \bar{p}q\bar{r}, q\bar{r}\}$$

- **Subsunção (subsumption)**

$$C1 \subseteq C2$$

$$S' - \{C2\}$$

$$S \approx S'$$

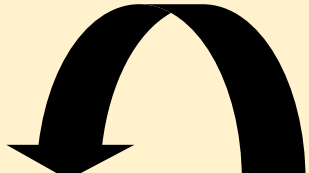

$$S = \{pr, \bar{p}q\bar{r}, \boxed{qr}\}$$

- **Subsunção (subsumption)**

$$C1 \subseteq C2$$

$$S' - \{C2\}$$

$$S \approx S'$$


$$S = \{pr, \bar{p}qr, qr\} \qquad S' = \{pr, qr\}$$

- **Subsunção (subsumption)**

$$C1 \subseteq C2$$

$$S' - \{C2\}$$

$$S \approx S'$$

$$S = \{pr, \quad q\bar{r}\}$$

- **Subsunção (subsumption)**

$$C1 \subseteq C2$$

$$S' - \{C2\}$$

$$S \approx S'$$

$$S = \{pr, \quad q\bar{r}\}$$

$$S' = \{pr, q\bar{r}\}$$

- ***Renomeação (Renaming)***

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- **Seja S um conjunto de cláusulas e U um conjunto de proposições atômicas $R_U(S)$**

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- **A renomeação de S por U é obtida de S substituindo cada literal em U pelo seu contraditório.**

- **Renomeação (Renaming)**

- **Seja S um conjunto de cláusulas e U um conjunto de proposições atômicas $R_U(S)$**

- **A renomeação de S por U é obtida de S substituindo cada literal em U pelo seu contraditório.**

- **$S \approx R_U(S)$**

$$S = \{pqr, \bar{p}q, \bar{q}\bar{r}, r\}$$

$$S = \{pqr, \bar{p}q, \bar{q}\bar{r}, r\}$$

$$\begin{aligned} p &= F \\ q &= F \\ r &= T \end{aligned}$$

$$S = \{pqr, \bar{p}q, \bar{q}\bar{r}, r\}$$

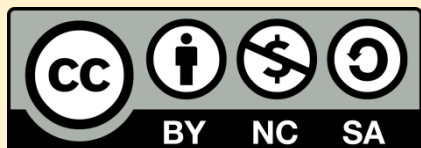
$$\begin{aligned} p &= F \\ q &= F \\ r &= T \end{aligned}$$

$$R_{[p,q]}(S) = \{\bar{p}\bar{q}r, p\bar{q}, q\bar{r}, r\}$$

$$\begin{aligned} p &= T \\ q &= T \\ r &= T \end{aligned}$$



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